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Emergence of cooperation in spatial social dilemmas with expulsion

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ABSTRACT

In social dilemmas cooperators pay costs for other individuals to receive benefits, whereas defectors are the benefactors but do not provide anything in return. What is more, except in the snowdrift game, defective strategies are evolutionarily stable against invasion by cooperative strategies under strong selection. It is thus of interest to determine under which conditions the spontaneous emergence of cooperation is possible in a population consisting entirely of defectors. To that effect, we here want to concentrate on exploring how expulsion influences the emergent dynamics of cooperation in the context of the spatial social dilemmas. Interestingly, expulsion can indeed support the emergence of cooperators for all classes of social dilemma games. By analyzing the spatial expansion and invasion processes of cooperative individuals, we further reveal that merely a 1×2 (or 2×1) cluster of expulsive cooperators, introduced for example by means of mutations, is required for invading and completely dominating a spatial population of expulsive defectors provided that the cost of cooperation is no larger than a constant value.

1. Introduction

In the real world, we are able to observe cooperative behaviors almost across all scales of life forms [1,2]. However, the evolution of cooperation seems to contradict Darwinism: Cooperators, who pay costs for other individuals to receive benefits, are doomed to be crowded out from the population by defectors, who have no costs and do not deal out benefits. As a result, natural selection would not favor cooperative behaviors unless additional mechanisms are considered. Until now, five fundamental rules that support cooperation have been discovered by researchers [3]. Noteworthy, not only evolutionary biologists but also a large number of scientists from many other research fields have been interested in such an evolutionary puzzle of cooperation. Especially, physicists have a significant interest in the evolution of cooperation within structured populations where the relationships among individuals are organized by network structures [4,5]. In the past few decades, the macro and micro dynamical behaviors of evolutionary games of social dilemma on networks have already been exten-

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Table 1
Four classes of two-person games on $T-S$ parameter space: parameter ranges and dilemmas arisen.

Two-person games	Parameter ranges	Dilemmas arisen
Harmony game	$R=1>T,S>P=0$	No dilemma arises.
Snowdrift game	$T>R=1>S>P=0$	Players prefer unilateral defection to mutual cooperation.
Stag-hunt game	$R=1>T>P=0>S$	Players prefer mutual defection to unilateral cooperation.
Prisoner's dilemma	$T>R=1>P=0>S$	Both dilemmas arisen in snowdrift and stag-hunt game are incorporated.

sively studied from the viewpoint of complex systems [6–30]. On the other hand, we notice that most of these studies have mainly focused on what complex behaviors these dynamical systems exhibit when the evolutionary competition between cooperators and defectors starts from an initial state, in which non-negligible numbers of cooperative and defective agents are distributed on a network. Nevertheless, it is still largely unexplored how such collaborators can emerge in a network-structured population composed merely of defectors in the first place under the condition of strong selection. As a matter of fact, the emergence of collective behaviors such as cooperation, what parts of a system do together that they would not do alone, has been an important research issue of complex systems until now.

In this paper, we investigate how expulsive behavior influences the emergent dynamics of cooperation by comparing the emergent dynamics of the spatial games of traditional social dilemmas with that of the spatial games of social dilemmas with expulsion [31,32]. In the latter game, we assign both cooperators and defectors with expulsive ability (i.e., expulsive cooperators vs. expulsive defectors in the social dilemma game with expulsion) so as to study the unbiased effects of expulsion on the emergence of cooperative behavior. Furthermore, a sufficiently small rate of mutation is also introduced into both evolutionary games so as to constantly disturb both spatial systems, which thus makes us be able to ask how expulsive behavior can help cooperators to emerge in a spatially structured population. Here we show that expulsion can indeed support the emergence of cooperation from the spatial population consisting merely of defective individuals in the social dilemma games.

2. Model

Let us consider a diluted square lattice of size $L \times L$ with the periodic boundary condition as well as the von Neumann neighborhoods, where N $(\leq L^2)$ randomly chosen nodes are occupied by N individuals and the rest $L^2 - N$ ones are set to be vacant. This means that the population density is given by a time-invariant parameter $\rho = N/L^2$. In our model, all individuals initially adopt the defective strategy, i.e., expulsive defection in the social dilemma game with expulsion or defection in the traditional social dilemma game (i.e., $\rho_{EC} = \rho - \rho_{ED} = 0$ in the social dilemma game with expulsion or $\rho_C = \rho - \rho_D = 0$ in the social dilemma game at the start of evolution). For both games, the interactions among individuals are modelled based on the two-person social dilemma game, the payoff matrix of which is given by

