



# Chaos in temporarily destabilized regular systems with the slow passage effect

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## Abstract

We provide evidences for chaotic behaviour in temporarily destabilized regular systems. In particular, we focus on time-continuous systems with the slow passage effect. The extreme sensitivity of the slow passage phase enables the existence of long chaotic transients induced by random pulsatile perturbations, thereby evoking chaotic behaviour in an initially regular system. We confirm the chaotic behaviour of the temporarily destabilized system by calculating the largest Lyapunov exponent. Moreover, we show that the newly obtained unstable periodic orbits can be easily controlled with conventional chaos control techniques, thereby guaranteeing a rich diversity of accessible dynamical states that is usually expected only in intrinsically chaotic systems. Additionally, we discuss the biological importance of presented results.

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## 1. Introduction

Since the introduction of chaos control algorithms [1–3], chaotic behaviour has been recognized as a desirable phenomenon in several research fields [4–6]. In living organisms, chaotic signature in the dynamics of vital functions might be used to distinguish between health and disease [7–9], while in artificial systems, chaos can be exploited to assure optimal functioning of devices in given circumstances [10–12], encode secret messages [13–15], stabilize nonlinear communication schemes [16,17], or explain the physical mechanism of extended-range atmospheric prediction in geophysical systems [18,19]. This overwhelming applicability of chaos control techniques has motivated several studies on how to make intrinsically regular systems chaotic [9,20–35]. Algorithms were first developed for discrete-time systems [9,20–22], while afterwards also continuous-time systems become available for chaotification [24–28,30]. However, chaotifying an arbitrary continuous-time system is more difficult than inducing chaos in a discrete map. Recently, a semiglobal technique for polynomial continuous-time systems and rational forms was introduced [35].

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In the present paper, we extend the existing theory of chaotifying continuous-time systems by introducing a simple chaotification method that is based on temporal destabilizations of regular systems. However, rather than concentrating on developing a mathematically rigorous algorithm, we focus on specific properties of regular continuous-time systems, which may facilitate the desired task. In particular, we focus on systems with the slow passage effect [36–44]. Several authors report that dynamical systems with the slow passage effect are extremely sensitive to small perturbations [38,42,44] and even to precision of numerical algorithms [37]. Thus, it is natural to consider such properties as possible sources of complex unpredictable behaviour in regular systems. We demonstrate how to exploit this hidden system instability to evoke chaotic behaviour in an otherwise regular system.

Furthermore, we introduce a new algorithm for calculating the largest Lyapunov exponent of the system. In particular, we modify the algorithm developed by Wolf et al. [45], which can be used to determine the largest Lyapunov exponent from a time series, i.e. without using the differential equations. We show that the temporarily destabilized system has a positive largest Lyapunov exponent, which is a strong indicator for chaotic behaviour, hence confirming the successfulness of chaotification in the studied system. Moreover, we show that the unstable periodic orbits embedded in the newly obtained chaotic attractor can be successfully controlled with conventional chaos control techniques [2,46,47], thereby additionally justifying our approach, as well as indicating the possibility of applying chaos control techniques also to originally regular systems.

The paper is structured as follows. Section 2 is devoted to the bifurcation analysis of the slow passage phase in the examined mathematical model. In Section 3 we present the chaotification algorithm and calculate the largest Lyapunov exponent of the resulting attractor, whereas in Section 4 the control of newly obtained periodic orbits is presented. In the last section we summarize the results and outline the biological importance of our findings. The examined mathematical model with the complete set of model equations and parameter values is given in [Appendix A](#).

## 2. Slow passage effect

The slow passage effect is characterized by a slow transition of the system through a Hopf bifurcation [36–44]. Because the passage through the bifurcation point is slow a delayed transition of the system from an unstable foci branch to a stable steady state or periodic solution occurs. Thus, the system stays close to the unstable foci branch for a considerable amount of time, before it eventually shifts to the stable solution. Since during the slow passage phase the system wanders on an unstable solution, it is at that time very susceptible to any external perturbations, ranging from environmental changes and random inputs [38], signals from neighbouring oscillators [42,44], or even precision of numerical algorithms [37]. Therefore, we argue that systems with the slow passage effect possess a hidden intrinsic instability, which might be exploited for chaotifying an initially regular system.

To study the slow passage effect in the examined mathematical model [48] (see [Appendix A](#)) in more detail, we apply the fast–slow subsystem bifurcation analysis, which was originally proposed by Rinzel [49]. The virtue of the fast–slow subsystem method is to extract the fast changing variables of the system and then use the slow changing variables as bifurcation parameters. The fast changing variables in the examined model were identified to be  $x(t)$  and  $y(t)$ , whereas the slow changing variable is  $z(t)$ . Hence, we can reduce the 3D system  $(x(t), y(t), z(t))$  to a 2D system  $(x(t), y(t))$  and use the variable  $z(t)$  as the bifurcation parameter. Results presented in [Fig. 1](#) were obtained with the software package AUTO97 [50]. It can be well observed that the trajectory stays close to the unstable foci branch well after the Hopf bifurcation is exceeded in the clockwise direction. Only slowly, the trajectory starts to diverge from the unstable foci branch, to eventually end up on the lower stable periodic branch. Thus, for the set of system parameters given in the [Appendix](#), the examined mathematical model expresses regular oscillatory behaviour with a well-expressed slow passage effect, which qualifies the system for further analyses.

