

Frontiers

Public goods games on random hyperbolic graphs with mixing

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ABSTRACT

Understanding the evolution of cooperation in structured populations remains one of the fundamental challenges of the 21st century, with far-reaching implications for the wellbeing of modern human societies. Studies over the past two decades showed that the structure of the network of contacts plays a crucial role in determining whether cooperation prevails or not. An important step to more realistic networks was made with the shift from regular grids and lattices to complex social networks at the turn of the century. Real networks exhibit a high mean local clustering coefficient, short average path lengths, and community structure. Recent studies have revealed that random geometric graphs in hyperbolic spaces exhibit properties that are frequently found in real networks. We here study the public goods game on random geometric graphs in hyperbolic spaces, and we consider assortative and disassortative mixing with different frequencies. We show that in hyperbolic spaces heterogeneous networks promote the evolution of public cooperation in comparison to the more homogeneous networks. We also confirm that assortative and disassortative mixing on random hyperbolic networks both impair the evolutionary success of cooperators, regardless of the network architecture. The differences between the two mixing protocols are most expressed at low mixing frequencies, whilst at high mixing frequencies the two almost converge.

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1. Introduction

Understanding the emergence and survival of cooperative behaviour among selfish individuals in diverse environments is one of the major challenges that continues to attract researchers from diverse fields including sociology, economics, and biology [1]. Evolutionary game theory provides a powerful theoretical framework for studying the evolution of cooperation, using models based on social dilemmas, where individual best interests are at odds with what is best for what societies as a whole [1–4].

It is well known that each individual in a population does not interact with everybody else, but with a small group in their neighborhood. Therefore, populations are structured and individuals interact repeatedly mostly with the same individuals. Almost three decades ago, Nowak and May introduced this idea in their seminal paper [5] and showed that spatial structures generally promote the evolution of cooperation [6–8]. The so-called network reciprocity relies on pattern formation in a structured population,

which provides that cooperators can survive [9,10]. Inspired by this work, the impact of different spatial structures on the evolution of cooperation has been intensively studied during recent years [11,12]. In the simplest case, structured interactions among particles are described by a square lattice [5,11,13]. Another important step towards more realistic representation of evolutionary games was the shift to evolutionary games on complex networks [14–17], examples of which include scale-free [18–30], small-world [31–37], hierarchical [38,39], real empirical social networks [40–42], dynamical coevolving [43–46] as well as multilayer and interdependent networks [47–54].

In the past couple of years, much effort has been invested into the development of new theoretical models that are able to describe the most important structural properties of real-world networks. Namely, several real-world networks share some common structural properties like scale-free degree distributions, community structure and strong clustering. Recent studies have shown that a high clustering coefficient implies the existence of an underlying geometry [55,56]. Krioukov et al. [57] called these spaces hidden metric spaces. Specifically, real heterogeneous networks can be modelled as random geometric graphs in hyperbolic rather than

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Euclidean spaces, where node coordinates abstract the popularity and similarity of nodes [58–63]. Zuev et al. [64] showed, that hyperbolic network geometry coupled with preferential attachment of nodes to this geometry induces scale-free, strongly clustered growing networks with emergent soft community structure. These inherent similarities with real-world networks provide a promising groundwork for collective social behaviour research.

Only few studies, however, investigated the evolution of cooperation in terms of different evolutionary games on hyperbolic networks [56,65–68]. For example, Kleineberg [56] studied the influence of metric clusters on the underlying hyperbolic metric spaces based on the two-player prisoner’s dilemma game and showed, that these metric clusters can be more efficient than the most connected nodes (hubs) at sustaining cooperation and that heterogeneity does not always favor cooperation in the prisoner’s dilemma. These studies focused on games on static networks where individuals can update their strategies, nonetheless, without the possibility to change their location. In most real-world situations, however, individuals can move. Inspired by the work of Vainstein et al. [69], where the effect of dilution and mobility were studied in the weak form of the prisoners dilemma game, several studies have paid attention to the impact of mobility on the evolution of cooperation. Perc et al. [7] distinguished two possible scenarios how mobility can be implemented in social dilemmas. In the first scenario the movement of players is independent of the evolutionary dynamics and the movements correspond to a random walk [70–74], while in the second scenario the mobility is driven by the evolutionary dynamics [75–81]. In previous works, different mobility scenarios have been explored, in which the mobility can be strategy-dependent [76,82,83] or driven by payoff [77,78], success [75], or reputation [80].

