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Impact of density and interconnectedness of influential players on social welfare

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ABSTRACT

We show that in defection prone environments influential players must be rare and weakly interconnected to optimally promote cooperation in the prisoner's dilemma game. Conversely, low temptations to defect warrant a high level of social welfare even if influential players are common, yet still demand the latter be weakly interconnected for cooperation to thrive.

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1. Introduction

Cooperation between competitive players or firms is a perplexing puzzle faced by scientists across fields of research as different as economics and anthropology [10]. The core of the problem lies in the fact that cooperation implies working for the common good of a society on the expense of individual prosperity, which is in contradiction with the premise of Darwinian evolution and results in a social dilemma. The most commonly adopted theoretical framework for addressing the issue is the evolutionary game theory, and the prisoner's dilemma game in particular [1], which succinctly captures the unadorned scenario of social downfall due to the egoistic and heedless urge of individuals to outperform competitors by temporarily harvesting the highest possible profit. In particular, the game originally consists of two players or firms who have to decide simultaneously whether they want to cooperate or defect as they venture into a joint enterprise. The dilemma is given by the fact that although mutual cooperation yields the highest collective payoff, a defector will do better if the opponent cooperates. In the long run this fact inflicts mutual defection that ultimately results in an irreversible economic decline and social poverty, as reviewed by Crawford [4].

The breakthrough discovery promoting cooperation arguably came in the form of spatial games [11,16], where the participating players no longer abide to the principles of well-mixed dynamics, but instead, cooperators are able to survive via clustering that protects the inner individuals against the exploitation by invading defectors. This approach has recently been extended further by considering structured populations where interactions amongst individuals are defined by complex networks [17], and also, by the introduction of asymmetry in influence [18] as well as payoff uncertainties [12,13], thus revealing additional mechanisms promoting mutually beneficial alliances not just in economy (e.g., [9] or [3], but also in ecology and biology in general.

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Presently, we study the impact of two crucial factors on the evolution of cooperation within the prisoner's dilemma game; namely the density and interconnectedness of influential players. The basic premise of our study is that influence is seldom equally distributed amongst members of human societies. Throughout history, many phrases and titles have been invented to distinguish influential individuals from those having little impact, and more often than not "being influential" is reserved for few selected individuals only. To take this fact into account we introduce a fraction ρ. of influential players on the grid, which are much more likely to enforce their strategy on the opponent than the rest. In addition, amongst all the possible connections existing between the influential players, we introduce a fraction κ that actually exist, and study how different values of ρ and κ affect the evolution of cooperation by different temptations to defect b. First, we show that the introduction of influential players and shortcut connections amongst them in general facilitates cooperation, which suggests that the role of leaders in societies might have emerged spontaneously in order to warrant optimal conditions for cooperative alliances and with it related social welfare. Yet more precisely, we reveal that in defection prone environments governed by large b influential players must be rare ($\rho << 1$) and weakly connected ($\kappa << 1$) to sustain cooperation, whereas on the other hand, by small b a high level of social welfare is warranted even if ρ is substantial, but still κ must remain small for cooperation to flourish best. Importantly, all results are obtained by full anonymity of players and incognito actions (for related literature see e.g., [5] or [15], as well as without the aid of additional strategies (see e.g., [14]. Finally we discuss that, although large differences in status may evoke dissatisfaction and rebellion amongst the deprived, our study suggests that such social states might have appeared spontaneously from an initially nonpreferential setup and are in fact optimal for the evolution of cooperation.

