Applied Mathematics and Computation 418 (2022) 126835

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Coupling group selection and network reciprocity in social dilemmas through multilayer networks

Zhenyu Shi[a,b,c,d], Wei Wei[a,b,c,d,*], Matjaž Perc[e,f,g,h], Baifeng Li[a,b,c,d], Zhiming Zheng[a,b,c,d]

[a] School of Mathematical Sciences, Beihang University, Beijing, China
[b] Key Laboratory of Mathematics Informatics Behavioral Semantics, Ministry of Education, China
[c] Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang
[d] Peng Cheng Laboratory, Shenzhen, Guangdong, China
[e] Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, Maribor 2000, Slovenia
[f] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 404332, Taiwan
[g] Alma Mater Europaea, Slovenska ulica 17, Maribor 2000, Slovenia
[h] Complexity Science Hub Vienna, Josefstädterstraße 39, Vienna 1080, Austria

ARTICLE INFO

Article history:
Received 30 October 2021
Revised 19 November 2021
Accepted 24 November 2021

Keywords:
Cooperation
Spatial evolutionary game
Network reciprocity
Group selection
Social dilemmas

ABSTRACT

Group selection and network reciprocity are two well-known mechanisms for cooperation. And while they have separately received ample attention for their ability to promote cooperation in social dilemmas, their joint effects in this regard are much less explored. Here we propose a multilayer network model that takes into account that not only individuals are connected by means of networks, but that also groups can be connected in much the same way. The model thus couples together network reciprocity and group selection, and it allows us to study their joint effect on the evolution of cooperation. We use the prisoner's dilemma game as the paradigmatic social dilemma example, showing that the fine-tuning of evolution frequency and the imitation rate in the group network play key roles in determining the survival thresholds of cooperators. We also explore the importance of deterministic and stochastic updating rules, showing that the former provides more options to further promote cooperation. We discuss the importance of our findings for cooperation in structured interactions among groups and in higher-order networks.

1. Introduction

Cooperative behavior could be widely observed in nature and human society, which means individuals sacrifice part of their own interests in exchange for greater interests of their group [1]. Although cooperative behavior is beneficial for the population, it violates Darwin's theory that only the fittest survive [2]. Therefore, how cooperative behavior evolves has become a question of widespread concern to biologists, economists, physicists, and mathematicians [3]. Evolutionary game theory, based on traditional game theory, bounded rationality hypothesis, and biological evolution theory, have become a powerful mathematical framework for studying the emergence, maintenance, and stability of cooperation under social dilemmas [4,5]. Among all kinds of social dilemmas, the prisoner's dilemma game is the most representative because it

* Corresponding author at: School of Mathematical Sciences, Beihang University, Beijing, China.
E-mail address: weiw@buaa.edu.cn (W. Wei).

https://doi.org/10.1016/j.amc.2021.126835

describes the contradiction between individual rationality and collective rationality best [6]. Prisoner's dilemma game(PDG) is a 2-player game in which both players could choose to be either a cooperator or a defector. Both of them will get a reward R or a punishment P if they choose to cooperate or defect together, respectively. But if one chooses to cooperate and another chooses to defect, the former will get a sucker's payoff S and the latter will get a temptation to defection T. If the above parameters satisfy $T > R > P > S$ and $2R > T + S$, it is a prisoner's dilemma game. In addition to the prisoner's dilemma game, there are other types of social dilemmas, such as snow-drift game satisfying $T > R > S > P$ and stag-hunt game satisfying $R > T > P > S$.

In the prisoner's dilemma game, defection is the Nash equilibrium, in other words, it is the rational choice for both players. But since the pioneering work of Nowak and May [7], it is found that spatial structure could promote cooperation by allowing cooperators to form clusters. Since then, researchers have explore the impact of different kinds of network structures and they are proven to promote cooperation, such as lattice [7,8], small world networks [9,10], scale-free networks [11,12], two-dimensional space [13,14], multilayer networks [15], and temporal networks [16]. Besides, many social mechanisms have been studied to have better explanations for the evolution of cooperation, such as punishment and reward [17–19], heterogeneous investments [20], exit rights [21], alliance [22], game organizers [23,24], stochastic interactions [25], payoff perturbations [26,27], and so on. Besides of using payoff-driven imitation as strategy updating rule, several other rules are also proposed, such as reinforcement learning [28], conformity-driven imitation [29,30], and aspiration [31–34].

In 2006, Nowak summarized five key mechanisms for promoting cooperation, which are kin selection, direct and indirect reciprocity, network reciprocity, and group selection [35]. Considering all of the five mechanisms, the most popular of them among researchers is network reciprocity, which means cooperators could expand by forming clusters that enable reaping the benefits of cooperation despite of exploitation by defectors at cluster boundaries [36]. Group selection is that an altruistic group would gain evolution, which also received much attention [37]. However, it seems that few researchers combine network reciprocity with group selection and propose a coupling model. In practical scenarios, not only the interactions between individuals have a network structure. In fact, interactions and selections between groups might also have a network structure. For instance, on the international stage, the overall mutual interactions between neighboring countries will be more frequent. Inspired by this, we propose a multilayer network model coupling network reciprocity and group selection. In the multilayer network, individuals play with their neighbors on individuals' networks and group selections work in groups' networks.

The remainder of our paper is organized as follows. First, we introduce our multilayer network model in detail. Then we show the Monte Carlo simulation results and make discussion on them, which are divided into two subsections. At last, we make a summary about our main conclusion, the meaning of our work, and the future research directions.

2. Model

We show our model by some schematic diagrams in Fig. 1. In this model, a population of $N = L^2$ individuals which are located in an $L * L$ square lattice is considered. This $L * L$ square lattice is called *Individual Network*. The strategy of individual i in step t is denoted as $\vec{s_i}(t)$, where $\vec{s_i}(t) = (1, 0)$ if i is a cooperator and $\vec{s_i}(t) = (0, 1)$ if i is a defector. In addition, The Individual Network is divided into $n * n$ groups and each group is a $d * d$ square lattice with periodic boundary conditions, satisfying $d * n = L$. So that we can define an $n * n$ square lattice with periodic boundary called *Group Network*. In a Group Network, each node represents a group with $d * d$ individuals. The Monte Carlo steps are described as follows:

(a)**Individual Network's evolution.** In each step t, each individual plays PDGs with all its four neighbors in the same group to get a payoff $P_i(t) = \sum_{j \in \Omega_i} P_{ij}(t)$, where Ω_i is the set of all neighbors of individual i. $P_{ij}(t) = \vec{s_i}(t) * P * \vec{s_j}(t)^T$ represents the payoff when i plays a PDG with j where P is the payoff matrix of PDG. Following [7], P is set as

$$\begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}, \tag{1}$$

where b representing the temptation value is the only parameters of this kind of PDG. Then each individual i chooses a neighbor j from Ω_i at random and imitates j's strategy with the probability:

$$W_{ij}(t) = \frac{1}{1 + \exp[(P_i(t) - P_j(t))/K]}, \tag{2}$$

which is called Fermi updating rule and K stands for the amplitude of noise[8]. In this paper, it is set to 0.1 in order to characterize appropriate noise [38,39].