$$M = \begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} R & S \\ T & P \end{bmatrix} \end{array}, \tag{1}$$

which allows us to study the impacts of expulsion in the emergence of cooperation for different classes of social dilemmas. Here mutual defection results in the punishment P whereas the reward R is resulted from mutual cooperation. Following common practice, we make the punishment $P = 0$ and the reward $R = 1$, which normalizes the advantage of the overall payoff of mutual cooperation over that of mutual defection to 2 in all kinds of social dilemma games [7]. Otherwise, if one player defects and the other one cooperates, the defector and the cooperator obtain the temptation T $(\in [0, 2])$ and the sucker's payoff S $(\in [-1, 1])$, respectively. In accordance with the relative ordering of $P = 0$, $R = 1$, T and S, we can naturally classify the two-dimensional $T-S$ parameter region into four different classes of games: the prisoner's dilemma, the stag-hunt game, the snowdrift game and the harmony game, wherein only the former three games fall into the category of the social dilemma games whereas the harmony game does not because cooperation is the dominant strategy in this case (see Table 1).

Every time step of the evolutionary process includes the following two or three elementary procedures in our model. First, all individuals accumulate their respective payoff P by playing the two-person game [for its payoff matrix, see Eq. (1)] with, if any, other ones in their neighboring sites synchronously. Next, for the social dilemma game with expulsion where individuals, be it cooperators or defectors, are endowed with the expulsion ability, each expulsive defector is chosen one time to be banished to an empty site on the square lattice in a random sequence manner, provided that at least one expulsive cooperator or one expulsive defector is in its neighboring site. Lastly, each individual updates its strategy by either mutation or imitation in a synchronous manner. In order to explore the emergence of cooperation from a population consisting merely of defectors initially, we introduce a negligible rate of mutation into the model: With an extremely low probability $\mu = 0.0001$, the strategy of an individual locating at site, e.g., i, would stochastically mutate into any strategy available. With the remaining probability (i.e., $1 - \mu$), this individual wants to update its strategy by imitation. Depending on the state of the neighboring site, e.g., j, which is randomly chosen by the focal individual, this individual can or cannot complete the