## 3. Chaotification

In order to induce chaotic behaviour in a continuous-time system with the slow passage effect, we must find a way to exploit the hidden instability that is intrinsically incorporated in the dynamics. The most natural and simple way of doing this is to temporarily destabilize the system with an external perturbation. Since the slow passage phase is very susceptible to external perturbations [38], the induced short-term destabilization will have a long lasting impact on the temporal evolution of the system. Note that during the slow passage phase even a small deviation of the trajectory from the original limit cycle may greatly affect the system's dynamics at a latter time [37,38,42,44]. By repeating the destabilization procedure several times, we obtain an enriched dynamics made up of unstable transients, which combined yield chaotic behaviour in the originally regular system.

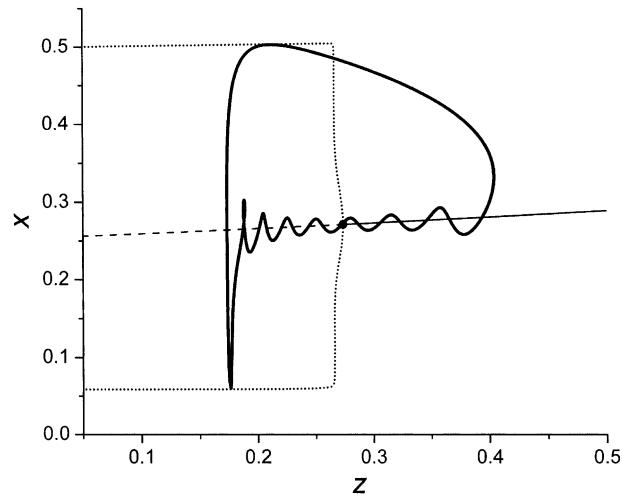


Fig. 1. Fast–slow subsystem analysis of the studied system. Presented is the bifurcation diagram of the fast variable  $x$  in dependence on the slow variable  $z$ , which was used as the bifurcation parameter. Thick solid line depicts the 2D projection of the limit cycle attractor in the phase space. Thin solid (dashed) line represents stable (unstable) foci, while the dotted lines originating from the Hopf bifurcation (circle) represent stable periodic solutions.

To temporarily destabilize the system, we introduce a random symmetric impulsive input function as an additional term to one of the fast variables of the system. In our case, Eq. (A.1) obtains an additional term of the form

$$f(t) = \begin{cases} a & \text{if } (t \bmod h) \geq (h - d), \\ -a & \text{if } ((t - d) \bmod h) \geq (h - d), \\ 0 & \text{else,} \end{cases} \quad (1)$$

where  $a$  is the amplitude,  $d$  the duration, and  $h$  the time interval between consecutive destabilizations. Parameters  $a$  and  $d$  determine the strength of destabilizations and are selected randomly from intervals with defined maximum values. Both maximums have to be determined carefully with respect to the system dynamics, i.e. fluxes that govern the temporal evolution of the unperturbed dynamics. It should be noted that for each randomly selected  $d$  the amplitude  $a$  is held constant. The time interval between consecutive destabilizations  $h$  is also variable, but not selected randomly. In fact,  $h$  equals the time the system needs to recover from each destabilization. Thus, each time the system is about to re-settle onto the limit cycle attractor, the next perturbation takes place. Since, however, the system recovers always at different moments, the temporal destabilizations are random in their strength as well as in time at which they are initiated. Thereby, we are able to fully exploit the extreme sensitivity of the slow passage phase, and thus exploit the system’s hidden instability for chaotification. Note also that  $f(t)$  is a symmetric function, which assures that the system is, after the destabilization and before completely re-settling onto the limit cycle, determined by the autonomous, i.e. unperturbed dynamical system. Noteworthy, we previously applied a similar method to enhance the number of controllable states in an intrinsically chaotic attractor [51].

The results obtained with the proposed method are presented in Fig. 2. We emphasize that the attractor presented in Fig. 2 is made up solely of autonomous system transients. Thus, those parts of the attractor that were directly subjected to destabilizations, i.e. when  $f(t) \neq 0$ , were discarded. It can be well observed that temporal destabilizations indeed enrich the system’s dynamics, yielding an apparently chaotic attractor.