(b)**Group Network's evolution.** In each step Gt, where G is introduced to measure the evolution frequency of the groups relative to the individuals, each group I accumulates its total payoff $P_I(t) = \sum_{i \in \Omega_I} P_i(t)$, where Ω_I denotes the set of individuals in group I and it contains $d * d$ individuals. Then each group I chooses one of its neighbors in Group Network randomly with equal probability. Based on their payoffs, we introduce two kinds of rules to decide whether I will imitate J's state.

(i)Deterministic rule. If $P_I \geq P_J$, I will not imitate J's state. If $P_I < P_J$, I will imitate J's state.

(ii)Stochastic rule. I will imitate J's state with the probability:

$$p_{IJ}(t) = \frac{P_J(t) - P_I(t)}{8d^2} + \frac{1}{2}, \tag{3}$$

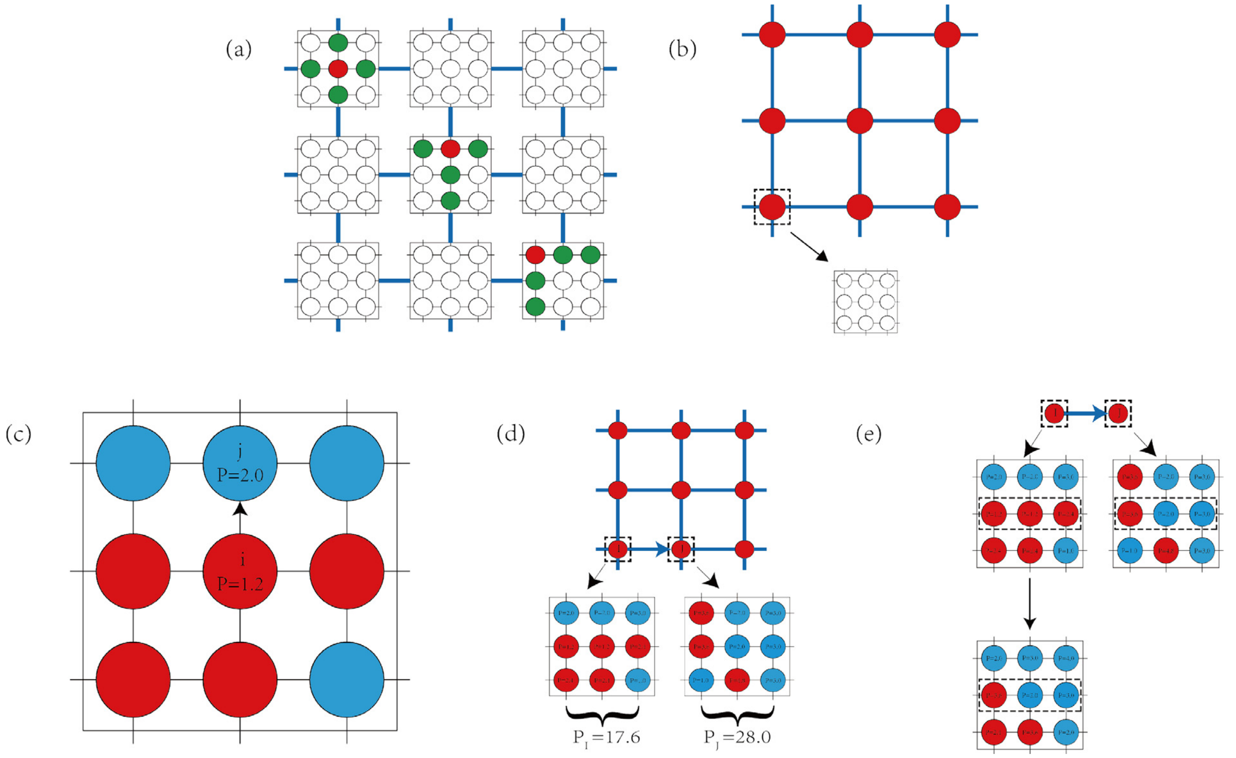

Fig. 1. We show our model by some schematic diagrams, with $L = 9$, $d = 3$, $L = 3$ and $K = 0.1$. (a)Individual Network. Individuals are located in a $9 * 9$ square lattice. They are divided into $3 * 3$ groups and each group is a $3 * 3$ square lattice with periodic boundary conditions. The red nodes denote central individuals and the green nodes denote their direct neighbors in Individual Network. (b)Group Network. Each red node represents a group and their links are denoted by blue thick lines. Each group consists of a $3 * 3$ square lattice with periodic boundary conditions, and all groups also form a $3 * 3$ square lattice with periodic boundary conditions. (c)Individual Network's evolution. Individual i chooses one of its neighbors j randomly, then they accumulate their payoffs. $P_i(t) = 1.2$ and $P_j(t) = 2.0$, so i will imitate j's strategy with the probability $W_{ij}(t) = \frac{1}{1+\exp[(1.2-2.0)/0.1]} = 0.9997$. Each individual does like this synchronously. (d) and (e)Group Network's evolution. Group I choose one of its neighbors J randomly, then they sum up their total payoffs. $P_I(t) = 17.6$ and $P_J(t) = 28.0$. Under Deterministic rule), $P_I(t) < P_J(t)$, so I will imitate J's state. Under Stochastic rule, I will imitate J's state with the probability $p_{IJ}(t) = \frac{28.0-17.6}{8*3^2} + \frac{1}{2} = 0.6444$. If I chooses to imitate J's state, we set $\alpha = \frac{1}{3}$, so 3 individuals in I are chosen randomly and they will imitate individuals' strategies in J with the same index. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