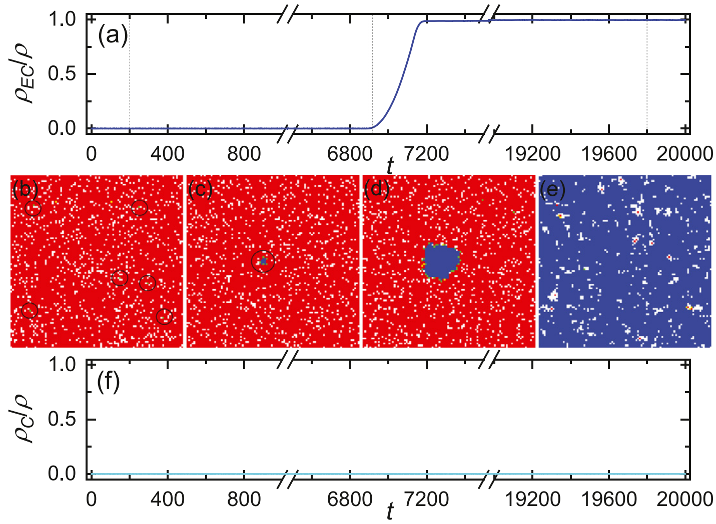

Fig. 1. Representative emergent dynamics of (a-e) the spatial prisoner's dilemma game with expulsion as well as of (f) the spatial prisoner's dilemma game for $T = 1.5$ and $S = -0.3$. The Monte Carlo simulations are performed on a 200×200 square lattice for population density $\rho = 0.9$, where all individuals adopt defective strategies at the start of evolution (i.e., $t = 0$). For the spatial patterns of strategies [i.e., Fig. 1(b–e)], they show a 100×100 portion of a larger 200×200 spatial network, where red denotes defective strategies, blue cooperative strategies, white vacant sites, green cooperative strategies which are turned from defective strategies in the last time step, and yellow defective strategies which are transitioned from cooperative strategies in the last time step. (a) In the prisoner's dilemma game with expulsion, the spontaneous emergence of prevalent cooperation can be achieved despite the presence of severe social dilemma. The subsequent four snapshots were recorded at (b) $t = 200$, (c) $t = 6894$, (d) $t = 6918$ and (e) $t = 19800$ [see also the dashed time lines in Fig. 1(a)]. (b) Initially, a very few expulsive cooperators are randomly dispersed by mutations of strategy with a considerably small probability $\mu/2 = 0.00005$ [see the green dots in small black circles]. (c) After a long time, there suddenly occurs a supercritical cluster of expulsive cooperators, which originates by random coincidence of favorable strategy mutations as well as the expulsive behaviors of individuals towards defectors, as indicated by a black circle. Note that the level of cooperation nearly does not vary at this moment. (d) Then, the small cluster of expulsive cooperators gradually expands, and (e) expulsive cooperators can almost completely dominate the whole spatial system eventually. The expansion, despite the destructive effects of mutations, requires the evolutionary mechanism of expulsion to form a growing cluster of expulsive cooperators. (f) In sharp contrast, in the prisoner's dilemma without expulsion, the severe social dilemma disables cooperation to spontaneously emerge by forming compact clusters of cooperators in a population consisting merely of defectors initially. Only a handful of cooperators randomly scatter in the spatial network during the evolutionary process, which is due to random mutations of strategy. In this case, the typical spatial patterns of strategies are similar to those presented in Fig. 1(b). Because of the same reason, the resulting level of cooperation reaches values close to $\mu/2 = 0.00005$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

imitation event: (i) If the chosen site j is empty, the focal individual at site i just keeps its current strategy; (ii) If the chosen site j is occupied, the focal individual at site i updates its strategy by imitating the individual at site j with a probability:

$$T\left(P_j - P_i\right) = \frac{1}{1 + \exp\left[-\left(P_j - P_i\right)/K\right]}, \tag{2}$$

where the amplitude of noise K is set to 0.1 in this study. This implies that worse-performing individuals are also possible to be imitated, although individuals are more willing to adopt the strategies of individuals who perform better.

3. Results

In social dilemma games, cooperators confront the severest challenge in the prisoner's dilemma game in comparison with they do in the stag-hunt game as well as in the snowdrift game (see Table 1). Therefore, we first study the emergent dynamics of cooperation in both the prisoner's dilemma game and the prisoner's dilemma game with expulsion. Monte Carlo simulations of our model show that, in the spatial prisoner's dilemma game but without expulsion, the emergence of cooperation cannot be realized, and the negligible fraction of cooperators ($\approx \mu/2 = 0.00005$) is mainly due to mutations [see Fig. 1(f)]. In general, mutations of strategy in spatial games of social dilemmas favor the short-term payoff-maximizing behavior, i.e., defection, over cooperation, which is a sort of long-term payoff-maximizing behavior [33,34]. Therefore, under the adverse condition of both severe social dilemma and a small rate of mutation, a tiny amount of cooperators mutating from defectors are not able to self-organize into spatial clusters, and thus cannot sustain or even expand into the spatial territories of defectors. On the contrary, we observe that a substantial fraction ($\approx 1 - \mu/2 = 0.99995$) of expulsive cooperators do spontaneously emerge from the population composed merely of expulsive defectors [see Fig. 1(a)]. Initially, the evolutionary process of the prisoner's dilemma game with expulsion is similar to that of the prisoner's dilemma game: Defection prevails for a considerably long time. In each time step, only a tiny fraction ($\approx \mu/2 = 0.00005$) of all expulsive defectors turn into expulsive cooperators as a result of random mutations. These few expulsive cooperators are usually dispersed randomly on the spatial network, which is thus extremely unfavorable for the spreading of cooperation [see Fig. 1(b)]. However,

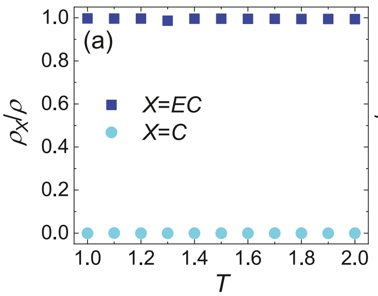

Fig. 2. Emergent fraction of cooperative strategies as a function of (a) T for $S = -0.3$ and of (b) S for $T = 1.5$ when $\rho = N/L^2 = 0.9$ for the spatial prisoner's dilemma game with expulsion as well as for the spatial prisoner's dilemma game. In both games, the initial state is set to consist merely of defective strategies. Filled blue squares represent expulsive cooperators in the prisoner's dilemma with expulsion, and filled cyan circles denote cooperators in the prisoner's dilemma. Each data point is obtained by averaging over 100 independent runs with different random realizations for a square lattice of size $L \times L = 200 \times 200$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