To verify if the attractor presented in Fig. 2 is indeed a chaotic attractor, we calculate the largest Lyapunov exponent pertaining to the newly obtained attractor with the modified algorithm originally developed by Wolf et al. [45]. The original algorithm can be briefly summarized as follows. Find two points of the attractor which fulfil  $\|\mathbf{p}(t) - \mathbf{p}(g)\| = \varepsilon \ll \zeta$ , where  $\mathbf{p}(t) = (x(t), y(t), z(t))$  is a point at time  $t$ ,  $\mathbf{p}(g)$  is a point at time  $g$ , and  $\varepsilon$  is the Euclidean distance between them that should be much smaller than the extend of the attractor  $\zeta$ . Then iterate both points forward in time for a fixed evolution time  $v$  and calculate the distance between them, i.e.  $\|\mathbf{p}(t + v) - \mathbf{p}(g + v)\| = \varepsilon_v$ . If the system is chaotic, the distance after the evolved time will typically be larger than the initial  $\varepsilon$ , while in case of regular behaviour  $\varepsilon \equiv \varepsilon_v$ . After each evolution time  $v$  a so-called replacement step is attempted in which we look for a new point  $\mathbf{p}(k)$  of the attractor, whose distance to the evolved point  $\mathbf{p}(t + v)$  should equal  $\varepsilon$ , under the constraint that the angular separation

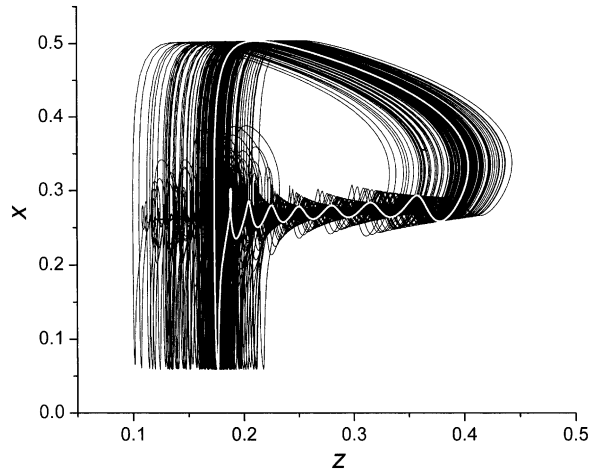


Fig. 2. Impact of temporal destabilizations on the studied system. Presented are 50 autonomous transients and the original limit cycle attractor, which is depicted with white. For both  $a$  and  $d$  used in Eq. (1), values were selected randomly from the unit interval  $[0, 1]$ . In accordance with more or less strong temporal destabilizations  $h$  varied from 15 s (16.7 s is the oscillatory period of the regular system) to maximally 80 s.

between the phase space vectors constituted by the pairs  $(\mathbf{p}(t + v), \mathbf{p}(g + v))$  and  $(\mathbf{p}(t + v), \mathbf{p}(k))$  is small. This procedure is repeated until the initial point of the attractor, i.e.  $\mathbf{p}(t)$  at  $t = 0$  s, reaches the last one. Finally, the largest Lyapunov exponent  $\Lambda$  can be calculated according to the equation

$$\Lambda = \frac{1}{Mv} \sum_{j=0}^M \ln \frac{\varepsilon_j}{\varepsilon}, \quad (2)$$

where  $M$  is the total number of replacement steps, and thus  $Mv$  equals the sum of all different  $h$  used in Eq. (1).

In order to calculate the largest Lyapunov exponent of our temporarily destabilized system we have to modify the above-described algorithm, since the attractor is made up of 50 autonomous transients that are not directly connected with each other. Note that we have discarded those parts of the attractor where  $f(t) \neq 0$ , which induces a discontinuity for each attempted destabilization. Hence, each time the above algorithm encounters a point  $\mathbf{p}(n)$  at time  $n$  such that  $[h - (n \bmod h)] < v$  an error would be introduced since the trajectory would go through a discontinuity during  $v$ , and thus  $\varepsilon_j$  would essentially be overestimated. To correct this, we exploit the fact that an arbitrarily small neighbourhood of every point that forms the chaotic attractor is repeatedly visited by the trajectory during the temporal evolution of the system. Hence, each time the trajectory is about to go through a discontinuity, i.e.  $(n \bmod h) = 0$ , a new search routine is initiated where we search for a close neighbour  $\mathbf{p}(m)$ , which fulfils  $\|\mathbf{p}(n) - \mathbf{p}(m)\| = \delta$ , where  $\delta$  should be smaller than  $\varepsilon$  in Eq. (2), and  $\mathbf{p}(m)$  is not the  $n + 1$ st point in time. Henceforth,  $\mathbf{p}(m)$  replaces  $\mathbf{p}(n)$  and the discontinuity is thereby avoided. Since the dynamics of chaotic attractors is ergodic, the probability of finding a suitable  $\mathbf{p}(m)$  equals 1. Note that although only 50 transients were used in Fig. 2, it is impossible to observe any discontinuity in the attractor. Results obtained with the modified algorithm are presented in Fig. 3. It can be well observed that the largest Lyapunov exponent converges well to a positive value  $\Lambda = 0.041 \text{ s}^{-1}$ , which is a strong indicator for chaotic behaviour in the system, and thus fully confirms the successfulness of the chaotification procedure. Although the obtained Lyapunov exponent is rather small in comparison to some other chaotic systems, the result is fully convincing because even intrinsically chaotic states of the mathematical model under consideration do not exceed values of  $0.05 \text{ s}^{-1}$  [52].