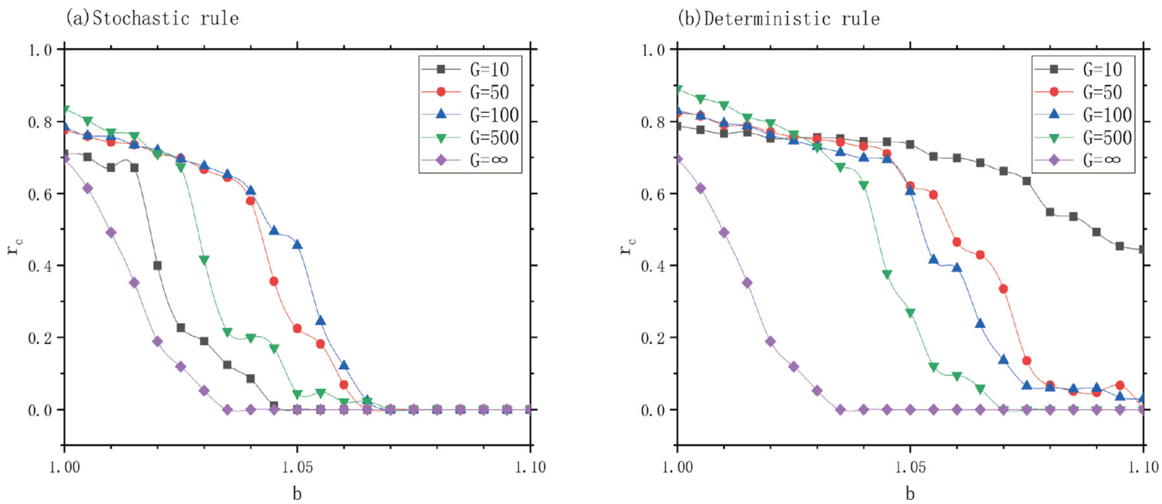

Fig. 2. The fraction of cooperators at the stable state as a function of b at several values of G under the Stochastic rule(a) and Deterministic rule(b). In initial, individuals locates in Individual Network randomly with $r_{\mathcal{C}}(0) = r_{\mathcal{D}}(0) = 0.5$, with $\alpha = 1$, $L = 100$, $n = 10$ and $d = 10$ being set. The black, red, blue, and green line represent $G = 10$, 50, 100, and 500 respectively. The purple line is the check experiment, that is, no Group Network's evolution. It could be found that for all values of G, Deterministic rule has better effects on promoting cooperation than Stochastic rule, and the lower G is, the higher $r_{\mathcal{C}}$ and b_c are. Under Stochastic rule, moderate values of G have the most significant effect on promoting cooperation, but under Deterministic rule, low values of G have the most significant effect on promoting cooperation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

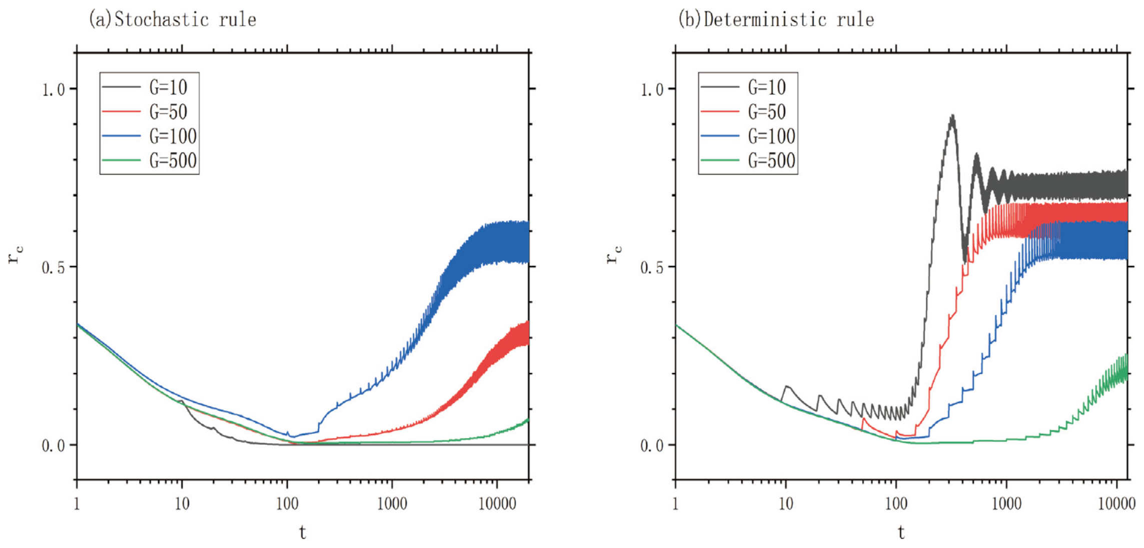

Fig. 3. Time series of the fraction of cooperators for different values of G under the Stochastic rule(a) and Deterministic rule(b). The black, red, blue, and green line represent $G = 10$, 50, 100, and 500 respectively. It is found that the lower G is, the earlier network becomes stable. Besides, the network becomes stable faster under Deterministic rule than under Stochastic rule with the same G except for $G = 10$, where cooperation vanishes soon under Stochastic rule. When $t = 10$, the Group Network's evolution step accelerates the extinction of cooperation under the Stochastic rule compared with $G = 500$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where $p_{IJ}(t) \in [0, 1]$: when all individuals in I are cooperators and all individuals in J are defectors, $P_I(t) = 4d^2$ and $P_J(t) = 0$, so $p_{IJ}(t) = 0$, vice versa.

If I chooses to imitate J's state, then a proportion of individuals in I will be randomly chosen. This proportion is denoted as imitating rate α, and the set of chosen individuals in I is denoted as I_α. We denote I_{xy} as the individual in the xth row and yth column of group I, and that I imitates J's state means for each individual in group I,

$$s_{\vec{I_{xy}}}(t+1) = s_{\vec{J_{xy}}}(t), \qquad I_{xy} \in I_\alpha. \tag{4}$$

Above Monte Carlo steps will carry out until the Individual Network is at a steady state. The fractions of cooperators and defectors in Individual Network at step t are denoted as $r_{\mathcal{C}}(t)$ and $r_{\mathcal{D}}(t)$, and the fractions of them when at steady state are denoted as $r_{\mathcal{C}}$ and $r_{\mathcal{D}}$ for convenience. For each simulation, the total steps t are 30,000 to ensure that Individual Network will be at a steady state, and $r_{\mathcal{C}}$ for one simulation is calculated by averaging $r_{\mathcal{C}}(t)$ in the last 2000 steps. In order to ensure the accuracy of the results, for each set of parameters, 100 independent simulations are carried out.

