Options for mobility and network reciprocity to jointly yield robust cooperation in social dilemmas

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\textbf{A B S T R A C T}

Collective cooperation and social mobility are ubiquitous in human societies. Due to information sharing and the complexity of everyday life, multidimensional mobility is also common, for example when moving into different districts of a city for better education or when settling permanently abroad due to better job prospects. Nevertheless, it is not clear how such complex mobility might affect cooperation in situations that constitute social dilemmas, where individual and public interests are at odds. Here, as an initial step to understand the impact of multidimensional mobility, we investigate one of its significant dimensions, namely the range of mobility. We propose an updating algorithm where individuals either move adaptively in the area bounded by a mobility radius or stay put for social learning. We use this on the prisoner’s dilemma and the snowdrift game, and we find that an increase in either probability or radius of mobility may weaken network reciprocity, simply by decreasing the odds of meeting old interaction partners. However, if mobility is free, there is a window of parameters where synergies with network reciprocity are possible, and where indeed cooperation can be robust and significantly elevated. Local mobility in particular may favorably affect cooperation. In fact, even if mobility is costly, the failure of local mobility can often be associated with the risk caused by the shortage of available empty sites. We also find that the synergistic effects of mobility superposed upon network reciprocity are best expressed for small flow rates. Overall, we hope that our research will promote the better understanding of the complex interplay between networks reciprocity and mobility and their coaction.

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1. Introduction

The civilization and advancement of human society could not be separated from cooperation in large scale [1,2]. On the other hand, according to Darwin’s theory, individuals are tempted to defect and get benefits from others’ efforts [3,4]. The factors and mechanisms that affect the output of cooperation has attracted substantial attention in recent decades [4]. Among them network reciprocity is an effective approach to enhance cooperation [5–9]. When the ratio of the benefit to the cost is larger than the average number of neighbors, to get the most benefit individuals are more likely opted to cooperate than behave as defectors [10–13]. And this mechanism may govern the stability of social networks [14–20].

Recent studies have shown that social mobility is of great significance for cooperation. Both humans and other animals often move to other regions because of various reasons. Due to information sharing, social mobility could be even observed in many aspects and is thus multidimensional. Some large-scale data of population movement have been observed to present a robust scaling law in both the temporal and spatial spectrum [21]. In addition, local mobility with low velocity promotes cooperation in the random walk dynamics [22]. However, the understanding to such complex mobility remains incomplete. On one hand, mobility may restore the mechanism which supports cooperation [23], or promote the evolution of cooperation through emergent self-assortment dynamics [24]. Specifically, medium density of individuals could promote cooperation when the network flow of mobile individuals is also at appropriate level [25]. On the other hand, the less frequently interaction with partners makes less stable social networks [14]. Actually, the synergy between mobility and network reciprocity may decide collective cooperation.

To elucidate this, here we propose to understand the impacts of multidimensional mobility through one of its crucial aspects — the covered range of mobility. Specifically, we investigate weak prisoner’s dilemma as well as snowdrift games, and study how different level of mobility range and mobility probability affects cooperation. The evolution of cooperation is obtained from Monte Carlo simulations. In our model, individuals either adaptively move in the area bounded by a mobility radius or stay put for social learning. Notice that there is no chance to update strategies anymore for individuals who have moved. This induces a mobility cost since some individuals forgo social learning for mobility opportunity. Further, even when individuals have paid mobility cost, they may still meet a situation where they could not actually move away since there is no available empty sites. This is the mobility risk in our model.

Our results indicate that the increase of either mobility probability or mobility radius may weaken network reciprocity by decreasing the odds of meeting old interactive partners, and then further weakens the synergy between mobility and network reciprocity. Mobility mechanisms make no odds to the result. And with the mobility cost, the cooperation frequency has been reduced. Particularly, only local mobility could promote cooperation remarkably as temptation to defect increases. Even if individuals have paid mobility cost, the failure of mobility, especially for local mobility, may appear due to mobility risk. We also find that the synergistic effects of both mobility and network reciprocity works best provided small flow rate. And under such scenario robust cooperation persists. We hope our results could help understand the pathways of mobility affecting cooperation.