2. Mathematical model

For the purpose of this study, we used the prisoner's dilemma game having temptation to defect $T = b$, reward $R = 1$, and both punishment P as well as the suckers payoff S equaling 0, whereby $1 < b \leqslant 2$ ensures a proper payoff ranking. The game is staged on a regular $L \times L$ square grid with nearest neighbor interactions and periodic boundary conditions, whereon initially each agent on site x is designated either as a cooperator (strategy $s_x = C$) or defector ($s_x = D$) with equal probability. Forward iteration is performed in accordance with the Monte Carlo simulation procedure comprising the following elementary steps. First, a randomly selected agent x acquires its payoff P_x by playing the game with all its neighbors (here neighbors are assumed to be all players that are directly connected to agent x). Next, one randomly chosen neighbor, denoted by y, also acquires its payoff P_y by playing the game with all its neighbors. Last, agent x tries to enforce its strategy s_x on player y in accordance with the probability

$$W(s_y \to s_x) = w_x \frac{1}{1 + \exp[(P_y - P_x)/K]}, \tag{1}$$

where $K = 0.5$ denotes the level of uncertainty by strategy adoptions due to imperfect information (see e.g., [15] and w_x determines the influence of agent x. The parameter w_x is assigned to each agent at the beginning of the game and remains fixed during the evolutionary process. In particular, amongst all $N = L^2$ players, and irrespective of their initial strategies, a fraction ρ is chosen randomly and designated as having $w_x = 1$, whereas the remaining $N(1 - \rho)$ are assigned the influence $w_x = 0.01$. In accordance with Eq. (1), players characterized by $w_x = 1$ are 100 times more likely to enforce their strategy on the opponent than those having $w_x = 0.01$, and are thus termed as the influential ones. Furthermore, we introduce a probability κ that determines the fraction of all possible $(\rho N - 1)\rho N/2$ connections linking the influential individuals that actually exist. If $\kappa = 1$ all possible pairs of influential players are connected with one another as depicted in Fig. 1(a), whereas if $0 < \kappa < 1$ the corresponding fraction of some of these links is randomly removed as shown in Fig. 1(b). Finally, the network characterizing the interconnectedness of influential players is merged with the underlying nearest-neighbor grid to form the final interaction structure for the considered evolutionary prisoner's dilemma game.

Results obtained via Monte Carlo simulations presented below were obtained on populations comprising 400×400 individuals, whereby the stationary fraction of cooperators f_C was determined within 10^6 full Monte Carlo steps after sufficiently long transients were discarded. In what follows, we will focus on different values of density ρ and interconnectedness κ of influential players to determine their impact on cooperation and social welfare by different values of the temptation to defect b.

3. Results and discussion

We start by presenting results obtained with the classical version of the prisoner's dilemma game when all players have the same probability of enforcing their strategy on the opponents and there are no additional shortcut links defining the interaction structure amongst them. In accordance with the above description of the game, this setup is obtained by setting $\kappa = 0$ and $\rho = 0$. Results presented in Fig. 1 (circles) evidence that under such conditions cooperators vanish already by $b = 1.065$. In sharp contrast, by setting $\kappa = 0.0012$ and $\rho = 0.15$, cooperators persist virtually throughout the whole range of $b \in (1, 2]$ applicable for the prisoner's dilemma game, as depicted by squares in Fig. 1. Noteworthy, by introducing this fairly small fraction of weakly linked influential players, the cooperators are able to dominate the game completely by $b \approx 1$ and outnumber defectors up to $b = 1.15$. Clearly, the presently studied additions to the game in form of positive κ