3. Results

3.1. Effects of group network's evolution frequency G

Compared to previous work, the most important highlight in our work is Group Network's evolution. So we start from the effects of Group Network's evolution frequency G on the fraction of cooperation $r_{\mathcal{C}}$ when steady. In initial, individuals locate in Individual Network randomly with $r_{\mathcal{C}}(0) = r_{\mathcal{D}}(0) = 0.5$, with $\alpha = 1$, $L = 100$, $n = 10$ and $d = 10$ being set. Figure 2(a) and (b) show $r_{\mathcal{C}}$ as functions of b for different values of G under two kinds of group selection rules. It is shown that the group selection models we proposed have a significant effect on promoting cooperation. We denote the lowest value of b where cooperation vanishes as b_c. For Stochastic rule (shown in Fig. 2(a)), compared to no group selection, where $b_c = 1.03$ as [8] showed, all values of G could promote cooperation. For small values of G ($G = 10$), cooperation could be enhanced in the range of $b \le 1.04$ and $b_c = 1.045$. With G increasing ($G = 50$ and $G = 100$), cooperation is further enhanced and b_c increases to 1.07. When G is high ($G = 500$), cooperation could still be enhanced and b_c is also 1.07, but the level of b_c is lower than that when $G = 50$ or 100. So it is found that moderate G has the most significant effect on promoting cooperation under Stochastic rule. But the phenomenon is quite different for Deterministic rule (shown in Fig. 2(b)). It could be found that for all values of G, Deterministic rule has better effects on promoting cooperation than Stochastic rule, and the lower G is, the higher $r_{\mathcal{C}}$ and b_c are. Especially when $G = 10$, cooperators will occupy the majority of Individual Network if $b \le 1.09$.

Now we further discuss why Deterministic rule has better effects on promoting cooperation than Stochastic rule and why low values of G have the most significant effect on promoting cooperation under Deterministic rule while moderate G has the most significant effect on promoting cooperation under Stochastic rule. For a mixed well group I, assuming that its fraction of cooperators is c, then the average payoff of the cooperators is $c * 1 + (1-c) * 0 = c$, and the average payoff of the defectors is $c * b + (1-c) * 0 = bc$, so the average payoff of I is $c * c + (1-c) * bc = (1-b)c^2 + bc$. Since $b \in [1, 2]$, P increases

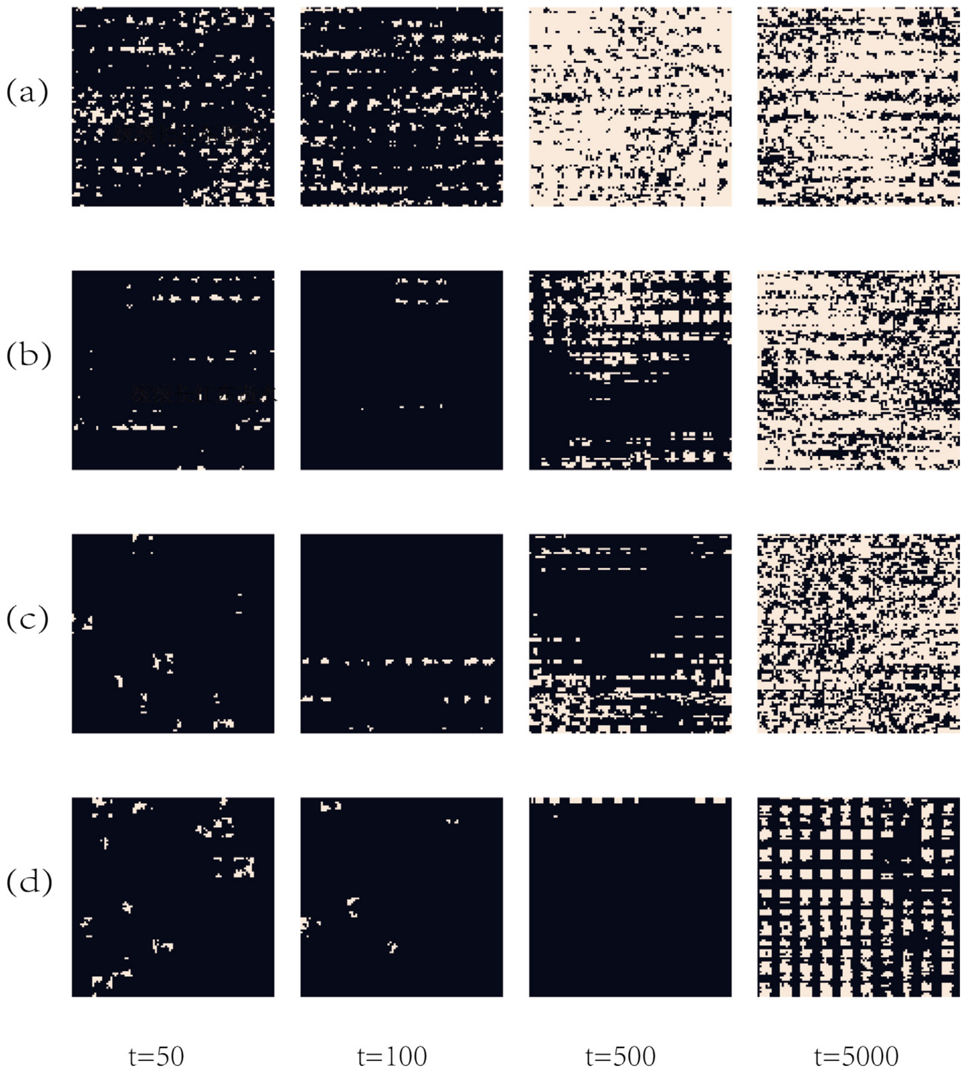

Fig. 4. The evolution of snapshot with time-evolution. Cooperators and defectors in the network are denoted as white and black respectively. From left to right $t = 50$, 100, 500, 5000. From top to bottom $G = 10$, 50, 100, 500. For all values of G, the fraction of cooperators will decrease at the start. At the same time, survival cooperators form clusters and with the help of Group Network's evolution step, they will expand to their neighboring groups.