2. Model

Here, the evolution is carried out on a $M \times M$ regular square lattice with periodic boundary conditions, where 70.0 percent of the sites are occupied by population initially and others are empty. Interactive relationships are characterized by links. Thus, each player interacts with linked neighbors by cooperating or defecting to gain his/her payoff (fitness) according to the following payoff matrix.

$$
\begin{pmatrix}
C & D \\
C & (R & S) \\
D & (T & P)
\end{pmatrix}
$$

We characterize weak prisoner’s dilemmas (PD) by fixing $R = 1$, $S = P = 0$ and adjusting $T = b$ (temptation factor, $1 < b < 2$) alone to represent the intensity of dilemmas. In addition, we set $R = 1$, $P = 0$, $S = 1 – r$, $T = 1 + r$ (cost to benefit ratio, $0 < r < 1$) for snowdrift dilemmas (SG). The bigger $b$ or $r$ is, the higher the intensity of a dilemma is. Initially, each player is set as either a cooperator or a defector with equal probability. Confronting survival competition, the player with strategy gaining highest payoff in a community has evolving advantage in the process of evolution. In the process of updating, a player compares his/her own payoff with all his/her direct neighbors and then copies the strategy with the highest payoff. If there is more than one neighbor who has the highest payoff, the player would adopt a strategy randomly among them. In case of his/her own highest payoff in the community, the player keeps the current strategy unchanged. In order to mirror the possible occasional uncertainty in nature, the strategy is randomly reset with probability $\delta$, or updated by social learning with $1-\delta$.

In our model, there are two ways that players raise their fitness: either implementing adaptive mobility based on the probability of $p$, or updating strategies with $1-p$. Here, we define a parameter mobility radius $L$ to set the area where a player could move into. Once a player determines to move, he/she would locate himself/herself as the center, and search empty sites in the range of the circular plane (marked as CP) with mobility radius $L$. For convenience, we define $L = 1$ as local mobility, and $L \geq 2$ as wide-range mobility). Given several empty sites, one will be picked randomly. In case of no available one, the player stays put with no strategy updating either. Besides, we define per capita mobility rate $\gamma$ as flow...
rate, the number of individuals who have moved actually divided by the number of whole population. To counteract the
effect of \( p \), \( \eta \) is also introduced to normalize \( \gamma \), three parameters satisfy \( \eta = \gamma / p \) theoretically. To show the potentials of
networks, we also run the simulations on well-mixed population, where players interacts with partners randomly among
the whole population in each round.

The evolution proceeds with Monte Carlo method, which is the widely used numerical simulations performed in evolution-
ary game theory [26]. Every player makes a choice with a equal probability. To mirror the cooperation level of the
system [27–30], we define cooperation frequency \( f_c \), which is the ratio of the number of cooperators to whole population.
Each data point is obtained by averaging over 1000 generations after a relaxation time of 9000 generations. To neutralize
the impact of initialization, we run 10 realizations in all with \( M = 128 \).

### 3. Simulation results

An important insight is that the fine-tuned interplay between the mobility and network reciprocity facilitates cooperation
remarkably. In this situation, mobility should be appropriate, neither too low to work effectively, nor too high to
destroy network reciprocity. For adaptive mobility mechanism, we investigated how cooperation frequency \( f_c \) changes over
the probability of mobility \( p \), the results are shown in Fig. 1(A). As \( b \) grows, it is observed that the effective region of \( p \) on
enhancing cooperation effectively narrows. Particularly, a peak appears when \( b \) is as high as 1.50. Despite different mobility
mechanism works actually [27], the conclusion that finite mobility facilitates cooperation is robust.

In reality, not all individuals could determine exactly the most profitable strategy due to uncertainty in nature. To reflect
this, we ran another simulation with a small noise \( (\delta) \), such as \( \delta = 0.01 \). And here in Fig. 1(B), it is observed that \( f_c \) increases
steadily as \( p \) rises within the optimum 0.84 for \( b = 1.30 \). For \( p \) over the optimum, \( f_c \) falls sharply. However, as \( b \) grows, it is
observed that the optimum of \( p \) declines from 0.84 to about 0.30 or so. Apparently, the optimum of \( p \) on promoting
cooperation declines as the intensity of dilemma mounts, and also drops along with the decrease of uncertainty, such as
\( p = 0.01 \) when \( \delta = 0.0 \) in panel (A).

