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Suppression of spiral wave turbulence by means of periodic plane waves in two-layer excitable media

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ABSTRACT

Spiral waves are relatively common, yet fascinating, visually appealing, and important phenomena in many nonlinear dynamical systems. The emergence of spiral waves in the heart's atrium, for example, signals abnormality that can lead to arrhythmias such as atrial flutter and atrial fibrillation. Spiral waves have also been associated with the disruption of resting states in the human brain, which are crucial for unimpaired cognitive ability and information processing. Here we consider two-layer excitable media, where spiral wave turbulence is triggered as the initial state. We study the effects of periodic plane waves on the dynamics of spiral wave turbulence, in particular by varying their spatial frequency. Our research shows that planes waves with low spatial frequency are in general too weak to overcome spiral wave turbulence. But when the spatial frequency is sufficiently increased, the plane waves can overcome spiral wave turbulence and impose a stripped spatial pattern over the excitable media. By increasing the spatial frequency of the plane waves even further, we show that it is possible to minimize the time needed to destroy spiral wave turbulence, although we also observe an upper limit beyond which the recurrence of turbulence is likely. This is linked to residual spirals that remain following a too rash elimination attempt, which then gradually regain footing across the whole medium.

1. Introduction

An excitable medium is a distributed nonlinear dynamical system composed of a large number of interconnected excitable unites and can support different forms of waves [1]. Modeling of the dynamics of excitable media has attracted much attention as they are omnipresent in nature, having several applications in physical, chemical or biological systems [2–4]. The excitable medium preserves a stationary homogeneous state in the absence of external perturbation. However, it can initiate wave propagation as a perturbation, which exceeds a certain threshold capable of disrupting its stable regime [5], is applied to a limited area of the medium [6]. Therefore, a wave front starts from that limited area and moves to the rest of the medium in all possible directions. As it reaches the boundary of the medium, it disappears and then the homogeneous state occupies the whole medium again. This homogeneous state will be preserved unless another wave front is generated. The interaction of different wave fronts with their particular refractory period can bring different patterns for the excitable medium [7], e.g., target-like patterns [8], solitary waves [9], and spiral waves [6].

Pattern formation in excitable media has been studied frequently as it has intimate relationship with the function of the system [10–12]. Among all possible dynamical behaviors of the excitable media, the spiral dynamics has attracted much attention [13,14]. This is basically because of far-reaching applications of the spiral pattern and also, its unique characteristic of being self-organized and self-sustained [15,16]. These two attributes are fundamentally important and can be affected by some factors including noise induction [17,18], network topology [17,19], and the delay of information transmission [20]. A self-sustained wave pattern [21] does not allow the media to return to its resting-state but rather, it generates successive wave fronts over and over again. The continuance of the resting-state, in turn, plays an essential role especially in different biological applications. For example, in the brain, the resting-state is crucial for cognitive ability in human

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as it determines the information flow configuration [22,23]. Also, the brain's function relies on its synchronous behavior [24] and the spiral wave can play a key role in synchronization or desynchronization of the neural network. Not only does self-sustenance break the resting-state, but also, in the case of spiral patterns, in particular, it can reorganize the dynamical behavior of the media (just like a pacemaker, or an additional wave origin). For instance, in the heart, an unexpected spiral pattern is the sign of abnormality as it destroys the proper dynamical behavior of the heart muscle [25,26]. In other words, when the spiral wave emerges in an excitable media, it acts like a pacemaker [16] which is not necessarily in line with the proper function of the media and can intrude upon the normal rhythm of the media [8,27]. Accordingly, there have been some studies focusing on elimination of the synchronization patterns in coupled neurons [28] and, in particular, on the emergence, termination, and dislocation of the spiral waves [29,30].