monotonically with c. For a $d * d$ square lattice with periodic boundary conditions, each edge represents a PDG, so the total numbers of PDGs are $2d^2$. Edges could be divided into three types: C-C links, D-D links, and C-D links, the fractions of which in step t are denoted as $r_{CC}(t)$, $r_{DD}(t)$, and $r_{CD}(t)$ respectively, satisfying $r_{CC}(t) + r_{DD}(t) + r_{CD}(t) = 1$ and $\frac{2r_{CC}(t)+r_{CD}(t)}{2r_{DD}(t)+r_{CD}(t)} = \frac{r_C(t)}{r_D(t)}$. For each C-C link, D-D link, and C-D link, the sum of the two individuals are 2, 0, and $1 + b$ respectively. So the average payoff of a group in step t is $2r_{CC}(t) + br_{CD}(t)$. Besides, we define the clustering index $\beta(t)$ as $\frac{r_{CC}(t)+r_{DD}(t)}{r_{CD}(t)}$. Combining the above equations, it could be calculated that $r_{CC}(t) = \frac{\beta(t)}{2(1+\beta(t))} + r_C(t) - \frac{1}{2}$, $r_{DD}(t) = \frac{\beta(t)}{2(1+\beta(t))} - r_C(t) + \frac{1}{2}$, $r_{CD}(t) = \frac{1}{1+\beta(t)}$, and the average payoff of a group in step t is $P(t) = 2r_C(t) + \frac{b-1}{1+\beta(t)}$. So it is calculated that $P(t)$ increases monotonically with r_C and decreases monotonically with $\beta(t)$. It means that if a group has more cooperators, it will have a more total payoff and competitive advantages during Group Network's evolution. For Deterministic rule, a group is sure to imitate its neighbor if its neighbor has more total payoff, or in other words, has more cooperators. This provides more favorable conditions for the expansion of partners than Stochastic rule. The lower G is, the more frequently cooperators will expand among groups. But for Stochastic rule, the phenomenon is different. A group may also imitate its neighbor even if its neighbor has a lower payoff. So if G is too low, frequent imitation with noise and defectors' fast expansion will inhibit cooperation. However for moderate G, during about 50 or 100 steps, the payoff gap among groups becomes large and the imitation will be more accurate, causing the best cooperation level.

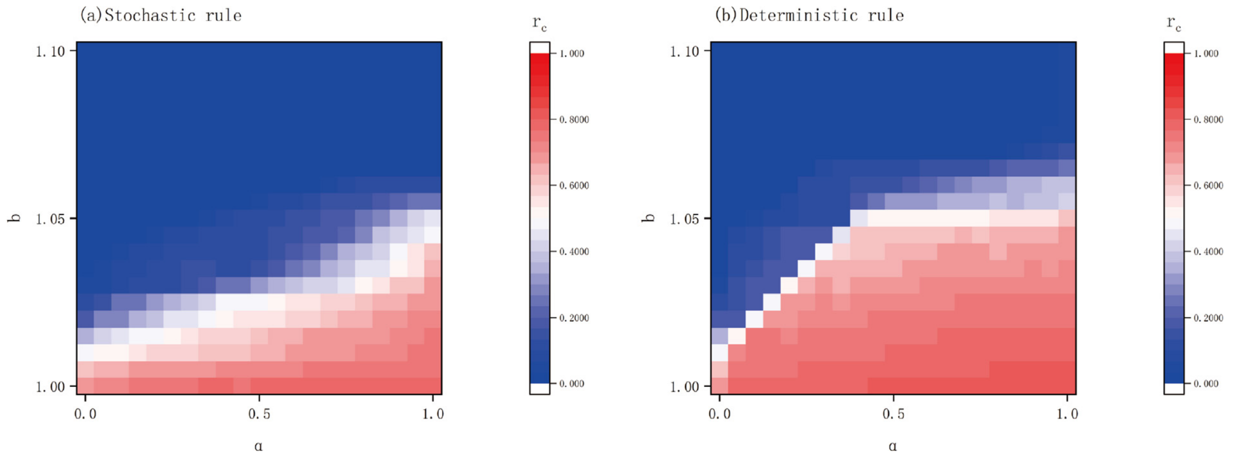

Fig. 5. The fraction of cooperators at the stable state as a function of α and b under the Stochastic rule(a) and Deterministic rule(b). In initial, individuals locate in Individual Network randomly with $r_{\mathcal{C}}(0) = r_{\mathcal{D}}(0) = 0.5$, with $L = 100$, $n = 10$ and $d = 10$ being set. Compared with $\alpha = 0$, which is equivalent to the absence of the Group Network's evolution, all positive values of α could promote cooperation when b is lower than the threshold for the extinction of cooperation. Besides, $r_{\mathcal{C}}$ is higher under Deterministic rule than under Stochastic rule for all pairs of α and b. Under Stochastic rule, the increase of $r_{\mathcal{C}}$ with the increase of α is more obvious. As a comparison, under Deterministic rule, a threshold α_b exists, roughly satisfied $\alpha_b = 10(b - 1.01)$ when $b < 1.05$.