#### 4. Chaos control

Finally, it is of interest to verify if the enriched dynamics of the initially regular system can be exploited by controlling one of the unstable periodic orbits (UPOs) embedded in the newly obtained chaotic attractor. The control of UPOs is carried out with the algorithm proposed by Boccaletti and Arcelli [46,47], which is based on the delayed feedback method originally proposed by Pyragas [2]. Here we will briefly summarize the algorithm, whereas its complete description can be found in [46,47]. Although the algorithm can be applied to an arbitrary dimensional system, we will, for

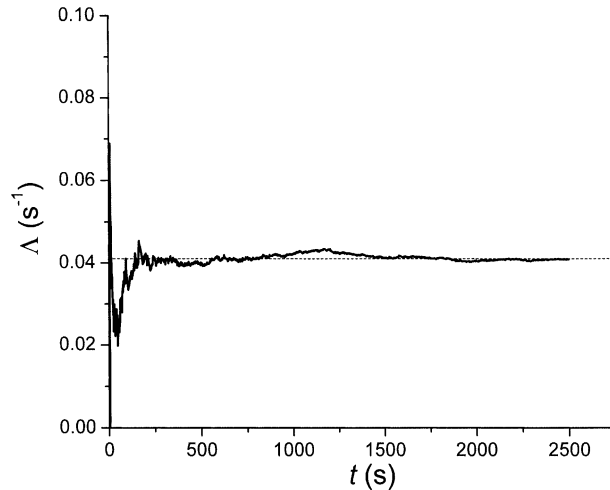


Fig. 3. Largest Lyapunov exponent pertaining to the temporarily destabilized system. The value converges well to  $\Lambda = 0.041 \text{ s}^{-1}$  that is indicated by the thin dashed line. Parameter values used for the algorithm were:  $\varepsilon = 0.01$ ,  $\nu = 20 \text{ s}$ , and  $\delta = 0.00005$ .

simplicity reasons and consistence with the studied mathematical model, summarize it for a 3D system. Let us consider a dynamical system

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}(\mathbf{p}, \mu), \tag{3}$$

where  $\mathbf{p}(t) = (x(t), y(t), z(t))$  and  $\mathbf{F} = (F_x, F_y, F_z)$  are a 3-dimensional vector and the governing vector field, respectively, and  $\mu$  is a set of fixed system parameters. After determining the period  $T$  of a particular UPO, the latter can be stabilized by introducing an additive correction term ( $\mathbf{U} = (U_x, U_y, U_z)$ ) to each of the three differential equations of the system. The relative weight of the correction term applied to a particular differential equation, i.e.  $U_w$  where  $w$  denotes either  $x$ ,  $y$  or  $z$ , is calculated according to the equation

$$U_w(t) = \frac{1}{\tau_s} (w(t - T) - w(t)), \tag{4}$$

where  $\tau_s$  is the minimum of all  $\tau_s^{(i)}$ , calculated for all dimensions of the dynamical system according to the equation

$$\tau_s^{(i)} = \tau_t^{(i)} / (1 - \tanh(\sigma \lambda_w(t))), \tag{5}$$

whereby  $s = t - dt$ ,  $dt$  being the time step for numerical integration. In Eq. (5)  $\sigma$  is a strictly positive constant chosen so as to forbid  $\tau_t^{(i)}$ , from going to zero. The local expansion and contraction rates  $\lambda_w(t)$  are calculated with respect to the period  $T$  of a particular UPO that is to be stabilized according to the equation:

$$\lambda_w(t) = \frac{1}{\tau_s} \ln \left| \frac{w(t) - w(t - T)}{w(s) - w(s - T)} \right|. \tag{6}$$

Note that this local variation rates also determine the time interval during which the correction term has a constant value, thereby reflecting the necessity to perturb the dynamics more or less often in order to stabilize the desired UPO.

We apply the above-described algorithm to the studied temporarily destabilized system. In Fig. 2 we show that the applied short-term destabilizations enrich the system’s dynamics, yielding a vast diversity of unstable dynamical states that is usually expected only in intrinsically chaotic systems. From the time course of variable  $x$  presented in Fig. 4(a), it can be well observed that the temporal destabilizations indeed have a large impact on the systems dynamics. Note that in a very short time insert following the impact of  $f(t)$ , a total of four different UPOs can be identified. Results presented in Fig. 4(b) and (c) show that these UPOs can be successfully controlled with the described algorithm for controlling chaotic behaviour. Herewith, we confirm our prediction that traditional chaos control techniques are applicable also to temporarily destabilized regular systems, and that the latter possess a rich diversity of accessible dynamical states that is usually expected only in intrinsically chaotic systems.