Motivated by the above mentioned observations, in this work we implement random geometric graphs in hyperbolic spaces to study the evolution of cooperation in terms of multi-players public goods game in which players exchange places in an assortative or a disassortative manner. First, following the geometric preferential attachment model [64], we will generate different network architectures embedded into hyperbolic space. By this means, networks of various levels of heterogeneity, efficiency and clustering can be generated with this procedure. The resulting networks will be than used for studying the evolution of cooperation of the public goods game. As already shown in the seminal study by Santos et al. [27], we now likewise confirm that heterogeneous networks promote the evolution of public cooperation in comparison to the more homogeneous networks also in hyperbolic spaces. We will also investigate the effect of assortative and disassortative mixing, by introducing a mixing process performed after each m -th Monte Carlo step, and quantify precisely the differences between the two mixing protocols on different network architectures.

The rest of the paper is organized as follows. In the next section we present the procedure for the construction of different network architectures embedded into hyperbolic space, the public goods game and the mixing protocols. We then proceed with the presentation of the main results and finally present our conclusions.

2. Public goods game on random hyperbolic graphs with mixing

We use random hyperbolic graphs to model interactions between individual players. These types of networks have been well analyzed theoretically and have been shown to exhibit features of realistic networks, such as a heavy tailed degree distribution, small diameter, and strong clustering [56,59,84,85]. Specifically, we utilize the geometric preferential attachment model, where each node i is mapped into the hyperbolic disc and is represented by the

polar coordinates r_i and θ_i , which attribute to the popularity and similarity of nodes [64]. We start with $n = 2$ interconnected nodes and at each further time step, a new node is generated. To each new node, polar coordinates are randomly assigned:

$$\theta_i = 2\pi u_1, \tag{1}$$

$$r_i = \frac{1}{\alpha} \cos^{-1} [1 + \cosh(\alpha R_{hd} - 1)u_2], \tag{2}$$

where R_{hd} is the radius of the hyperbolic disc, α is the internal growth parameter, and u_1 and u_2 are independent random variables sampled from the uniform distribution on a unit interval. The new i -th node connects then to n existing nodes with a probability proportional to the hyperbolic distance d_{ij} between the i -th and the existing j -th node:

$$d_{ij} = \cosh^{-1}[\cosh(r_i) \cosh(r_j) - \sinh(r_i) \sinh(r_j) \cos(\Delta\theta_{ij})], \tag{3}$$

where $\Delta\theta_{ij} = \pi - |\pi - |\theta_i - \theta_j||$ is the angular distance. Note that the hyperbolic the space curvature parameter is set to 1. Network structures for three selected values of α (0.1, 0.17 and 0.95) are visualized in Fig. 1(a) and in (b) the corresponding degree distributions are presented. It can be observed that small values of α lead to very heterogeneous networks, while $\alpha = 0.95$ yields a more homogeneous architecture with an exponential degree distribution. Figs. 1(c)-(e) feature the average clustering coefficient $\langle C \rangle$, the average shortest path $\langle L \rangle$, and the assortativity coefficient r as a function of α . Evidently, with increasing α , $\langle C \rangle$ slightly decreases, whereas the average shortest path increases. Moreover, the network structure switches from a disassortative to an assortative organization at $\alpha \approx 0.125$. Apparently, by modifying the parameter α , networks of various levels of heterogeneity, efficiency and clustering are generated, thereby enabling exploring a broad class of networks that are representative for real-life networks [56,64].