At the personal level, interaction with defectors is the major obstacle for players being out of the woods. Actually, a
cooperator, even a defector may adaptively move away to avoid interacting with defectors in real life. Mobility is one of
effective ways to escape from being exploited situation. In reality, despite of different purposes or reasons, various destina-
tions are open for each mobile individual. Thus, multidimensional mobility actually occurs. Here, we mainly focus on the
macroscopic effect of networks caused by mobility, particularly the positive impacts on cooperation.

It is worth mentioning that the insight is obtained based on the fact that mobility actually occurs in a community. It is
interesting to explore whether it is robust when the range of mobility extends. In order to figure out this, we ran the
simulation where PD players move in the CP with different \( L \), the results of color-coded cooperation frequency in \( (p - L) \)
parameter space are revealed in Fig. 2. In left two panels, shaded area is L-shaped obviously. For fixed \( L \), it is observed
that the color of \( f_c \) turns darker as \( p \) grows from 0.0. And yet it fades when \( p \) continue to rise. Despite of slight difference
between both panels \( b = 1.50 \) and \( b = 1.40 \), such as the dark region of \( p \) is much wider for the latter than that for the
former, both panels show obviously \( p \) has optimal values of promoting cooperation for a given \( L \). Remarkably, with the
increase of \( L \), the effective region of \( p \) enhancing cooperation narrows down. The conclusion that limited mobility promotes
cooperation is thus robust when mobility occurs beyond the realm of a community.

For a given \( p \), small \( p \) provides an effective approach to maintaining high level of cooperation. In this situation, it is not as
sensitive to values of \( L \) as large \( p \). There seems a threshold of \( p \), over which only local mobility could enhance cooperation.

![Fig. 1. Cooperation frequency \( (f_c) \) as a function of probability of mobility \( (p) \). As decrease of uncertainty \( (\delta) \) or increase of temptation factor \( (b) \), the optimal value of \( p \) on enhancing cooperation becomes small. The parameters: \( \delta = 0.0(A) \), and for \( \delta = 0.01(B) \).](image-url)
below which the effective $L$ on enhancing cooperation widens, whereas shortens as $p$ grows. The results with snowdrift game are as shown in right panels in Fig. 2. Different with prisoner’s dilemma where the Nash equilibrium strategy is to defect, the best strategy is to cooperate (defect) when interacting with a defector (cooperator) in snowdrift game. Thus, the cooperation strategy is much more often in snowdrift dilemmas than prisoner’s dilemmas in theory. Accordingly, besides the dark L-shaped area, one observes the condition for preserving cooperation is more tolerant of both $p$ and $L$ in SG than that in PD.

We try to explain this phenomenon from the perspective of network reciprocity. For comparison, we investigate it on a well-mixed population, a square lattice, and evolutionary networks caused by mobility with different $L$ respectively in Fig. 3. In left panels, without networks, all players trapped in prisoner’s dilemma defect regardless of $b$. In case of square lattice, some cooperators could survive in clusters by helping each other to gain rewarding payoff. Although the system could maintain a low level of cooperation even for far high temptation, network reciprocity alone is far from getting system off the hook.

However, together with mobility, the situation is remarkably optimistic, particularly for $b < 1.35$. Whereas for higher temptation, like $b > 1.5$, the combination of both mechanisms fails to reverse trend that all players defect for short-term high payoff. Even worse, the network reciprocity is also destroyed by mobility, which wipes out cooperators completely. However, For medium temptation $b$, like the region $1.35 \leq b \leq 1.5$, it seems too complicated to conclude because of two factors $p$ and $L$ working together. In order to figure out this, we first create a high-viscosity network by fixing $p = 0.05$, where the impact of $L$ is explored. As shown in top left panel, the system ends with all players cooperate when $L = 1$. As $L$ mounts, $f_c$ drops obviously. Fixing $L = 4$ as shown in bottom left panel, $f_c$ drops as $p$ rises too.