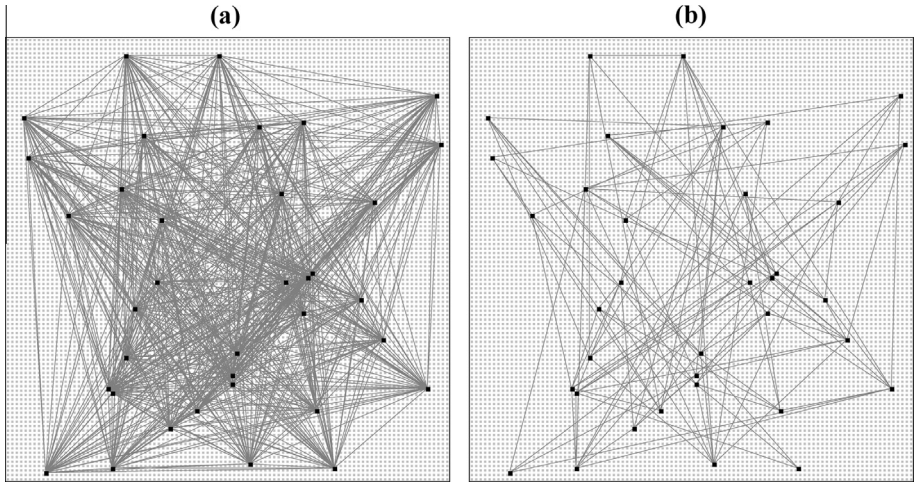

Fig. 1. Interconnectedness of influential players obtained by different κ. In panel (a) $\kappa = 1$ whereas in (b) $\kappa = 0.2$. Gray lines connect only those having $w_x = 1$ (marked with larger black squares), whereas the rest (marked with smaller light-gray squares) is connected only via nearest-neighbor links. For clarity only 100×100 spatial grids with $\rho = 0.004$ randomly distributed influential players are depicted.

and ρ may have a notable impact on the evolution of cooperation and markedly facilitate altruistic behavior and social welfare.

To quantify the ability of interconnected influential players to facilitate and maintain cooperation in the studied spatial prisoner's dilemma game more precisely, we calculate the fraction of cooperators f_C over a broad range of κ and ρ by three different values of $b \in (1, 2]$. The left panel of Fig. 3 features results obtained by $b = 1.2$. Remarkably, there exists a double resonance-like dependence of f_C on κ and ρ, implying that both parameters can be fine-tuned to yield an optimal environment for cooperation. In particular, by setting $\kappa = 0.0003$ and $\rho = 0.2$ almost a complete dominance of cooperators can be warranted despite $b = 1.2$. This is rather astonishing in view of the fact that by $\kappa = 0$ and $\rho = 0$ cooperators never dominate completely and go extinct already by $b = 1.065$ (see Fig. 2). Features observed by $b = 1.2$ prevail also by higher b, although expectedly the maximally attainable f_C decreases as the environment becomes increasingly prone to defection. In particular, by $b = 1.5$ (note that this is the middle of the interval spanned by b) cooperators may still become widespread provided $\kappa = 0.0004$ and $\rho = 0.11$, as depicted in the middle panel of Fig. 3. Finally, even by the largest possible temptation to defect given by $b = 2$ cooperators may still survive and reach respectable $f_C = 0.07$ if $\kappa = 0.001$ and $\rho = 0.04$, as shown in the right panel of Fig. 3. This more precise analysis reveals that the facilitative effect on cooperation (with respect to the classical version of the game) depicted in Fig. 2 is by no means maximal and may be further refined and enhanced with respect to the governing temptation to defect.

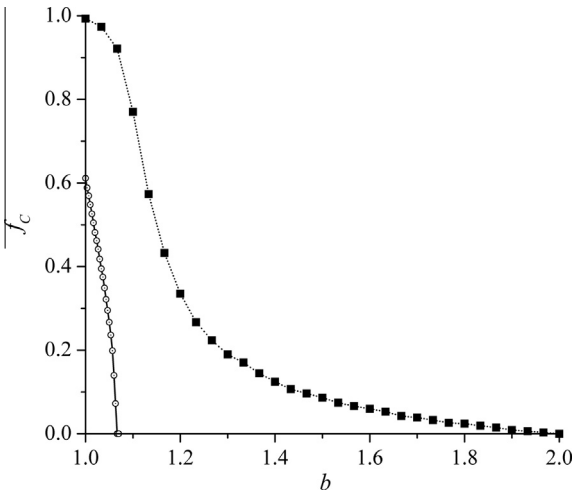

Fig. 2. Fraction of cooperators f_C in dependence the temptation to defect b by $\kappa = 0$, $\rho = 0$ (circles) and $\kappa = 0.0012$, $\rho = 0.15$ (squares). Lines are just to guide the eye. The presented results were obtained on a square lattice, but are expected to be robust also on the honeycomb and triangular lattice.