The resulting networks are used for the classical public goods game, where nodes represent individual players, whereas the edges characterize the interactive relationships between them. Initially, each node is being appointed as a defector (D) or cooperator (C) with equal probability. Each player i has k_i direct neighbors and participates in $G = k_i + 1$ overlapping groups. Cooperators contribute a fixed cost ($s_i = 1$) to the common pool of each group, while defectors contribute nothing ($s_i = 0$). The total contributions are multiplied by the synergy factor $R > 1$ and the resulting amount is equally divided among the $k_i + 1$ group members. The payoff of a player x in one group g , where N_C^g players cooperate, is given by:

$$P_i^g = \frac{RN_C^g}{k_i + 1} - s_i, \tag{4}$$

respectively. The overall payoff P_i of player i from all the G groups is the sum: $P_i = \sum_g P_i^g$. The number of direct neighbors k_i varies between individuals. To ensure a relevant comparison of the results, the multiplication factor R is therefore normalized with the size of the corresponding group [27]. Employing the Monte Carlo simulation procedure, each elementary step involves randomly selecting one player i and one of its direct neighbors j . After calculating their payoffs P_i and P_j as described above, we consider the traditional strategy-updating rule where the player j adopts the strategy of player i with a probability determined by the Fermi function

$$W(s_i \rightarrow s_j) = \frac{1}{1 + \exp[(P_j - P_i)/K]}, \tag{5}$$

where K quantifies the uncertainty by strategy adoptions and is set to 0.5 without loss of generality [86].

To incorporate mixing of players, a mixing process is performed after each m -th Monte Carlo step. More specifically, within one