In order to check the robustness of the results, we also ran another simulation with SG, as shown in right panels. Despite of different dilemmas that players are trapped, similar results are also found particularly for $0.65 < r < 0.75$. Paying such high cost yet without extra profit, it is difficult for strangers to establish trust in each other. Cooperation is often seen among “old” friends due to tacit agreement (direct reciprocity). No matter whether dilemma is PD or SG, network reciprocity shows unique advantage on enhancing cooperation particularly when cooperation falters owing to high $b$ or $r$.

Apparently, the effects of network reciprocity is positive but limited. Theoretically, mobility makes no sense in well mixed population due to the fact that a random player interacts with random neighbors in each round. However, based on
networks, there’s ample scope enhancing cooperation for mobility mechanism, which is significant in real world filled with various social networks. Players change interactive neighbors by mobility. Higher \( p \) increases flow rate \( \gamma \), which reduces the impact of networks. The range of mobility extends with the increase of \( L \). Accordingly, the number of available empty sites is proportional to \( L^2 \). The actual mobility distance between current location and potential destination varies from 1 to \( L \). Within the CP with a given \( L \), the number of available empty sites steps upward as actual mobility distance increases from 1 to \( L \), and also the odds of migrating into distant destinations increase accordingly. Actually, the prospect of potential location is hard to predict based on known information. However, the wide-range mobility is a big probability event. In general, the farther mobility occurs, the less odds of encountering old partners are. Therefore, the odds of meeting old friend decreases with increase of either \( p \) or \( L \), which inhibits the influence of networks actually. Further, the synergistic effect of network reciprocity and migration on improving cooperation is destroyed too.

In our model, there is no chance of updating strategy for players who have moved in a round. Actually, the alternative forgone is opportunity cost, which is considered as mobility cost here. In case of free mobility cost, what it would be like if individuals have chance to update strategy right after adaptive mobility? As shown in Fig. 4, for small temporal payoff like \( b = 1.3 \), there is a wide effective region of \( L \) on promoting cooperation. However, the curve of \( f_c \) declines as \( L \) continues to grow. As \( b \) grows, facing increasing temptation to defect, only cooperators implementing rather short-trip mobility have advantage to survive. Extremely, only local mobility promotes cooperation effectively at \( b = 1.50 \) where \( f_c \) reaches about 1.0 for \( L = 1 \) in the condition of zero mobility cost.

As to adaptive mobility mechanism, the mobility is driven by defectors. The odds of a player moving away are in proportion to the number of defectors among his/her direct neighbors. For a player \( i \), the probability of his/her mobility depends on his/her environment parameter \( w = N_D/(N_C + N_D) \), where \( N_C \) and \( N_D \) are the number of cooperators and defectors in all direct neighbors respectively. On the whole, \( \gamma = \eta \times p \), we thus obtain \( \eta = N_{DW}/(N_{CW} + N_{DW}) \), where \( N_{CW} \) and \( N_{DW} \) are the number of cooperators and defectors in the system respectively. Since \( f_c = N_{CW}/(N_{CW} + N_{DW}) \), \( f_c + \eta = 1.0 \) is thus obtained. In general, the higher cooperation frequency is, the less mobile players are. As shown on panel (B), one observes \( f_c \) seems inversely proportional to \( \gamma \) at a \( L \) for a given \( b \).
Fig. 4. Cooperation frequency is inversely proportional to $\gamma$ without paying mobility cost. $f_c$ as a function of $L$ and $\gamma$ as a function of $L$ for PD on left panels and for SG on right panels. Instead of forgoing social learning for mobility, individual are free to updating strategy after mobility here. $\delta = 0.0$.