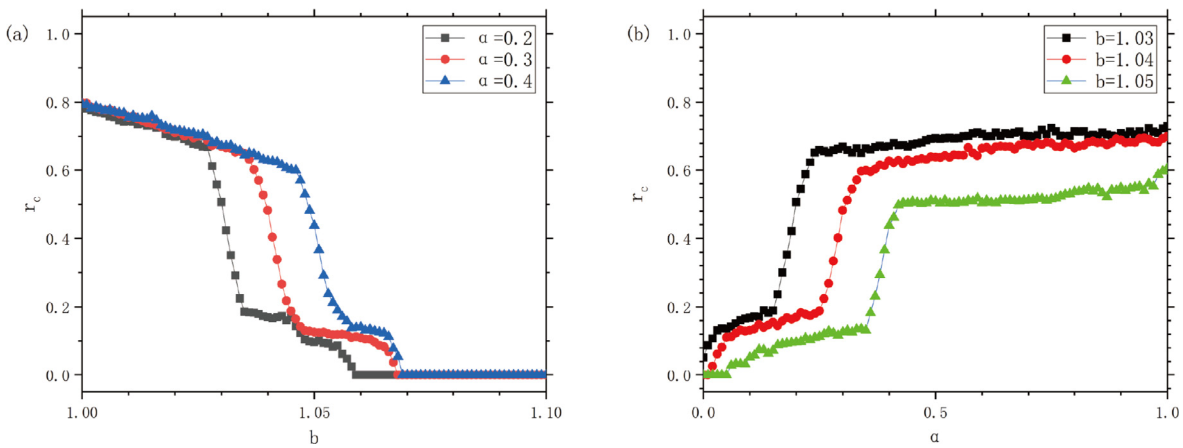

Fig. 6. (a)The fraction of cooperators at the stable state as a function of b under the Deterministic rule. The black, red, and blue line are $\alpha = 0.2$, 0.3, and 0.4 respectively. (b)The fraction of cooperators at the stable state as a function of α under the Deterministic rule. The black, red, and green line are $b = 1.03$, 1.04, and 1.05 respectively. The threshold α_b could be easily found. When $b < \alpha_b$ or $b > \alpha_b$ with a distance, $r_{\mathcal{C}}$ increases with α and deceases with b monotonously and gently. But when b is close to α, $r_{\mathcal{C}}$ changes sharply. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Figure 3 shows the fraction of cooperators with time-evolution for different values of G when $b = 1.05$. It is found that the lower G is, the earlier network becomes stable. Besides, the network becomes stable faster under Deterministic rule than under Stochastic rule with the same G except for $G = 10$, where cooperation vanishes soon under Stochastic rule and from Fig. 3(a) it could be observed that when $t = 10$, the Group Network's evolution step accelerates the extinction of cooperation under the Stochastic rule. Figure 4 presents snapshots of strategy pattern with time-evolution for different values of G when $b = 1.05$ under the Deterministic rule. It could be observed that for all values of G, the fraction of cooperators will decrease at the start. At the same time, survival cooperators form clusters and with the help of Group Network's evolution step, they will expand to their neighboring groups. When $G = 10$, cooperators begin to expand when $t = 50$. When $G = 50$ and 100, cooperators begin to expand when $t = 100$. As for $G = 500$, cooperators begin to expand when $t = 500$, which is the first Group Network's evolution step, and when it comes, cooperation is on the verge of extinction. When $t = 5000$, it could be observed that the spatial distributions of individuals' strategies on neighboring groups are similar, which is powerful evidence that the selections between groups play a role.

It is concluded that for Deterministic rule, the lower G is, the more effectively our model could promote cooperation. But for Stochastic rule, there is a moderate value of G at which our model performs the best. Besides, Deterministic rule has better effects than Stochastic rule for all values of G.

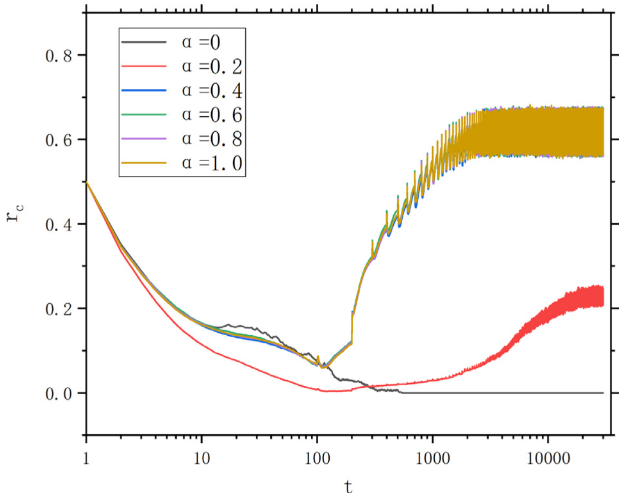

Fig. 7. Time series of the fraction of cooperators for different values of α under the Deterministic rule. The black, red, blue, green, purple, and yellow line represent $\alpha = 0$, 0.2, 0.4, 0.6, 0.8, and 1.0 respectively. Three different evolutionary phenomena are shown, which is consistent with the result shown in Fig. 6. The values of steps required for the network to reach the stable state for different α are almost the same if α is large enough($\alpha = 0.4$, 0.6, 0.8, and 1.0). But when α is moderate($\alpha = 0.2$), $r_C(t)$ is even lower than that when $\alpha = 0$ in the first 300 steps,. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

3.2. *Effects of imitating rate α*

In the above subsection, imitating rate α is set to a constant value of 1.0. However, imitations between groups are not always sufficient. So we should also consider the condition in which groups imitate their neighbors partially in Group Network, and the imitating rate is measured by α. It is still kept $r_{\mathcal{C}}(0) = r_{\mathcal{D}}(0) = 0.5$, with $L = 100$, $n = 10$ and $d = 10$ being set. Besides, G is fixed to 100 to measure a moderate Group Network's evolution frequency.

Figure 5 shows how the imitating rate α and b affect $r_{\mathcal{C}}$ under Stochastic rule and Deterministic rule. The results show that the imitating rate has a significant impact on cooperation. Compared with $\alpha = 0$, which is equivalent to the absence of the Group Network's evolution, all positive values of α could promote cooperation when b is lower than the threshold for the extinction of cooperation. Besides, $r_{\mathcal{C}}$ is higher under Deterministic rule than under Stochastic rule for all pairs of α and b. Under Stochastic rule, the increase of $r_{\mathcal{C}}$ with the increase of α is more obvious. As a comparison, it could be observed that under Deterministic rule, a threshold α_b exists, roughly satisfied $\alpha_b = 10(b - 1.01)$ when $b < 1.05$. When $b < \alpha_b$ or $b > \alpha_b$ with a distance, $r_{\mathcal{C}}$ increases with α and deceases with b monotonously and gently. But when b is close to α, $r_{\mathcal{C}}$ changes sharply. Figure 6(a) and (b) shows how $r_{\mathcal{C}}$ changes with α when b is fixed and changes with b when α is fixed, respectively. The threshold α_b could be easily found in Fig. 6.