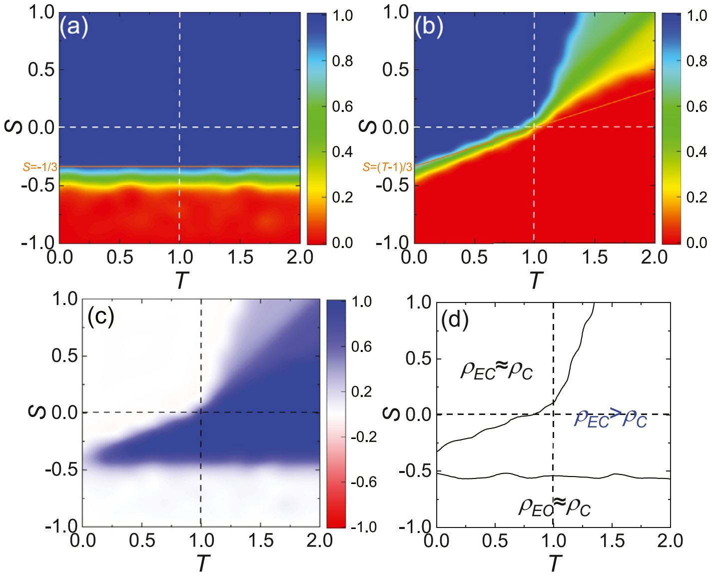

Fig. 3. Emergence of cooperation in the spatial games of social dilemmas. Dependence of the emergent fraction of cooperative strategies [(a): Emergent fraction of expulsive cooperation ρ_{EC}/ρ in the spatial games of social dilemmas with expulsion; (b): Emergent fraction of cooperation ρ_C/ρ in the spatial games of social dilemmas] on the parameters T and S. The full $T-S$ parameter area on Figs. 3(a-d) is separated into four regions by the horizontal and vertical lines (compare with Tab. 1): the harmony game (upper-left quadrant), the snowdrift game (upper-right quadrant), the stag-hunt game (bottom-left quadrant) and the prisoner's dilemma (bottom-right quadrant). The orange solid line in Fig. 3(a) represents the equation $S = -R/3 = -1/3$, while that in Fig. 3(b) denotes the equation $S = (T - R + 3P)/3 = (T - 1)/3$. (c) Difference between the emergent fraction of expulsive cooperators ρ_{EC}/ρ and that of cooperators ρ_C/ρ. (d) Full phase diagram on the $T-S$ parameter plane, depicting two distinct parameter areas satisfying $\rho_{EC}/\rho > \rho_C/\rho$ and $\rho_{EC}/\rho \approx \rho_C/\rho$, respectively. The simulations are performed on a square lattice with $L \times L = 200 \times 200$ sites and $N = 0.9 \times L^2$ individuals, corresponding to a population density $\rho = N/L^2 = 0.9$. At time $t = 0$, all individuals adopt defective strategies. The results are averaged over 100 independent runs with different random realizations.

due to the expulsive behaviors of individuals towards defectors as well as mutations, a small, supercritical cluster of expulsive cooperators has suddenly arisen by coincidence [see Fig. 1(c)]. Thereafter, this cluster of expulsive cooperators expands quickly [see Fig. 1(d)], and soon expulsive cooperators dominate the spatial population [see Fig. 1(e)].

Fig. 2 shows that the expulsive behavior of individuals does determine the emergent behaviors of cooperation at least for the cases of $T = 1.5$ and $S = -0.3$: Cooperative strategies are able to emerge in the spatial prisoner's dilemma game with expulsion, but cannot in the spatial prisoner's dilemma game. Furthermore, one can observe that, while the sucker's payoff S has a significant effect, the temptation T seems to have no obvious impacts in the emergence of cooperation in the spatial prisoner's dilemma game with expulsion. Indeed, we confirm that such an interesting phenomenon is universal for the emergence of cooperation in the spatial games of social dilemma with expulsion [see Fig. 3(a)]. The simulation results show that expulsive cooperators are able to emerge and then expand to the whole population as long as $S > -1/3$ in the spatial games of social dilemma with expulsion. In comparison, for the spatial games of social dilemma, cooperators may emerge and then dominate the whole population only if $S > (T - 1)/3$, the parameter area of which is strictly smaller than that in the former case [see Figs. 3(a) and (b)]. In addition, we discover that the beneficial effect of expulsion on the emergence of