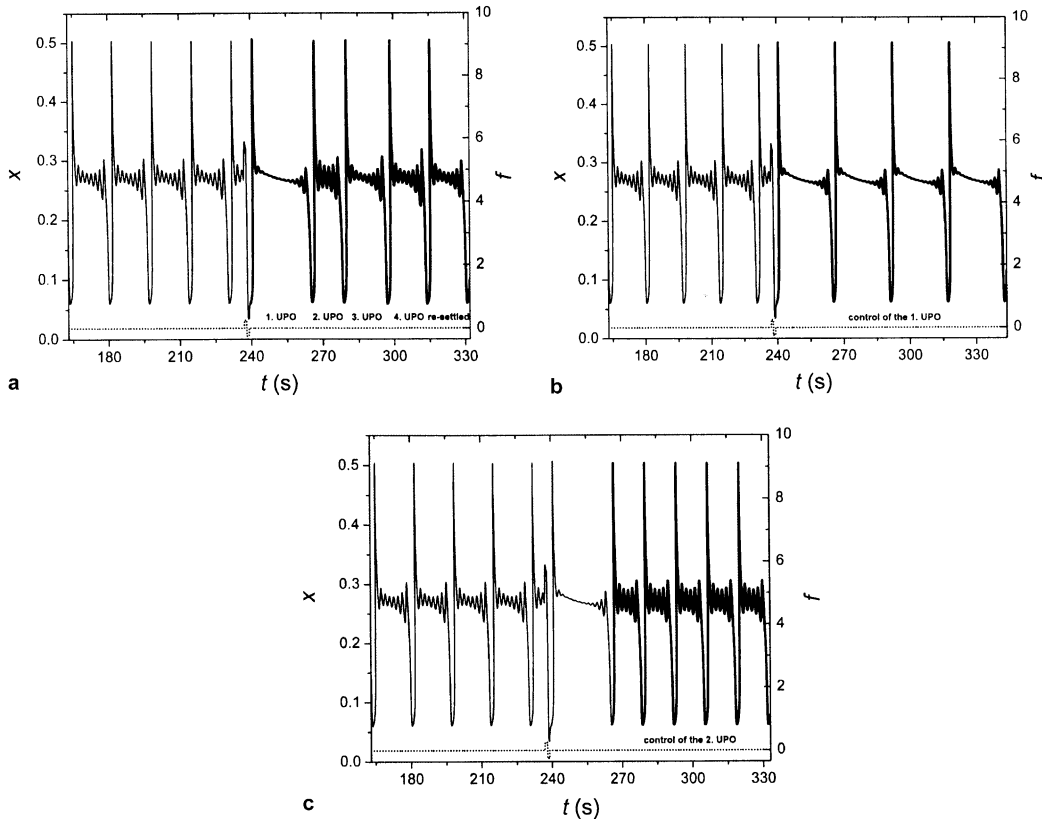


Fig. 4. Chaos control of the temporarily destabilized regular system: (a) impact of destabilization—parameter values used in Eq. (1) were  $a = 0.25$  and  $d = 1.0$ ; (b) control of the first UPO following the impact of  $f(t)$ ; (c) control of the second UPO following the impact of  $f(t)$ .

## 5. Conclusions

In the present paper, we study a regular system with the slow passage effect under the influence of symmetric impulsive perturbations that were applied to temporarily destabilize the system's dynamics. We show that due to the extreme sensitivity of the slow passage phase, short pulsatile perturbations have a large impact on the dynamics, evoking long unstable autonomous transients that ultimately yield chaotic behaviour in the originally regular system. To confirm the chaotic behaviour in the newly obtained attractor, we introduce a modified algorithm originally developed by Wolf et al. [45] to overcome the problem with discontinuities that emerge because parts of the attractor where  $f(t) \neq 0$  were omitted. We confirm the chaotic behaviour of the temporarily destabilized system by calculating a positive largest Lyapunov exponent. Moreover, we show that the newly obtained unstable periodic orbits can be easily controlled with conventional chaos control techniques, thereby guaranteeing a rich diversity of accessible dynamical states that is usually expected only in intrinsically chaotic systems. In summary, we extend the existing theory of chaotifying time-continuous systems by applying a simple yet effective procedure that is able to evoke chaotic behaviour in a regular system with the slow passage effect.