The above phenomenon could be explained that for a fixed value of b, without other reciprocity mechanisms, cooperators are exploited by defectors in Individual Network's evolution process. Cooperators could only survive by forming some small clusters and expanding to other groups in Group Network's evolution step. When α is much larger than α_b, more individuals will imitate strategies from their neighboring groups in Group Network's evolution step, so cooperators' expansion rate is higher than its extinction rate. $r_{\mathcal{C}}(t)$ will increase with t until it reaches the steady state, is related to the value of b but unrelated with α. On the contrary, when α is much smaller than α_b, fewer individuals will imitate strategies from their neighboring groups in Group Network's evolution step, so cooperators' expansion rate is lower than its extinction rate. $r_{\mathcal{C}}(t)$ will decrease to the steady state fast. But when α is close to α_b, the expansion rate is also close to its extinction rate, the steady state $r_{\mathcal{C}}$ is sensitive to the value of α. Figure 7 shows the fraction of cooperators with time-evolution for different values of α when $b = 1.03$. Three different evolutionary phenomena are shown in Fig. 7, which is consistent with the result shown in Fig. 6. Besides, we could further achieve two conclusions from Fig. 7. One is the values of steps required for the network to reach the stable state for different α are almost the same if α is large enough($\alpha = 0.4$, 0.6, 0.8, and 1.0). The other is when α is moderate($\alpha = 0.2$), $r_{\mathcal{C}}(t)$ is even lower than that when $\alpha = 0$ in the first 300 steps, which is because that during these beginning steps, $r_{\mathcal{D}}(t)$ is much larger than $r_{\mathcal{C}}(t)$, so the relatively low imitating rate α makes defection more frequently to be imitated at the beginning few Group Network's evolution step($t = 100$ and 200) and destroys the clusters of cooperators to some extent. With the time growing($t > 300$) and some clusters stable enough, cooperators begin to expand in Group Network's evolution step. The snapshots of the strategy pattern shown in Fig. 8 could also support the above discussions.

It is concluded that group selection does not require a very large value α to be fully effective, but only needs to exceed a certain threshold α_b. And when α is small but larger than 0, group selection can also partly play a role in enhancing

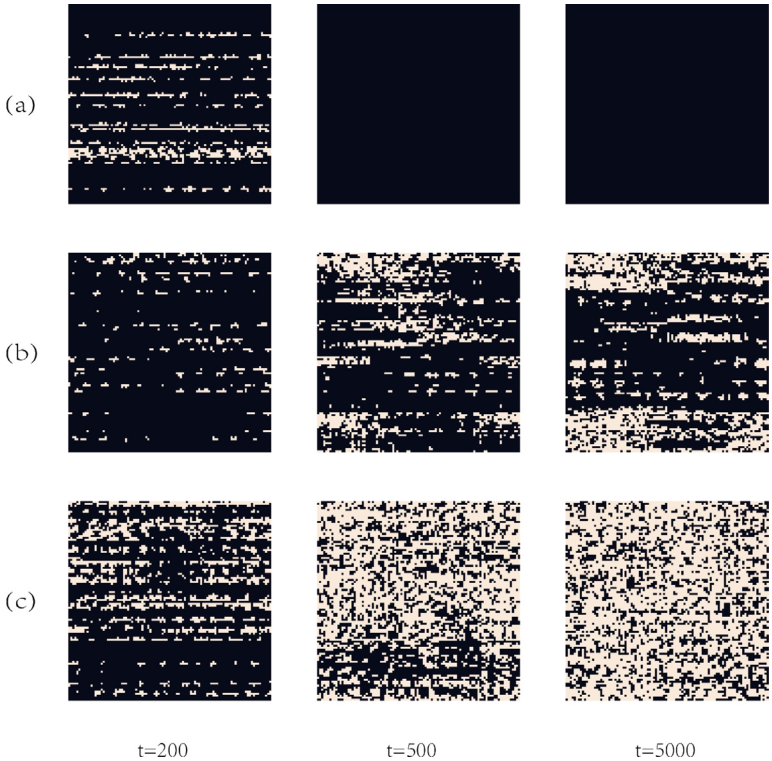

Fig. 8. The evolution of snapshot with time-evolution. Cooperators and defectors in the network are denoted as white and black respectively. From left to right $t = 200$, 500, 5000. From top to bottom $\alpha = 0$, 0.2, 0.8. When $\alpha = 0$, cooperators vanish before $t = 500$. When $\alpha = 0.2$, $r_C(t)$ decreases faster than when $\alpha = 0$ before $t = 200$, then some clusters stable enough, cooperators begin to expand in Group Network's evolution step. When $\alpha = 0.8$, cooperators expand continuously until the network is stable.

cooperation. When α is moderate and close to α_b, r_C changes sensitively with the change of α, and the beginning of the cooperators' expansion will need more steps.

4. Conclusion

In summary, combining two classical mechanisms called group selection and network reciprocity, both of which have already been proved to be the key mechanism to promote cooperation, we proposed a network-based group selection model by using a multilayer network, in which not only individuals but also groups have a spatial structure. It is found that Group Network's evolution process has a significant effect on promoting cooperation. In our model, Group Network's evolution is controlled by two key parameters called Group Network's evolution frequency G and imitating rate α whose functions are explored in Sections 3.1 and 3.2 respectively. For all values of G and α, Deterministic rule performs better than Stochastic rule because of its determinacy. For Stochastic rule, there is a moderate value of G at which our model performs the best, and the increase of r_C with the increase of α is more obvious. For Deterministic rule, the lower G is, the more effectively our model could promote cooperation, and it could be observed that a threshold α_b exists, roughly satisfying $\alpha_b = 10(b - 1.01)$ when $b < 1.05$. It is also found that group selection does not require a very large value α to be fully effective, but only needs to exceed a certain threshold α_b.

Our work combines group selection and network reciprocity to create a new perspective to solve the social dilemma, and it has a broad space for further exploration. For example, how heterogeneity of attributes among groups affects the spatial evolution process could be explored under our model, and the interaction between groups could not only be the imitation of the strategies. It is hoped that our work provides a powerful framework and could be applied to researches with practical background, such as insurance [40], climate change [41], or some other problems.

Acknowledgments

This work is supported by the Fundamental Research Funds for the Central Universities, the Research and Development Program of China (Grant No. 2018AAA0101100), the Beijing Natural Science Foundation (Grant Nos. 1192012, Z180005), National Natural Science Foundation of China (Grant No. 62050132), and Slovenian Research Agency (Grant Nos. P1-0403 and J1-2457).