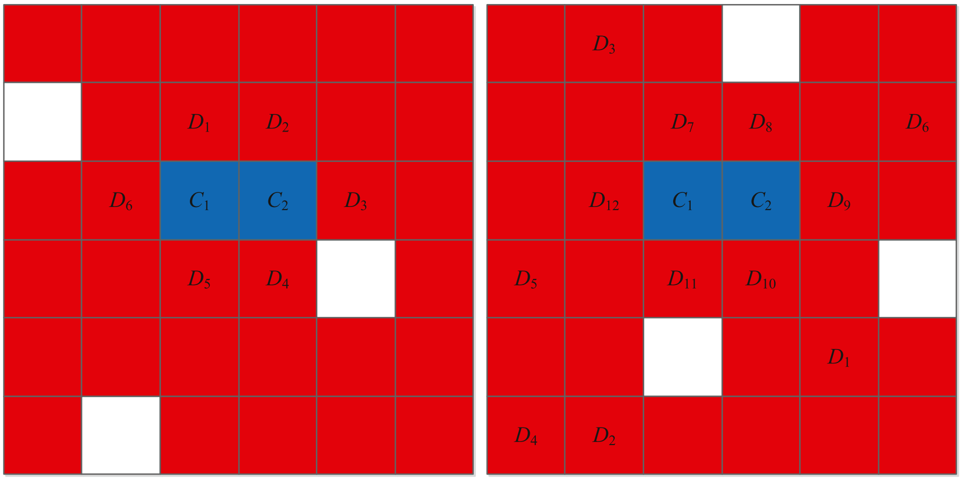

Fig. 4. The critical 1×2 (or 2×1) cluster of cooperative individuals in the emergent dynamics of cooperation from a spatial population consisting merely of defective individuals. For both plots, red denotes defective strategies, blue cooperative strategies and white vacant sites. This example shows a square lattice of size $L \times L = 6 \times 6$, where the population density $\rho = 33/36$. For both the spatial games of social dilemmas with expulsion and the spatial games of social dilemmas, individuals interact with their neighbors, if any, on the spatial networks, and then collect their own payoffs (see the left panel). If individuals have expulsive ability (i.e., in the spatial games of social dilemmas with expulsion), there is still an expulsion stage: Each expulsive defector is chosen one time to be banished to an empty site on the square lattice in a random sequence manner, provided that at least one expulsive cooperator or one expulsive defector is in its neighboring site. Due to the expulsion events, the previous six defective neighbors (i.e., D_1, D_2, D_3, D_4, D_5 and D_6) of the 1×2 (or 2×1; see the left panel) cooperative cluster are replaced by six other defective individuals (i.e., D_7, D_8, D_9, D_{10}, D_{11} and D_{12}; see the right panel) expelled from other sites with extremely high probability. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

cooperation is robust against the variations of both T and S: The difference on the full $T-S$ parameter region between the emergent fraction of expulsive cooperators ρ_{EC}/ρ and that of cooperators ρ_C/ρ is generally no smaller than zero [compare Fig. 3(a) with Fig. 3(b); see Fig. 3(c)]. Notably, the phase diagram reveals that a substantial part of the $T-S$ parameter area, where social dilemma is present, satisfies $\rho_{EC}/\rho > \rho_C/\rho$ [see Fig. 3(d)].

Finally, it still requires to reveal the evolutionary micro-mechanism that enables expulsion to support the emergence of cooperation. Due to the very small rate of mutations (i.e, $\mu = 0.0001$), there is a non-negligible but extremely small possibility with which two cooperative individuals can form a 1×2 (or 2×1) cluster. In other words, it usually requires a long time for cooperators to constitute such a cluster in an attempt to realize the emergence of cooperation [see, e.g., Fig. 1(a)]. In this case, it is nearly impossible to form clusters of cooperative individuals which are larger than 1×2 (or 2×1). Therefore, it is reasonable to consider the 1×2 (or 2×1) cluster of cooperative individuals as the critical cluster. Here we aim to analyze the condition for such a critical cluster to expand into the spatial network for both the spatial games of social dilemmas with expulsion and the spatial games of social dilemmas (see Fig. 4). Without loss of generality, only the $K \to 0$ and $\mu \to 0$ limits are considered in the following simplified analysis. In this case, for the 1×2 (or 2×1) cluster of cooperative individuals, this cluster is able to expand into the spatial population provided that the payoff of each cooperative individual in this cluster is larger than that of each defective neighbors. For the spatial games of social dilemmas with expulsion, the payoffs of expulsive cooperators and expulsive defectors along the boundary are respectively given by