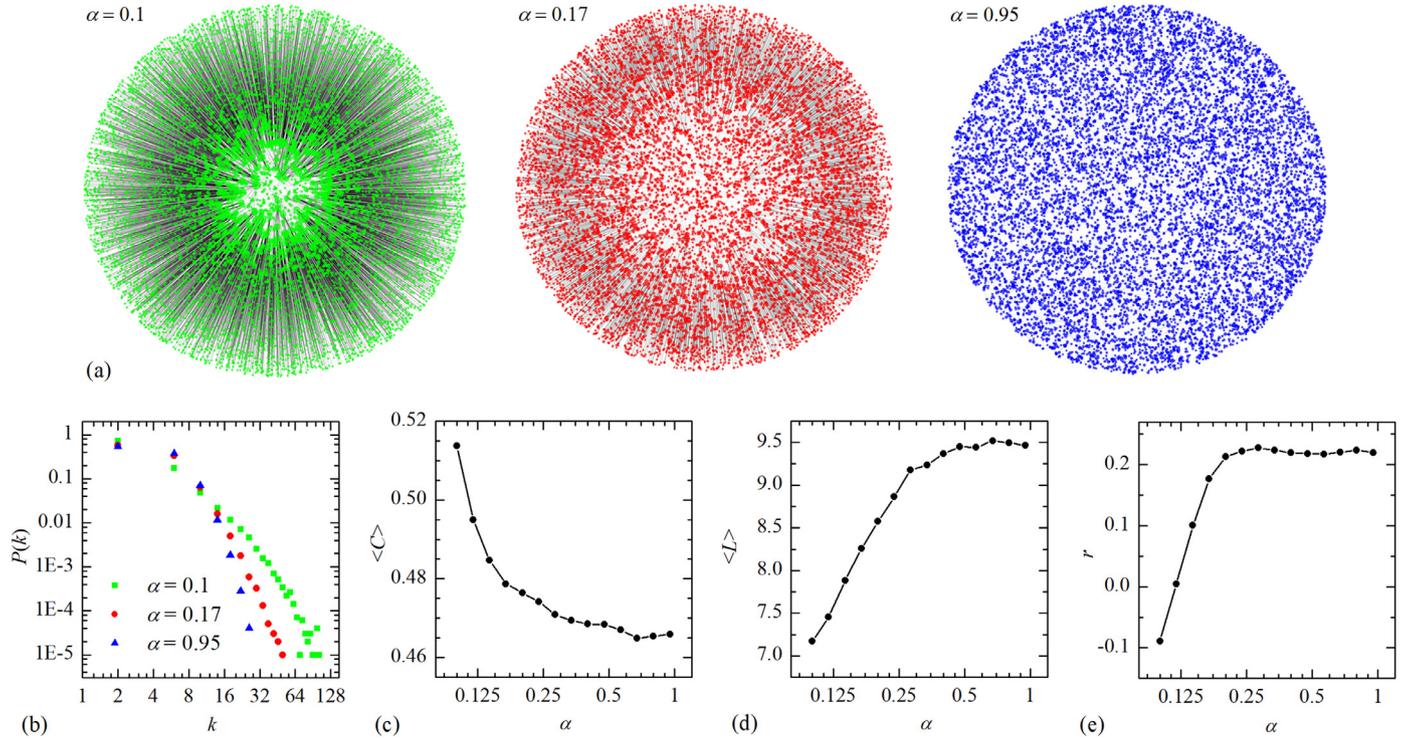


Fig. 1. (a) Modelled network architectures embedded into hyperbolic space for three different values of the growth parameter α (0.1, green; 0.17 red; 0.95 blue). Sizes of nodes are proportional to their degrees. Network size was $N = 10000$. (b) Degree distributions for three different values of α . (c) The average clustering coefficient as a function of α . (d) The average shortest path as a function of α . (e) The assortativity coefficient as a function of α . (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

mixing step each player x has a chance to exchange his site with the selected player y once on average. For example, if the mixing frequency is $m = 16$, N randomly selected pairs are switched after every 16th full Monte Carlo step. We consider two mixing protocols: assortative and disassortative mixing. In the former, places are exchanged among players with similar degrees, while the later refers to switching of positions of players with very different numbers of connections. To that purpose, nodes are ranked in accordance with their degrees. In each mixing step, the x -th player with rank RD_x exchanges the place with the y -th player with rank RD_y , as specified by the type of mixing. In the scenario of assortative mixing, rank RD_y is randomly selected from the interval $[RD_x - \delta N, RD_x + \delta N]$, whereas in the disassortative mixing protocol, rank RD_y is randomly selected from the interval $[N - RD_x - \delta N, N - RD_x + \delta N]$. Parameter $\delta \in [\frac{1}{N}, 0.5]$ specifies the dispersion of the selection. In our calculations we set $\delta = 0.025$, unless stated otherwise. In this manner, the rank of the selected y -th node RD_y is in the protocol of assortative mixing at most 5% different from the rank of the x -th node RD_x . Importantly, if rank $RD_x < \delta N$ or $RD_x > N - \delta N$, the interval for the selection of the y -th player is properly adjusted, so that the width of the selection is always $2\delta N$. For example, if the rank of x -th player is $RD_x = 1$, then the ranks of the y -th node are drawn from the intervals $[1 + \frac{1}{N}, 2\delta N]$ or $[N - \frac{1}{N}, N - 2\delta N]$ for the cases of assortative and disassortative mixing, respectively.

In all our calculations we have used a network with size $N = 10000$ and each node connected to $n = 2$ existing nodes, which yielded a network with an average degree 4. The equilibrium fraction of cooperators has been determined by averaging the last 10000 generations after a transient period of 10000 Monte Carlo time steps. Furthermore, the final results are averaged over 10 to 100 independent runs for each set of parameter values.

3. Results

Let us first consider the public goods game on random hyperbolic networks generated with the model described in Section II above. Panels in Fig. 2 show the evolution of cooperation in the public goods game as obtained on random hyperbolic networks generated with different values of the internal growth parameter α . The colour map presented in the left panel encodes the fraction of cooperators f_C in dependence on the internal growth parameter α and the multiplication factor R/G . Additionally, the right panel shows the fraction of cooperators in dependence on the group size normalized value of the multiplication factor R/G , as obtained for eight different values of the internal growth parameter α . As already mentioned in Section II and shown in Fig. 1, small values of α (e.g. $\alpha = 0.1$) lead to very heterogeneous networks, while at larger values (e.g. $\alpha = 0.95$) more homogeneous networks with an exponential degree distribution are established. In Fig. 2, it can be observed that heterogeneous networks promote the evolution of public cooperation, i.e., the larger the internal growth parameter, the larger values of the normalized multiplication factor R/G are required for cooperators to survive. Particularly, for $\alpha = 0.1$ cooperators emerge above $R/G = 0.31$, while for $\alpha = 0.95$ cooperators survive extinction for $R/G > 0.48$.