In the real world, the spiral wave can exist in multi-layer excitable media [31,32] such as in the cardiac [26,33] or in the cortical system [34]. In such systems, the interaction of the spiral pattern in the layers of a media is significant. Also, in addition to the spiral wave itself, some other wave forms – e.g., plane waves – may take place simultaneously, and affect the dynamics of the spiral wave in each layer. The co-existence of the spiral wave and the plane waves can lead to different dynamical behaviors. Depending on the circumstances, each of the spiral wave or the plane wave can overcome the other one and in each case, the function of the system will be different.

Given this, here we focus on the effect of spatial frequency of the plane waves and how it determines the ultimate spatiotemporal dynamics of the media. To do so, a model of two-layer excitable media is designed and the spiral wave is triggered as the initial state of the media. Each layer of the excitable media is composed of 100×100 coupled neurons. The local dynamics of the neurons is governed by the three-dimensional magnetic Fitzhugh-Nagumo (FN) neuronal model. In order to generate plane waves with controllable spatial frequency, we impose a periodic external force ($f = A\cos(\omega t)$) on the left boundary of the media. In the way the generated wave fronts move from the left boundary toward the right boundary of the media, they interact with the spiral waves that primarily existed in the media. Different frequencies of this external force result in different spatial different spatial frequency of the plane waves and thus, different spatial-temporal patterns in the media. The aim is to see whether the spiral-wave turbulence in a media can be eliminated by successive plane waves. The answer to this question, based on the simulation results of this study, is yes. We have found that, the higher the spatial frequency of the plane waves, the more they are capable of overcoming the turbulent spiral wave. However, although the high-spatial-frequency plane waves can guarantee the elimination of the spiral waves that existed as the initial state of the media, they are still associated with some drawbacks: there is a high chance that some secondary spiral cores form near the borders of the plane waves since the borders are very close to each other. In fact, one way for the spiral wave to emerge in a media is that the wave front and the wave back of two successive plane waves get together. So, the higher the spatial frequency of the plane waves, the closer are the wave fronts and therefore, it is more probable that the wave front meet the wave back. We particularly use four different angular frequencies for the external periodic force. For each case, the results are presented in some snapshots showing the pattern's development over the time.

2. Mathematical model of the two-layer excitable media

In this part, the mathematical model of the two-layer excitable media is explained. For each layer, the two-dimensional media is composed of 100×100 excitable units. Here, the new Fitzhugh–Nagumo (FN) neuronal model, in which the effect of magnetic flux is considered, is used for the local dynamics of each unit. This additional magnetic flux is the result of ion concentration across the membrane of the neuron. The time-varying accumulation of the ions generates time-varying electrical field, which, based on the Maxwell's equations, causes magnetic field [35]. Having said that, the three-dimensional differential equation model can be seen below:

$$\begin{aligned}
\frac{du}{dt} &= D_u \nabla^2 u - ku(u-a)(u-1) - uv + k_0 \rho(\phi) u \\
\frac{dv}{dt} &= \left(\varepsilon + \frac{v\mu_1}{u+\mu_2}\right)(-v - ku(u-a-1)) \\
\frac{d\phi}{dt} &= k_1 u - k_2 \phi \\
\rho(\phi) &= \frac{dq(\phi)}{d\phi} = \alpha + 3\beta\phi^2
\end{aligned} \tag{1}$$

where u, v and ϕ are the membrane potential, the ion current and the magnetic flux across the membrane, respectively. The term $-\mathrm{ku}(u-a)(u-1) - \mathrm{uv}$ is the total transmembrane current per unit area [35]. Parameter $a = 0.15$ is the excitation threshold. $\nabla^2 = \partial_{xx} + \partial_{yy}$ is the Laplacian operator in two-dimensional space. Parameter $D_u = 1$ implies the intra-layer connections. The term $k_0\rho(\phi)u$ is the electromagnetic induction current [35] and parameter $k_0 = 0.5$ regulates the effect of magnetic induction on the membrane potential. Parameter ε determines the excitability of the media. For the upper and lower layer it is fixed as $\varepsilon_1 = 0.005$ and $\varepsilon_2 = 0.008$, respectively. The parameters $\mu_1 = 0.2$, $\mu_2 = 0.3$, $k = 8$, $k_1 = 0.2$ and $k_2 = 1$ are constant.