The presented results may also have important biological implications. In particular, it has often been found that pulsatile perturbations may improve performance of biological systems, either by restoring normal, i.e. healthy, system functioning or enhancing their overall effectiveness. On the cellular level, it has been found that electrically stimulated  $\text{Ca}^{2+}$  influx can resume apparently normal fertilization and early embryonic development in human oocytes that fail to fertilize after intracytoplasmic sperm injection [53]. Examples of positive influences on functioning of a whole organ or tissue include reanimation of the human heart with electroshocks [54], treatments of psychical diseases with electric stimuli applied to the brain [55], or enhancing cartilage growth and bone healing with electromagnetic fields [56–60]. These phenomena have not yet been fully explained by the scientific community. While the reanimation of the human

heart with strong electroshocks may seem self-explanatory and has been used successfully for many decades, other phenomena seem to have more intriguing background and are subject of fearsome debate [61]. Here, we provide some interesting results showing that such perturbations, although applied in a seemingly random fashion, may have a large impact on the system's dynamics, and even shift the behaviour from regular to chaotic. Since it is a widespread belief that, especially in nature, chaos is a desirable system state enabling excellent adaptation possibilities and enhanced survival chance, such dramatic changes in the system's dynamics might be a good starting point for further investigations concerning these fascinating real-life phenomena.

## Appendix A

The mathematical model for intracellular calcium oscillations proposed by Marhl et al. [48] is described by the following differential equations:

$$\frac{dx}{dt} = J_{\text{ch}} - J_{\text{pump}} + J_{\text{leak}} + J_{\text{out}} - J_{\text{in}} + J_{\text{CaPr}} - J_{\text{Pr}}, \quad (\text{A.1})$$

$$\frac{dy}{dt} = \frac{\beta_{\text{er}}}{\rho_{\text{er}}} (J_{\text{pump}} - J_{\text{ch}} - J_{\text{leak}}), \quad (\text{A.2})$$

$$\frac{dz}{dt} = \frac{\beta_{\text{m}}}{\rho_{\text{m}}} (J_{\text{in}} - J_{\text{out}}), \quad (\text{A.3})$$

where

$$J_{\text{ch}} = k_{\text{ch}} \frac{x^2}{x^2 + K_1^2} (y - x), \quad (\text{A.4})$$

$$J_{\text{pump}} = k_{\text{pump}} x, \quad (\text{A.5})$$

$$J_{\text{leak}} = k_{\text{leak}} (y - x), \quad (\text{A.6})$$

$$J_{\text{Pr}} = k_+ x c_{\text{Pr}}, \quad (\text{A.7})$$

$$J_{\text{CaPr}} = k_- c_{\text{CaPr}}, \quad (\text{A.8})$$

$$J_{\text{in}} = k_{\text{in}} \frac{x^8}{x^8 + K_2^8}, \quad (\text{A.9})$$

$$J_{\text{out}} = \left( k_{\text{out}} \frac{x^2}{x^2 + K_1^2} + k_{\text{m}} \right) z, \quad (\text{A.10})$$

$$c_{\text{Pr}} = c_{\text{PrTot}} - c_{\text{CaPr}}, \quad (\text{A.11})$$

$$c_{\text{CaPr}} = c_{\text{CaTot}} - x - \frac{\rho_{\text{er}}}{\beta_{\text{er}}} y - \frac{\rho_{\text{m}}}{\beta_{\text{m}}} z. \quad (\text{A.12})$$

Parameters values are:  $k_{\text{ch}} = 4230 \text{ s}^{-1}$ ,  $k_{\text{leak}} = 0.05 \text{ s}^{-1}$ ,  $k_{\text{pump}} = 20 \text{ s}^{-1}$ ,  $k_{\text{in}} = 300 \mu\text{M s}^{-1}$ ,  $k_{\text{out}} = 150 \text{ s}^{-1}$ ,  $k_{\text{m}} = 0.00625 \text{ s}^{-1}$ ,  $k_+ = 0.1 \mu\text{M}^{-1} \text{ s}^{-1}$ ,  $k_- = 0.01 \text{ s}^{-1}$ ,  $K_1 = 5.0 \mu\text{M}$ ,  $K_2 = 0.8 \mu\text{M}$ ,  $c_{\text{PrTot}} = 90 \mu\text{M}$ ,  $c_{\text{CaTot}} = 120 \mu\text{M}$ ,  $\rho_{\text{er}} = 0.01$ ,  $\beta_{\text{er}} = 0.0025$ ,  $\rho_{\text{m}} = 0.01$ ,  $\beta_{\text{m}} = 0.0025$ . Although the mathematical model has a specific biological importance, and so the parameter values as well as the system variables are not dimensionless, we omit the use of physical units in the present paper, since biological particularities are presently not of special importance. For details regarding the biological meaning of variables and parameter values see [48].