$$P_C = 3S + R = 3S + 1 \quad \text{and} \; P_D = 0, \tag{3}$$

where $C \in \{C_1, C_2\}$ and $D \in \{D_7, D_8, D_9, D_{10}, D_{11}, D_{12}\}$ (see the right panel of Fig. 4). Thus for the critical cluster of cooperative individuals to expand in the spatial games of social dilemmas with expulsion, it requires

$$S > -1/3. \tag{4}$$

Interestingly, Eq. (4) confirms our observations in Figs. 2(a) and 3(a) that the emergent fraction of cooperation in the spatial games of social dilemmas with expulsion does not depend on the temptation T. For the spatial games of social dilemmas, the payoffs of cooperators and defectors along the boundary of the critical cluster can be respectively expressed as

$$P_C = 3S + R = 3S + 1 \quad \text{and} \; P_D = T + 3P = T, \tag{5}$$

where $C \in \{C_1, C_2\}$ and $D \in \{D_1, D_2, D_3, D_4, D_5, D_6\}$ (see the left panel of Fig. 4). Thus for the critical cluster of cooperative individuals to expand in the spatial games of social dilemmas, it needs

$$S > (T-1)/3, \tag{6}$$

where $T \in [0, 2]$. Comparing Eq. (4) with Eq. (6), one can find that expulsion does support the emergence of cooperation in the spatial games of social dilemmas. Noteworthy, the analytical approximations [i.e., Eqs. (4) and (6)] are just sufficient for the critical cluster to successfully expand into the spatial population of defective individuals but are not sufficient for the critical cluster to successfully invade the whole spatial population (for the definition of successful invasion, see Ref. [35] as an example) at the first glance. However, as the critical size of the cluster is 1×2 (or 2×1), which is the smallest possible size of cluster for two dimensional graphs, such analytical approximations [i.e., Eqs. (4) and (6)] can also be used to estimate the critical condition for the 1×2 (or 2×1) cluster to successfully invade or even dominate the whole spatial population as long as the social dilemma does not fall into the category of the snowdrift game, which is confirmed by the simulation results [see Fig. 3(a) and (b)]. Note that the best response for a player is to adopt a strategy that is different from its opponent in the snowdrift game, which hampers the formation of clusters of cooperative individuals.

4. Discussions

Overall, the impacts of expulsion in the emergence of cooperation have been explored by comparing the emergent dynamics of the spatial games of social dilemmas with expulsion with that of the traditional spatial games of social dilemmas. In the spatial games of social dilemmas with expulsion, individuals, be it cooperators or defectors, are endowed with expulsive ability towards defective individuals. We have shown that expulsion can support the emergence of cooperation in all kinds of social dilemma situations [compare Fig. 3(a) with Fig. 3(b); see Fig. 3(c)]. By analyzing the expansion and invasion ability of the critical 1×2 (or 2×1) cooperative cluster, we have provided further insights as to how the introduction of expulsion fundamentally alters the emergent dynamics of cooperative behavior.

As a matter of fact, a few mechanisms have already been found for the emergence of cooperation under the condition of strong selection by several earlier works. For example, Helbing and Yu have reported that success-driven migration can induce the sudden outbreak of predominant cooperation in a noisy world dominated by selfishness and defection [36]. By success-driven migration, they mean that individuals are able to acquire information concerning the quality of all adjacent empty sites within a certain range by fictitious play, and then to migrate to the one promising the highest payoffs. Put differently, individuals in their model require to obtain information of sites that are beyond their nearest neighborhoods before game interactions. However, individuals in our model merely need to acquire information of the nearest neighboring sites after game interactions, and thus do not require a high cognitive ability. In addition, Roca and Helbing have studied a generalized win-stay-lose-shift (strategy switching and/or site relocating by migration) learning rule in the networked public goods game. It has been found that moderate greediness favors the emergence of cooperation [37]. In this model, individuals do not need information of other sites, but require to remember their own previous experiences. Nevertheless, individuals in our model do not need such a memory ability. Our results have thus shown that neither extensive information nor memory of self-history is necessary for the emergence of cooperation in the spatial games of social dilemmas with expulsion.

Acknowledgments

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