In the next step we study the public goods game on different random hyperbolic networks with two mixing protocols and different mixing frequencies, expressed as every how many m full Monte Carlo steps the players are mixed. We consider three different network architectures characterized with different internal growth parameter values α ($\alpha = 0.1$ in the first row, $\alpha = 0.17$ in the second row, and $\alpha = 0.95$ in the third row) and two mixing protocols: assortative and disassortative mixing, as described in Section II above. In the former, a randomly selected player exchanges his site with a player with similar degree (for example

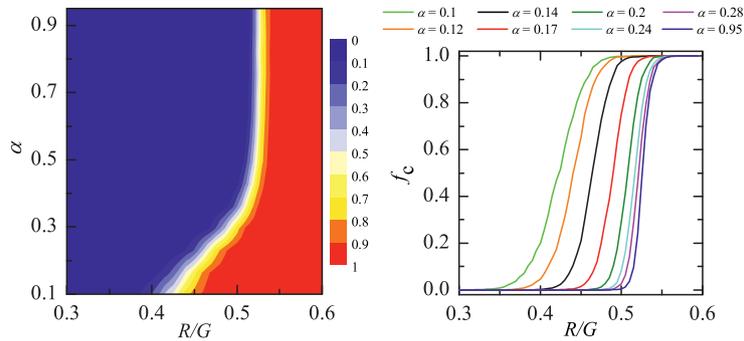


Fig. 2. The evolution of public cooperation on random hyperbolic graphs depends on the value of the internal growth parameter α . Heterogeneous networks, established with small values of α , promote the evolution of public cooperation in comparison to the more homogeneous networks. In panel (a) the fraction of cooperators f_c in dependence on the internal growth parameter α and the normalized multiplication factor R/G is depicted. The fractions of cooperators f_c are color-coded as specified by the color bar. In panel (b) the fractions of cooperators f_c in dependence on the normalized multiplication factor R/G for eight different values of the internal growth parameter α are shown, where α increases from left to right (see also the legend). The size of each network is $N = 10000$, and the average degree is 4. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