## References

- [1] Ott E, Grebogi C, Yorke JA. Controlling chaos. *Phys Rev Lett* 1990;64:1196–9.
- [2] Pyragas K. Continuous control of chaos by self-controlling feedback. *Phys Lett A* 1992;170:421–8.
- [3] Shinbrot T, Grebogi C, Ott E, Yorke JA. Using small perturbations to control chaos. *Nature* 1993;363:411–7.
- [4] Kapitaniak T. *Chaos for engineers: theory, applications and control*. Berlin: Springer; 1998.
- [5] Chen G, Dong X. *From chaos to order: methodologies, perspectives and applications*. Singapore: World Scientific; 1998.
- [6] Boccaletti S, Grebogi C, Lai Y-C, Mancini H, Maza D. The control of chaos: theory and applications. *Phys Rep* 2000;329:103–97.
- [7] Garfinkel A, Spano ML, Ditto WL, Weiss JN. Controlling cardiac chaos. *Science* 1992;257:1230–5.
- [8] Schiff SJ, Jerger K, Duong DH, Chang T, Spano ML, Ditto WL. Controlling chaos in the brain. *Nature* 1994;370:615–20.
- [9] Yang W, Ding M, Mandell AJ, Ott E. Preserving chaos: control strategies to preserve complex dynamics with potential relevance to biological disorders. *Phys Rev E* 1995;51:102–10.

- [10] Abarbanel HDI. Analysis of observed chaotic data. New York: Springer; 1996.
- [11] Schwartz IB, Georgiou IT. Instant chaos and hysteresis in coupled linear–nonlinear oscillators. *Phys Lett A* 1998;242:307–12.
- [12] Georgiou IT, Schwartz IB. Dynamics of large scale coupled structural mechanical systems: a singular perturbation proper orthogonal decomposition approach. *SIAM J Appl Math* 1999;59:1178–207.
- [13] Pecora LM, Carroll TL. Synchronization in chaotic systems. *Phys Rev Lett* 1990;64:821–4.
- [14] Hayes S, Grebogi C, Ott E. Communicating with chaos. *Phys Rev Lett* 1993;70:3031–4.
- [15] Peng JH, Ding EJ, Ding M, Yang W. Synchronizing hyperchaos with a scalar transmitted signal. *Phys Rev Lett* 1996;76:904–7.
- [16] VanWiggeren GD, Roy R. Communication with chaotic lasers. *Science* 1998;279:1198–200.
- [17] Zhu SQ, Lu X, Chen XF, Thornburg KS, VanWiggeren GD, Roy R. General synchronization of chaos in a system of two non-identical lasers. *Chin Phys* 2000;9:337–41.
- [18] Kapitaniak T. Continuous control and synchronization in chaotic systems. *Chaos, Solitons & Fractals* 1995;6:237–44.
- [19] Brindley J, Kapitaniak T, Kocarev L. Controlling chaos by chaos in geophysical systems. *Geophys Res Lett* 1995;22:1257–60.
- [20] Chen G, Lai D. Feedback control of Lyapunov exponents for discrete-time dynamical systems. *Int J Bifurcat Chaos* 1996;6:1341–9.
- [21] Chen G, Lai D. Making a dynamical system chaotic: feedback control of Lyapunov exponents for discrete-time dynamical systems. *IEEE Trans Circuits Syst* 1997;44:250–3.
- [22] Chen G, Lai D. Feedback anticontrol of discrete chaos. *Int J Bifurcat Chaos* 1998;8:1585–90.
- [23] Triandaf I, Schwartz IB. Tracking sustained chaos: a segmentation method. *Phys Rev E* 2000;62:3529–34.
- [24] Wang XF, Chen G, Yu X. Anticontrol of chaos in continuous systems via time-delay feedback. *Chaos* 2000;10:771–9.
- [25] Konishi K. Making chaotic behavior in a damped linear harmonic oscillator. *Phys Lett A* 2001;284:85–90.
- [26] Chen M, Han Z. An iteration method for chaotifying and controlling dynamical systems. *Int J Bifurcat Chaos* 2002;12:1173–80.
- [27] Yang L, Liu Z, Chen G. Chaotifying a continuous-time system via impulsive input. *Int J Bifurcat Chaos* 2002;12:1121–8.
- [28] Chen G, Yang L, Liu Z. Anticontrol of chaos in continuous-time system. *IEICE Trans Fundament* 2002;E85-A:1333–5.
- [29] Codreanu S. Desynchronization and chaotification of nonlinear dynamical systems. *Chaos, Solitons & Fractals* 2002;13:839–43.
- [30] Chen G, Yang L. Chaotifying a continuous-time system near a stable limit cycle. *Chaos, Solitons & Fractals* 2003;15:245–53.
- [31] Li C, Chen G. On the Marotto–Li–Chen theorem and its application to chaotification of multi-dimensional discrete dynamical systems. *Chaos, Solitons & Fractals* 2003;18:807–17.
- [32] Li C. On super-chaotifying discrete dynamical systems. *Chaos, Solitons & Fractals* 2004;21:855–61.
- [33] Danca M-F. Chaotifying discontinuous dynamical systems via time-delay feedback algorithm. *Int J Bifurcat Chaos* 2004;7:2321–39.
- [34] Chen H-K, Lee C-I. Anti-control of chaos in rigid body motion. *Chaos, Solitons & Fractals* 2004;21:957–65.
- [35] Starkov K, Chen G. Chaotification of polynomial continuous-time systems and rational normal forms. *Chaos, Solitons & Fractals* 2004;22:849–56.
- [36] Nejshtadt A. Asymptotic investigation of the loss of stability by an equilibrium as a pair of eigenvalues slowly cross the imaginary axis. *Usp Math Nauk* 1985;40:190–1.
- [37] Rinzel J, Baer M. Threshold for repetitive activity for a slow stimulus ramp: a memory effect and its dependence on fluctuations. *Biophys J* 1988;54:551–5.
- [38] Baer SM, Erneux T, Rinzel J. The slow passage through a Hopf bifurcation: delay, memory effects, and resonances. *SIAM J Appl Math* 1989;49:55–71.
- [39] Holden L, Erneux T. Slow passage through a Hopf bifurcation: from oscillatory to steady state solutions. *SIAM J Appl Math* 1993;53:1045–58.
- [40] Holden L, Erneux T. Understanding bursting oscillations as periodic slow passages through bifurcation and limit points. *J Math Biol* 1993;31:351–65.
- [41] Izhikevich EM. Neural excitability spiking and bursting. *Int J Bifurcat Chaos* 2000;10:1171–266.
- [42] Izhikevich EM. Synchronisation of Elliptic Bursters. *SIAM Rev* 2001;43:315–44.
- [43] Perc M, Marhl M. Different types of bursting calcium oscillations in non-excitable cells. *Chaos, Solitons & Fractals* 2003;18:759–73.
- [44] Perc M, Marhl M. Synchronization of regular and chaotic oscillations: the role of local divergence and the slow passage effect. A case study on calcium oscillations. *Int J Bifurcat Chaos* 2004;14:2735–51.
- [45] Wolf A, Swift JB, Swinney HL, Vastano JA. Determining Lyapunov exponents from a time series. *Physica D* 1985;16:285–317.
- [46] Boccaletti S, Arecchi FT. Adaptive-control of chaos. *Europhys Lett* 1995;31:127–32.
- [47] Boccaletti S, Arecchi FT. Adaptive recognition and control of chaos. *Physica D* 1996;96:9–16.
- [48] Marhl M, Haberichter T, Brumen M, Heinrich R. Complex calcium oscillations and the role of mitochondria and cytosolic proteins. *BioSystems* 2000;57:75–86.
- [49] Rinzel J. Bursting oscillations in an excitable membrane model. *Lecture Notes in Mathematic*, 1151. Berlin: Springer; 1985.
- [50] Doedel E, Champneys A, Fairgrieve T, Kuznetsov Y. AUTO97: continuation and bifurcation software for ordinary differential equations. Montreal: Concordia University; 1997.
- [51] Perc M, Marhl M. Detecting and controlling unstable periodic orbits that are not part of a chaotic attractor. *Phys Rev E* 2004;70:016204.
- [52] Haberichter T, Marhl M, Heinrich R. Birhythmicity, trirhythmicity and chaos in bursting calcium oscillations. *Biophys Chem* 2001;90:17–30.