Fig. 5. Sum of $f_c$ and $\eta$ is equal to 1.0 in theory. Results of the sum as shown for PD (A) and SG (B) varying with $L$. Here, $b = 1.5$(A), $r = 0.7$(B) and $\delta = 0.0$. The shortage of available empty sites to move into causes high mobility risk in local mobility ($L = 1$), which decreases flow rate $\gamma$ directly and also depriving them of social learning.
In order to get a clear image, we summed \( f_c \) and \( \eta \), which changes over \( L \) for different \( p \), as shown in Fig. 5(A). It is found that all cases perfectly fit with theoretical points except for \( L = 1 \), where all data points are lower than theoretical values. Particularly, the sum of \( f_c \) and \( \gamma \) falls with the increase of \( p \). Despite of different dilemmas, similar results are also observed for SG, as shown in panel (B). In spite of mobility cost that mobile players have paid, they actually may fail to move away due to shortage of available empty sites, which is considered as mobility risk. Initially, all players are evenly and randomly distributed on the lattice, with 70.0 percent of whole lattice. Theoretically, there are about 1.5 empty sites in a unit community. The obtained payoff in compact cooperative clusters is higher than that in loose ones, cooperators thus survive in compact clusters adaptively. The distribution of empty sites is thus neither evenly nor randomly at the end of evolution. Particularly, high population density comes with high risk of empty sites shortage.

Given a population density, mobility risk is highly relative with the parameter \( L \). Actually, the risk decreases with increase of \( L \). The higher \( L \) is, the more available empty sites in corresponding CP are, the less risk is. However, wide-range mobility is not as effective as local mobility on enhancing cooperation on the whole. Therefore, it is interesting to explore the impact of mobility risk on the evolution of cooperation. It occurs to us that high risk may reduce flow rate. Considering this, we further investigate how \( f_c \) evolves with \( \gamma \), as shown in Fig. 6. It is found that \( f_c \) falls sharply with increase of \( \gamma \) for different \( L \). Obviously, mobility mechanism does not give better results with the increase of \( \gamma \). The risk reducing \( \gamma \) actually has no negative effect on promoting cooperation. Remarkably, low \( \gamma \) together with local mobility has superiority over other cases due to the combination of mobility and network reciprocity working best.

4. Discussion

Actually, multidimension mobility occurs in reality. It is interesting yet significant to investigate whether obtained results of mobility on local mobility is robust in diverse situations. By extending mobility range \( L \), it is found that limited mobility still promote cooperation effectively regardless of whether dilemmas are either PD or SG. Despite of different reasons to move, all cases indicate that limited mobility promotes cooperation. And mobility mechanism itself make no odds to robustness of the result seemingly in theory.

Besides, local mobility \((L = 1)\) on promoting cooperation is found more effective than wide-range mobility \((L \geq 2)\), which seems irreconcilable with the result that adaptive long-range migration promotes cooperation when mobility range is proportional to environment parameter \( w \) [17]. Be more specific, the higher \( w \) is, the farther individuals moving away is. Similarly, it is \( w \) that reflects quantitatively how bad the situation is. Actually, we argue the effect of mobility on cooperation should be attributed to \( w \) rather than long-range mobility for following reasons. Whether the prospect of potential destination is more profitable or even worse is unpredictable by current mobility mechanism on one hand. As \( w \) increases, the potential destination is more profitable for an individual suffering severe exploitation than current site on the other hand. Last but not least, the prospect of destination does not depend on distance, but maybe old friends. The increase of mobility range may weaken network reciprocity by decreasing the odds of renewing acquaintance with old partners in social networks [31–34].

Actually, not all individual would move away successfully even if they had given up opportunity of social learning in a round, which is caused by shortage of available empty sites. Due to this, neither mobility nor social learning may come to nought. Although large \( L \) could reduce risk effectively, its enhancement on cooperation is far from satisfactory for high temptation. Obviously, the high mobility risk reduces flow rate \( \gamma \), whereas the combination of two mechanisms do not give better results with the increase of \( \gamma \). Mobility risk thus actually has no negative effect on promoting cooperation. Firstly,
only small flow rate could promote cooperation effectively in different $I$ remarkably. And secondly, local mobility plays a dominating role in preserving network reciprocity, as well as the synergy of mobility and networks reciprocity.

Despite people have investigated mobility deeply and widely with the approach of theory and experiments, an end to understand the essence of mobility seems a long way off. Based on diverse mobility mechanisms proposed, like mobility for success, hybrid migration mechanism, leaving bad situation or approaching good community etc, they show great potentials on enhancing cooperation provided prerequisites met. However, the mobility is complicated yet multidimensional in reality. In diverse condition, whether these mobility mechanisms are robust is significant in its applications. To advance theory research on mobility, the further studies based on higher-order networks mechanism is urgent [35–39], including its applications for social interactions [40–46].

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