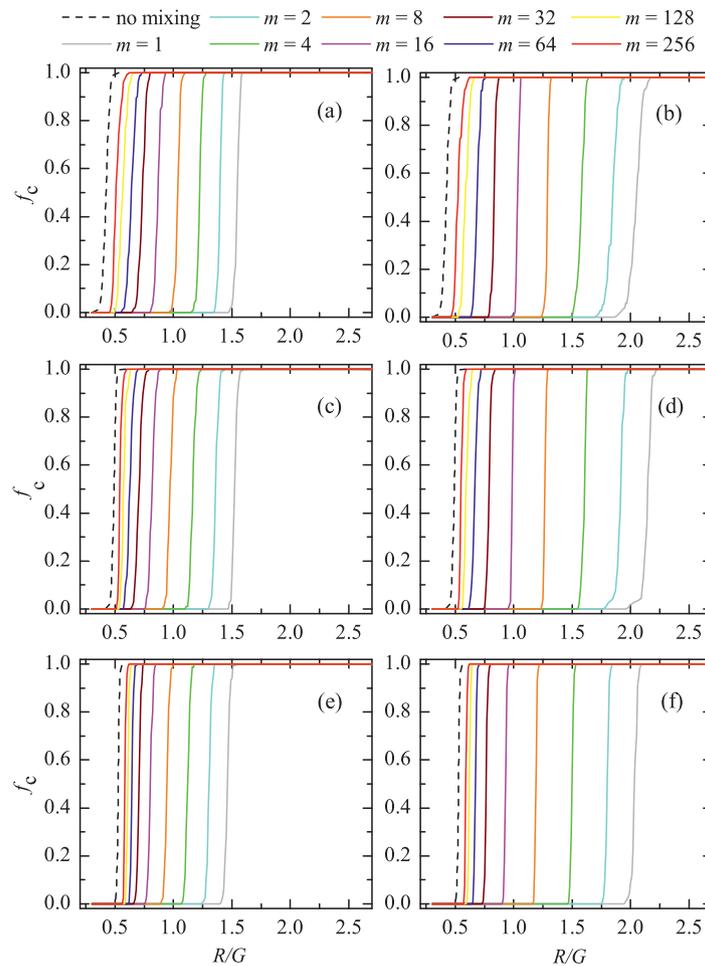


Fig. 3. Assortative and disassortative mixing on random hyperbolic networks both impair the evolutionary success of cooperators. Furthermore, disassortative mixing is more detrimental for cooperation regardless of the network architectures. Depicted is the fraction of cooperators f_c in dependence on the normalized multiplication factor R/G , as obtained on random hyperbolic graphs for three different values of the internal growth parameter α : $\alpha = 0.1$ (a) and (b), $\alpha = 0.17$ (c) and (d), and $\alpha = 0.95$ (e) and (f). The left three panels (a), (c) and (e) show results for assortative mixing, while the right three panels (b), (d) and (f) for disassortative mixing. In all panels results for nine different mixing frequency m (solid lines) are presented, where m increases from right to left (see also the legend). The dashed black line shows the corresponding result for the public goods game on a random hyperbolic graph without mixing. In all cases we used a network with $N = 10000$ nodes and an average degree 4.

hubs with hubs), while in the later, the player switches the position with a player with different degree (for example hubs with low-degree nodes). Panels in Fig. 3 show the results for assortative and disassortative mixing on the left and right, respectively. It can be observed that both types of mixing impair the evolutionary suc-

cess of cooperators, i.e., that larger values of R/G are required for cooperators to survive, and the more so the smaller the value of m . Large mixing frequencies m hardly evoke a visible difference in the fraction of cooperators compared with the case, where no mixing is applied (the dashed black line in the panels). By comparing

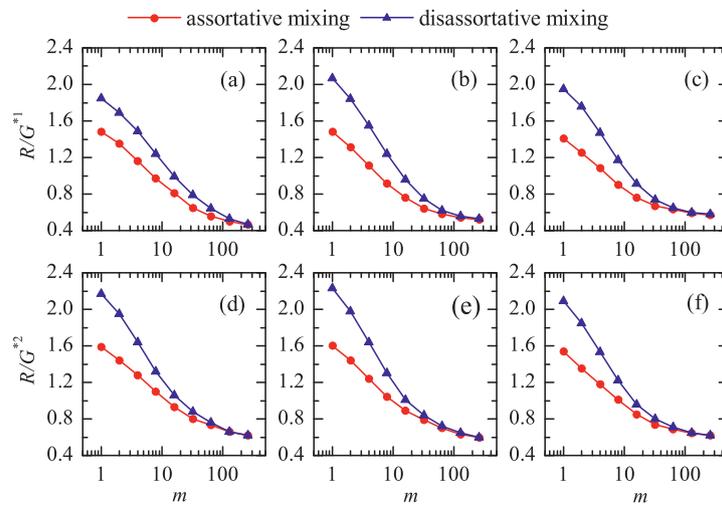


Fig. 4. A more specifically presentation of the differences between the effects of assortative and disassortative mixing on different network architectures shows noticeable differences between the assortative and disassortative mixing for $m \rightarrow 1$, which are most expressed in the heterogeneous network architecture ($\alpha = 0.17$). Depicted is the critical value of the normalized multiplication factor R/G as a function of the mixing frequency m , as obtained on three different random hyperbolic graphs defined with different values of the internal growth parameter α : $\alpha = 0.1$ (a) and (d), $\alpha = 0.17$ (b) and (e), and $\alpha = 0.95$ (c) and (f). In all panels both mixing protocols: assortative (red line with circles) and disassortative mixing (blue line with triangles) are presented. Panels in the first row (a), (b) and (c) show results for the critical value of the normalized multiplication factor R/G^{*1} , above which cooperators emerge. Panels in the second row (d), (e) and (f) show results for the critical value of the normalized multiplication factor R/G^{*2} , above which cooperators dominate. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

results in left panels with the results in the right panels in Fig. 3, we can observe that disassortative mixing is more detrimental for cooperation regardless of the network architectures, which is more expressed for smaller values of the mixing frequency m .