- [53] Zhang J, Wang CW, Blaszczyk A, Grifo JA, Ozil J, Haberman E, et al. Electrical activation and in vitro development of human oocytes that fail to fertilize after intracytoplasmic sperm injection. *Fertility and Sterility* 1999;72:509–12.
- [54] Temes G, Lantos J, Torok B. Possibility of brain recovery after electrically induced cardiac arrest and reanimation in dogs. *Acta Physiol Hung* 1987;70:93–103.
- [55] Beyer JL, Weiner RD, Glenn MD. *Electroconvulsive therapy: a programmed text*. Washington: American Psychiatric Press; 1998.
- [56] Torricelli P, Fini M, Giavaresi G, Cane V, Giardino R. In vitro evaluation of the effects of electromagnetic fields used for bone healing. *Electro Magnetobiol* 1998;17:335–42.
- [57] Ciombor DM, Lester G, Aaron RK, Neame P, Caterson B. Low frequency EMF regulates chondrocyte differentiation and expression of matrix proteins. *J Orthopaed Res* 2002;20:40–50.
- [58] Aaron RK, Wang S, Ciombor DM. Upregulation of basal TGF beta (I) levels by EMF coincident with chondrogenesis-implications for skeletal repair and tissue engineering. *J Orthopaed Res* 2002;20:233–40.
- [59] De Mattei M, Pasello M, Pellati A, Stabellini G, Massari L, Gemmati D, et al. Effects of electromagnetic fields on proteoglycan metabolism of bovine articular cartilage explants. *Connect Tissue Res* 2003;44:154–9.
- [60] De Mattei M, Pellati A, Pasello M, Ongaro A, Setti S, Massari L, et al. Effects of physical stimulation with electromagnetic field and insulin growth factor-I treatment on proteoglycan synthesis of bovine articular cartilage. *Osteoarthr Cartilage* 2004;12:793–800.
- [61] Electroconvulsive therapy, <http://www.psycom.net/depression.central.ect.html>.