The mathematical model of the two-layer excitable media is as follows:

$$\begin{aligned}
\frac{du_{1,ij}}{dt} &= -ku_{1,ij}\left(u_{1,ij} - a\right)\left(u_{1,ij} - 1\right) - u_{1,ij}v_{1,ij} + k_0\rho\left(\phi_{1,ij}\right)u_{1,ij} \\
&\quad + D_u\left(u_{1,i+1j} + u_{1,i-1j} + u_{1,ij+1} + u_{1,ij-1} - 4u_{1,ij}\right) \\
&\quad + D_l\left(u_{2,ij} - u_{1,ij}\right) + A\cos(\omega t)\ \delta_{i\alpha}\delta_{j\beta} \\
\frac{dv_{1,ij}}{dt} &= \left(\varepsilon_1 + \frac{v_{1,ij}\mu_1}{u_{1,ij} + \mu_2}\right)\left(-v_{1,ij} - ku_{1,ij}\left(u_{1,ij} - a - 1\right)\right) \\
\frac{d\phi_{1,ij}}{dt} &= k_1 u_{1,ij} - k_2\phi_{1,ij} \\
\rho\left(\phi_{1,ij}\right) &= \alpha + 3\beta{\phi_{1,ij}}^2
\end{aligned} \tag{2}$$

$$\begin{aligned}
\frac{du_{2,ij}}{dt} &= -ku_{2,ij}\left(u_{2,ij} - a\right)\left(u_{2,ij} - 1\right) - u_{2,ij}v_{2,ij} + k_0\rho\left(\phi_{ij}\right)u_{2,ij} \\
&\quad + D_u\left(u_{2,i+1j} + u_{2,i-1j} + u_{2,ij+1} + u_{2,ij-1} - 4u_{2,ij}\right) \\
&\quad + D_l\left(u_{1,ij} - u_{2,ij}\right) + A\cos(\omega t)\delta_{i\alpha}\delta_{j\beta} \\
\frac{dv_{2,ij}}{dt} &= \left(\varepsilon_2 + \frac{v_{2,ij}\mu_1}{u_{2,ij} + \mu_2}\right)\left(-v_{2,ij} - ku_{2,ij}\left(u_{2,ij} - a - 1\right)\right) \\
\frac{d\phi_{2,ij}}{dt} &= k_1 u_{2,ij} - k_2\phi_{2,ij} \\
\rho\left(\phi_{2,ij}\right) &= \alpha + 3\beta{\phi_{2,ij}}^2
\end{aligned} \tag{3}$$

where the index 1, ij shows the location of neuron in row i and column j in the first layer and the index 2, ij shows the location of the neuron in row i and column j in the second layer. $D_l = 0.024$ shows the strength of inter-layer connection. A and ω are the amplitude and the frequency of the external periodic current. $\delta_{i\alpha} = 1$ for $i = \alpha$, $\delta_{i\alpha} = 0$ for $i \neq \alpha$; $\delta_{j\beta} = 1$ for $j = \beta$, $\delta_{j\beta} = 0$ for $j \neq \beta$. Here, $\alpha = 1 : 100$ and $\beta = 1$ so that the periodic force is applied to the left boundary of the two layers of the media.