References

[1] R. Axelrod, The Evolution of Cooperation, Basic Books, New York, 1984.
[2] G. Claeys, The "survival of the fittest" and the origins of social darwinism, J. Hist. Ideas 61 (2000) 223–240.
[3] R. Axelrod, W.D. Hamilton, The evolution of cooperation, Science 211 (1981) 1390–1396.
[4] K. Sigmund, M.A. Nowak, Evolutionary game theory, Curr. Biol. 9 (1999) R503–R505.
[5] M.A. Javarone, Statistical Physics and Computational Methods for Evolutionary Game Theory, Springer, Hertfordshire, 2018.
[6] A. Rapoport, A.M. Chammah, C.J. Orwant, Prisoner's Dilemma: A Study in Conflict and Cooperation, University of Michigan Press, Ann Arbor, 1965.
[7] M.A. Nowak, R.M. May, Evolutionary games and spatial chaos, Nature 359 (1992) 826–829.
[8] G. Szabó, C. Tőke, Evolutionary Prisoner's dilemma game on a square lattice, Phys. Rev. E 58 (1998) 69–73.
[9] Z. Wu, X. Xu, Y. Chen, Y. Wang, Spatial Prisoner's dilemma game with volunteering in Newman-Watts small-world networks, Phys. Rev. E 71 (2005) 037103.
[10] J. Ren, W. Wang, F. Qi, Randomness enhances cooperation: coherence resonance in evolutionary game, Phys. Rev. E 75 (2007) 045101.
[11] Z. Rong, X. Li, X. Wang, Roles of mixing patterns in cooperation on a scale-free networked game, Phys. Rev. E 76 (2007) 027101.
[12] W. Du, X. Cao, L. Zhao, M. Hu, Evolutionary games on scale free networks with a preferential selection mechanism, Phys. A 388 (2009) 4509–4514.
[13] L. Zhang, C. Huang, H. Li, Q. Dai, J. Yang, Migration based on historical payoffs promotes cooperation in continuous two-dimensional space, EPL 134 (2021) 68001.
[14] L. Zhang, C. Huang, H. Li, Q. Dai, J. Yang, Effects of directional migration for pursuit of profitable circumstances in evolutionary games, Chaos Solitons Fractals 144 (2021) 110709.
[15] C. Xia, X. Li, Z. Wang, M. Perc, Doubly effects of information sharing on interdependent network reciprocity, New J. Phys. 20 (2018) 075005.
[16] A. Li, L. Zhou, Q. Su, S.P. Cornelius, Y.-Y. Liu, L. Wang, S.A. Levin, Evolution of cooperation on temporal networks, Nat. Commun. 11 (2020) y2259.
[17] B. Herrmann, C. Thöni, S. Gächter, Antisocial punishment across societies, Science 319 (2008) 1362–1367.
[18] D. Helbing, A. Szolnoki, M. Perc, G. Szabó, Punish, but not too hard: how costly punishment spreads in the spatial public goods game, New J. Phys. 12 (2010) 083005.
[19] X. Li, H. Wang, C. Xia, M. Perc, Effects of reciprocal rewarding on the evolution of cooperation in voluntary social dilemmas, Front. Phys. 7 (2019) 125.
[20] L. Zhang, Y. Xie, C. Huang, H. Li, Q. Dai, Heterogeneous investments induced by historical payoffs promote cooperation in spatial public goods games, Chaos Solitons Fractals 133 (2020) 109675.
[21] C. Shen, M. Jusup, L. Shi, Z. Wang, M. Perc, P. Holme, Exit rights open complex pathways to cooperation, J. R. Soc. Interface 18 (2021) 20200777.
[22] X. Li, H. Wang, G. Hao, C. Xia, The mechanism of alliance promotes cooperation in the spatial multi-games, Phys. Lett. A 384 (2020) 126414.
[23] A. Szolnoki, M. Perc, Conformity enhances network reciprocity in evolutionary social dilemmas, J. R. Soc. Interface 12 (2015) 20141299.
[24] A. Szolnoki, M. Perc, Leaders should not be conformists in evolutionary social dilemmas, Sci. Rep. 6 (2016) 23633.
[25] X. Li, G. Hao, Z. Zhang, C. Xia, Evolution of cooperation in heterogeneously stochastic interactions, Chaos Solitons Fractals 150 (2021) 111186.
[26] M.A. Amaral, M.A. Javarone, Heterogeneity in evolutionary games: an analysis of the risk perception, Proc. R. Soc. A 476 (2020) 20200116.
[27] M.A. Amaral, M.A. Javarone, Strategy equilibrium in dilemma games with off-diagonal payoff perturbations, Phys. Rev. E 101 (2020) 062309.
[28] D. Jia, H. Guo, Z. Song, L. Shi, X. Deng, M. Perc, Z. Wang, Local and global stimuli in reinforcement learning, New J. Phys. 23 (2021) 083020.
[29] H. Yang, L. Tian, Enhancement of cooperation through conformity-driven reproductive ability, Chaos Solitons Fractals 103 (2017) 159–162.
[30] M.A. Javarone, A. Antonioni, F. Caravelli, Conformity-driven agents support ordered phases in the spatial public goods game, EPL 114 (2016) 38001.
[31] M.A. Amaral, L. Wardil, M. Perc, J.K. da Silva, Stochastic win-stay-lose-shift strategy with dynamic aspirations in evolutionary social dilemmas, Phys. Rev. E 94 (2016) 032317.
[32] Z. Shi, W. Wei, X. Feng, X. Li, Z. Zheng, Dynamic aspiration based on win-stay-lose-learn rule in spatial Prisoner's dilemma game, PLoS ONE 16 (2021) e0244814.
[33] Z. Shi, W. Wei, X. Feng, R. Zhang, Z. Zheng, Effects of dynamic-win-stay-lose-learn model with voluntary participation in social dilemma, Chaos Solitons Fractals 151 (2021) 111269.
[34] Y. Liu, X. Chen, L. Zhang, L. Wang, M. Perc, Win-stay-lose-learn promotes cooperation in the spatial Prisoner's dilemma game, PLoS ONE 7 (2012) e30689.
[35] M.A. Nowak, Five rules for the evolution of cooperation, Science 314 (2006) 1560–1563.
[36] M.A. Nowak, R.M. May, The spatial dilemmas of evolution, Int. J. Bifurc. Chaos Appl. Sci. Eng. 3 (1993) 35–78.
[37] J. Paulsson, Multileveled selection on plasmid replication, Genetics 161 (2002) 1373–1384.
[38] G. Szabó, J. Vukov, A. Szolnoki, Phase diagrams for an evolutionary Prisoner's dilemma game on two-dimensional lattices, Phys. Rev. E 72 (2005) 047107.
[39] M. Perc, Coherence resonance in the spatial Prisoner's dilemma game, New J. Phys. 8 (2006) 022.
[40] F.P. Santos, J.M. Pacheco, F.C. Santos, S.A. Levin, Dynamics of informal risk sharing in collective index insurance, Nat. Sustain. 4 (2021) 426–432.
[41] S. Dhakal, R. Chiong, M. Chica, R.H. Middleton, Climate change induced migration and the evolution of cooperation, Appl. Math. Comput. 377 (2020) 125090.