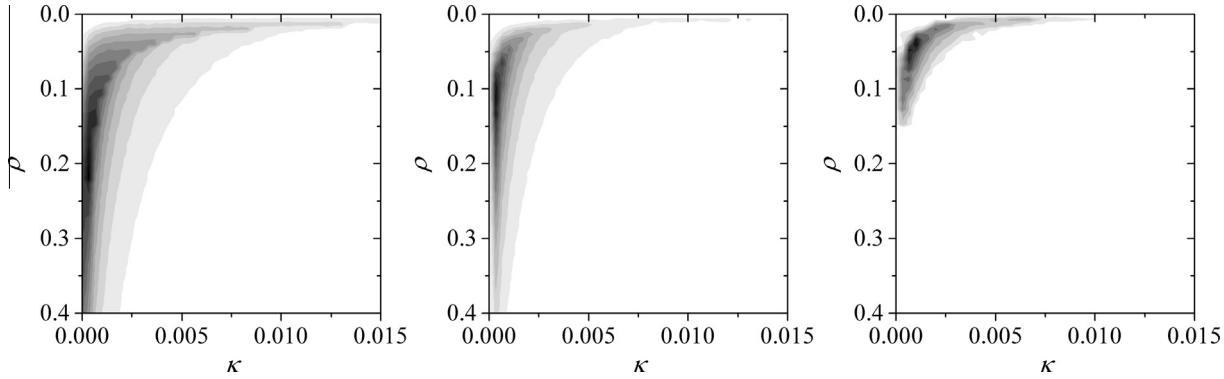

Fig. 3. Color-coded fraction of cooperators f_C in dependence on κ and ρ for $b = 1.2$ (left panel), $b = 1.5$ (middle panel) and $b = 2.0$ (right panel). The color profile is linear, white depicting 0.0 and black 0.9 (left panel), 0.4 (middle panel) and 0.07 (right panel) values of f_C.

In view of results presented in Fig. 3, we conclude that strongly defection prone environments characterized by $b >> 1$ require minute fractions of influential players ($\rho = 0.04$) in order to assure the best survival chances for cooperators. On the other hand, if b is close to 1, as much as a fifth of all participants may constitute just the optimal density of influential players needed to propel cooperators to dominance. Irrespective of b, however, influential players must always be weakly interconnected for cooperation to thrive best. This is constituted by the fact that no more than 0.1% of all possible links between influential players must exist if the socially optimal cooperative state is to be reached, whereby by small b even this small fraction may be preclusive, and indeed as low as 0.03% of all possible links constitute the best interaction structure for the studied prisoner's dilemma game.

As emphasized already in the Introduction, across human societies influence is seldom equally distributed amongst their members, and indeed, large segregations in status and wealth abound. Understandably, such states may evoke grief and despair by the less fortunate, leading to gatherings and riots that serve as opportunities for expressing discomfort and anger. Despite of this potentially disturbing fact, our study reveals that societies incorporating an innate segregation of influence are better adapted for sustaining unselfish cooperative alliances amongst their members, and should therefore be more successful in warranting a comfortable level of social welfare for all involved. It thus seems reasonable to argue that, although large differences in status may evoke dissatisfaction and rebellion amongst the deprived, such social states likely appeared spontaneously from an initially nonpreferential setup and are in fact optimal for the evolution of cooperation. Presented results may also provide insights as to why developed and successful economies, characterized by low temptations to defect, incorporate more influential players than those soaked with corruption and bribery.

Promising avenues of research further along this line concern the evolution of cooperation on interdependent and multilayer networks [20,6,21,22,7,19,23]. Indeed, several mechanisms have already been discovered by means of which the interdependence between different networks or network layers may help to resolve social dilemmas. Specific examples include interdependent network reciprocity [21,22], non-trivial organization of cooperators across the interdependent layers [6], and information transmission [19]. We hope that our study will help contribute to the continued vibrancy of this field of research (for recent reviews see [2] or [8].

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