To quantify the differences between the effects of assortative and disassortative mixing on different network architectures more accurately, we show in Fig. 4 the critical value of enhancement factor R/G^{*1} above which cooperators emerge (panels in the first row) and R/G^{*2} above which cooperators dominate (panels in the second row) for both mixing protocols as a function of the mixing frequency m . With the increment of m , both critical values of enhancement factor R/G^{*1} and R/G^{*2} decrease, and for $m \rightarrow \infty$ the two mixing types almost converge. Particularly, for $m \rightarrow 1$ the differences between the assortative and disassortative mixing become notable and are the most expressed in the heterogeneous network architecture characterized with the internal growth parameter $\alpha = 0.17$. In fact, assortative mixing on more heterogeneous network architectures is slightly more detrimental for cooperation in comparison with homogeneous network, as can be seen by comparing results in panels in the same row. For $m \rightarrow 1$, both critical values of enhancement factor R/G^{*1} and R/G^{*2} are slightly higher in more heterogeneous network structures compared with the homogeneous network structure ($\alpha = 0.95$). The same effect can be seen for disassortative mixing but only in bottom panels (d)-(f) where the critical values of enhancement factor R/G^{*2} are depicted.

4. Discussion

Various investigations over the last two decades have shown that particular properties, such as heterogeneous degree distribution, small characteristic path lengths, strong clustering, and community structure, are universally associated with social networks [87,88]. Those real social networks can be appropriately modeled as random geometric graphs in hyperbolic spaces [59,60]. While the evolution of cooperation in terms of different evolutionary games on hyperbolic networks has been studied on static networks, to the best of our knowledge, the impact of different types of mixing in such systems has not been addressed to date.

In this paper, we have investigated the evolution of cooperation in terms of multi-players public goods game on a random hyperbolic network, where players are allowed to switch their positions with other players. We have first done a benchmark analysis without mixing, where different random hyperbolic network structures has been generated by varying the internal growth parameter α . We have observed that networks of different levels of heterogeneity, efficiency and clustering can be generated with this procedure. Furthermore, heterogeneous networks promote the evolution of public cooperation in comparison to the more homogeneous networks. In the next step, a mixing process, where N selected pairs are switched, was performed after each m -th Monte Carlo step. In particular, assortative and disassortative mixing were considered, both with varying frequencies. The effects of degree-mixing patterns on the evolution of cooperation in the prisoner's dilemma game on scale-free networks have been studied in a seminal paper by Rong et al. [21]. They have shown that assortative mixing impairs cooperation and promotes the invasion of defectors, while disassortative mixing promotes cooperation due to the isolation among hubs. In contrast to this previous research, our investigation shows that assortative and disassortative mixing on random hyperbolic networks both impair the evolutionary success of cooperators. Particularly, the smaller the value of m (the mixing is applied very frequently), more significant differences can be observed between the case without mixing and different mixing protocols. Furthermore, disassortative mixing is more detrimental for cooperation regardless of the network architectures.

Random geometric graphs in hyperbolic spaces have revealed new paths to better understanding cooperation in structured populations, as well as opening up new questions that remain to be answered. We hope this work will inspire future research along similar lines, for example to explore different social dilemmas and multigames on random geometric graphs in hyperbolic spaces.

Declaration of Competing Interest

Matjaz Perc is Editor of Chaos, Solitons & Fractals. In keeping with Elsevier's guidelines on potential editorial conflicts of interest, manuscripts coauthored by one of the Editors will be handled fully by other Editors or the Editor-in-Chief in an undisclosed

review process. The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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