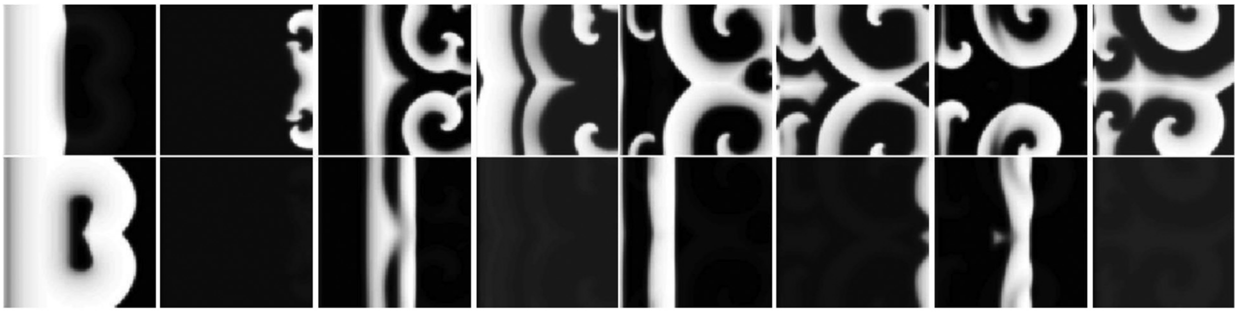

Fig. 1. The development of spatial pattern in the two-layer excitable media over the time for $\omega = 0.05$. The upper snapshots show the results for the upper layer of the media over the time, while the lower snapshots show the results for the lower layer of the media over the time.

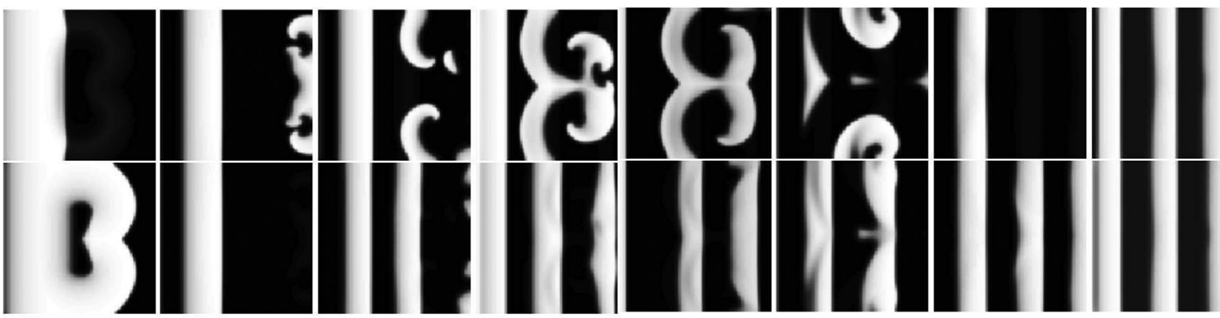

Fig. 2. The development of spatial pattern in the two-layer excitable media over the time for $\omega = 0.2$. The upper snapshots show the results for the upper layer of the media over the time, while the lower snapshots show the results for the lower layer of the media over the time.

3. Numerical analysis and discussion

In this part, the computational analysis and simulation of the excitable media with the co-existence of spiral waves and the successive plane waves is carried out. The results are shown in eight snapshots that show the development of the pattern of the two layers over the time.

The initial condition of all the neurons in the upper layer is $u = 0$, $v = 0$, $\phi = 0$. And the initial condition of the neurons in the lower layer is set as: $\mathrm{u}(30:70, 48:50) = 1$, $\mathrm{u}(30:70, 51:53) = 0.7$, $\mathrm{u}(30:70, 54:56) = 0$, $\mathrm{v}(30:70, 48:50) = 0$, $\mathrm{v}(30:70, 51:53) = 0.6$, $\mathrm{v}(30:70, 54:56) = 0.8$, $\phi(30:70, 48:50) = 0$, $\phi(30:70, 51:53) = 0.1$, $\phi(30:70, 54:56) = 0.2$ and $u = 0$, $v = 0$, $\phi = 0$ elsewhere. By this set of initial conditions in the lower layer, two spiral cores initiate in the middle of the media, starting to generate expanding circular wave fronts. Since the two layers are connected to each other, the circular wave front in the lower layer breaks the homogeneous state of the upper layer as well. Thereby, due to the level of excitability of the media and the strength of the inter-layer connection, multiple spiral cores initiate in the upper layer each of which rotates with a particular frequency. This leads to spiral-wave turbulence in the upper layer. In this study, in order to see the effect of successive plane waves on the spiral-wave turbulence that primarily exists in a media, a periodic external force is applied to the left boundary of the two layers of the media (after the generation of the spiral-wave turbulence). Four different angular frequencies (ω) are considered for this periodic force which results in successive plane waves with four different spatial frequencies.

For the first, $\omega = 0.05$ is taken (Fig. 1). In this case, the spatial frequency of the resulting plane waves is low. In the lower layer, the plane waves generated on the left boundary of the media, move to the right side of the media with very slight deformations. These deformations are because of the influence of the spatial-temporal pattern of the upper layer on the lower layer. In the upper layer, on the other hand, the traveling plane waves do not maintain in the media. In fact, the spiral turbulence existing in the upper layer gradually rules out the plane waves and eventually, covers the whole upper layer of the media (see Fig. 1).

Next, the spatial frequency of the successive plane waves is increased by increasing the angular frequency of the external current from $\omega = 0.05$ to $\omega = 0.2$. For this case, in the lower layer, the plane waves wove from the left side to the right side of the media with slight deformation (just like the previous case). However, based on our results, the higher-frequency plane waves can gradually overcome the spiral pattern, and therefore, the spiral turbulence existing in the upper layer gradually disappears very soon. Following that, the deformations in the plane waves become more and more sparse and finally, the striped pattern of the plane waves governs both upper and lower layer of the media (see Fig. 2). So, for this level of frequency, the spiral waves are well excluded and the dynamics of the two layers become synchronized as well.

For the next step, the frequency is set as $\omega = 0.7$ and the results are shown in Fig. 3. It can be observed that in this case, the resulting plane waves can eliminate the spiral pattern in a less time (compared to the case of $\omega = 0.2$). This is clear in the fourth snapshot of Fig. 3 in which the spiral cores on the right side of the media, where the primary spiral cores existed, are eliminated. However, with further pass of time, some spiral cores start to form on the left side of the media. The formation of these secondary cores is because the borders of the plane waves are very close to each other. In fact, when the wave front and the wave back get together, there is a high chance for the spiral cores to emerge [15]. Thereby, after some time the striped pattern of the plane waves is destructed by the secondary spiral waves growing from the left side of the media (see Fig. 3). So, for this level of frequency, the media still hosts spiral-wave turbulence which, in turn, is even stronger than the turbulence that primarily existed.

Finally, the frequency of the external periodic force is set as $\omega = 1$. For this case, the spatial frequency of the plane waves, generated on the left boundary, is very high. As described for the previous case, even though the primary spiral waves are excluded from the media, some secondary spiral cores find existence near

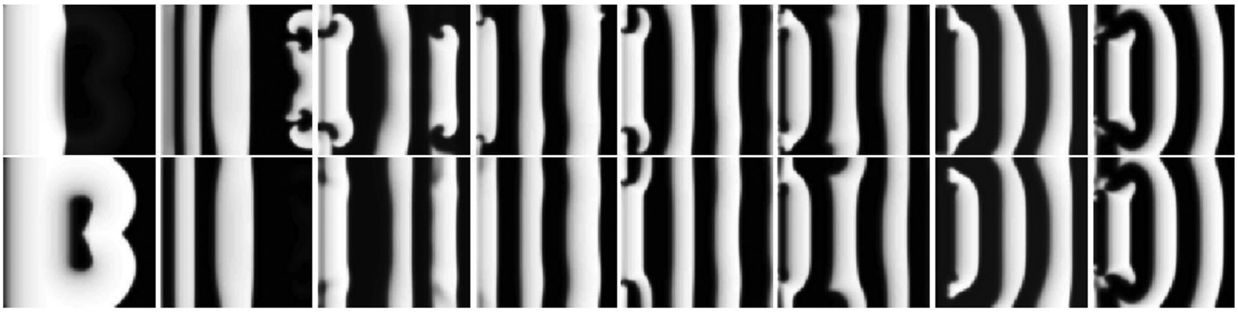

Fig. 3. The development of spatial pattern in the two-layer excitable media over the time for $\omega = 0.7$. The upper snapshots show the results for the upper layer of the media over the time, while the lower snapshots show the results for the lower layer of the media over the time.

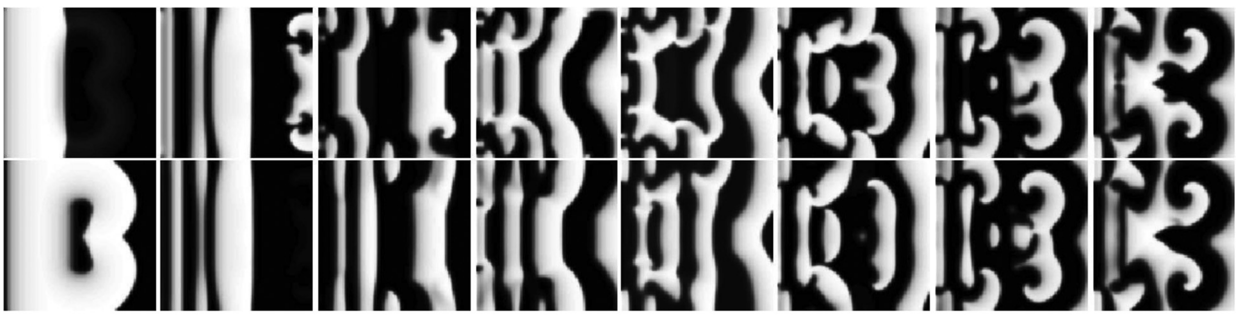

Fig. 4. The development of spatial pattern in the two-layer excitable media over the time for $\omega = 1$. The upper snapshots show the results for the upper layer of the media over the time, while the lower snapshots show the results for the lower layer of the media over the time.

the borders of the plane waves. Therefore, as is shown in Fig. 4, a complete chaotic pattern covers both upper and lower layer of the media. This spiral-wave turbulence is even stronger than the one that primarily existed in the media. Besides, the primary turbulence was only in the upper layer, but in this case, the spiral turbulence can be seen in both layers of the media (and they are actually synchronized with each other).

4. Conclusion

In this study, the effect of successive plane waves on the dynamics of the spiral-wave turbulence in a two-layer excitable media is investigated. The new three-dimensional magnetic Fitzhugh–Nagumo (FN) neuronal model is used for modeling of the excitable media and a periodic force was considered for generation of the successive plane waves with different spatial frequencies. The aim was to see whether the spiral-wave turbulence in a media can be eliminated by successive plane waves.

It is found that, for the case of low-spatial-frequency plane waves, the spiral waves could break the plane waves and gradually grow in the media and ultimately, govern the entire network. But when the spatial frequency increased, the spiral waves did not have the opportunity to grow and become stronger. The higher the spatial frequency of the plane waves, the more they were capable of overcoming the turbulent spiral wave. However, when the spatial frequency of the plane waves was too high, although it could guarantee the elimination of the spiral waves that initially existed in the media, there was a high chance that some secondary spiral cores form near the borders of the plane waves. This was because the borders were very close to each other. In fact, one way for the spiral wave to emerge in a media is that the wave front and the wave back of two successive plane waves get together.

Our study showed that the spatial frequency of the successive plane waves can greatly determine the dynamics of the spiral waves in an excitable media. Nevertheless, in the real-world distributed dynamical systems, the components are not rigidly connected. Therefore, for future studies, it is helpful to use time varying coupling strengths either for the inter-layer or the intra-layer links.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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