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Collective behavior in a two-layer neuronal network with time-varying chemical connections that are controlled by a Petri net

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ABSTRACT

In this paper, we propose and study a two-layer network composed of a Petri net in the first layer and a ring of coupled Hindmarsh–Rose neurons in the second layer. Petri nets are appropriate platforms not only for describing sequential processes but also for modeling information circulation in complex systems. Networks of neurons, on the other hand, are commonly used to study synchronization and other forms of collective behavior. Thus, merging both frameworks into a single model promises fascinating new insights into neuronal collective behavior that is subject to changes in network connectivity. In our case, the Petri net in the first layer manages the existence of excitatory and inhibitory links among the neurons in the second layer, thereby making the chemical connections time-varying. We focus on the emergence of different types of collective behavior in the model, such as synchronization, chimeras, and solitary states, by considering different inhibitory and excitatory tokens in the Petri net. We find that the existence of only inhibitory or excitatory tokens disturbs the synchronization of electrically coupled neurons and leads toward chimera and solitary states.

Multilayer networks have attracted ample attention in complexity science due to their ability to comprehensively model different levels of complex systems. Indeed, this framework is also compatible with the structural and functional properties of neural systems. With this motivation, we study the collective behavior of a multilayer neuronal network with different layers. The information transmitted between the neurons is achieved through the synapses. For modeling the circulation of information, however, we use the Petri net. Thus, the Petri net represents one layer of the network, while the other layer is composed of a ring of Hindmarsh–Rose neurons. The neurons are connected by means of constant electrical synapses and time-varying chemical synapses. It is assumed that the Petri net manages the chemical links by considering excitatory and inhibitory tokens. The behavior of the

neurons is studied by varying the parameters of the Petri net. We find that the existence of the chemical tokens causes the transition of the network from synchronization toward chimera and solitary states. Moreover, by increasing the timing of the Petri net, we observe that more neurons are able to escape from the synchronous group.

I. INTRODUCTION

Complexity science studies the mechanism and behavior of complex systems such as neural populations, climate change, social networks, etc.[1–7] To better understand these systems, it is necessary to investigate many different aspects of the interactions that connect many constituents.[8,9] A popular tool for mimicking these different characteristics is the multilayer framework.[10] In a multilayer network, the aspects of a complex phenomenon can be considered with each layer.[11] For instance, multilayer networks are widely used for modeling the features of human behaviors in both societies and social media spaces.[12] In these examples, the models consider one layer for the agents' behavior in physical environments and another for their behavior in virtual spaces. Thus, a multiplex framework is usually applied wherein the corresponding nodes in different layers refer to the same agent.[13] Multilayer structures have also been extensively selected for studying the neurons' behaviors.[14,15]

The neuronal behaviors have usually been modeled by coupling a group of dynamical neuronal models such as Hindmarsh–Rose and Hodgkin–Huxley equations.[16–19] Among the neurons, electrical, chemical, or magnetic connections exist. Chemical connections can be classified into two main categories: excitatory and inhibitory.[20] Electrical coupling is built by gap junctions.[21] The neurons are also able to interact with each other through magnetic fields.[22] In neuronal population models, the dynamical equations can be linked to each other with these different connections to structure the networks.[20] Consequently, the networks can show some well-known collective behaviors such as synchronization.[23] The synchronization is a universal concept and has many applications in different fields.[24,25] Studying the synchronization among neurons is crucial to understand some brain disease mechanisms such as epilepsies.[26] In addition, some brains' cognitive activities are also associated with neurons' synchronous state.[27]

A chimera state is another collective phenomenon that has grabbed the attention of many researchers.[28–30] A chimera state refers to the pattern in which some oscillators of the network are synchronous, while others are asynchronous. Chimera states can be found in natural phenomena such as dolphins' uni-hemisphere sleep, the human brain's local sleep, and also some brain disorders such as schizophrenia.[31,32] Recently, chimera-like states have also been observed in heterogeneous Kuramoto phase oscillators with attractive and repulsive couplings.[33] The existence of chimera has been investigated in multilayer neuronal networks with various kinds of coupling.[15] For instance, a study showed how the chimera state can exist in a network with electrical intra-layer and chemical inter-layer couplings.[34] The other interesting collective behavior is the solitary state that has recently received attention.[35–37] The solitary state describes a special state when most of the neurons are synchronous, but a few split off from them.[38] These few neurons that are scattered among the synchronous ones are named solitary neurons.[38] The emergence of solitary states in a multiplex network was investigated by Majhi *et al.*,[39] with considering positive inter-layer and negative intra-layer couplings.

Most studies on the neuronal population have been done by networks with constant connections among neurons. However, neuronal networks with time-varying links have recently received much attention from researchers.[40,41] As an example, the effect of changing the frequency of time-variant links on the neurons spiking behaviors has been investigated in Ref. 42. In another work, it was shown that the higher frequency increases the synchronization level among neurons in a multiplex network.[43] In a study, Belykh *et al.* introduced the blinking coupling that refers to the time-dependent on–off connections among neurons.[44] The variations in the connections can be referred to as the neuroplasticity.[45] The blinking coupling and its effect on the chimera and synchronization have also been studied in multilayer networks.[46] Besides, the advent of chimera has been studied in a network wherein the strength of the links depends on the bursting time.[47]

Inspired by multilayer networks that are mostly used for modeling human behaviors, this paper introduces a two-layer network to show different aspects of neurons' behavior. One layer of the network is composed of the coupled Hindmarsh–Rose neurons with constant electrical connections and time-varying chemical connections. The second layer is a Petri net that represents the circulation of information and determines which neurons' chemical connections should be on or off. Petri nets, which were first introduced by Carl Adam Petri,[48] are widely used in different branches of sciences including the behavior of the neurons.[49,50] Petri nets are suitable for modeling the consequential processes. Each Petri net is built of four main elements: tokens, places, arrows, and functions (Fig. 1).[51] Tokens move among places. In each step, a token may transfer from a place to the next one or may remain in its current place.[52] Each place may contain zero, one, or more numbers of tokens.[52] Tokens move from a place to another based on the directions of the arrows and conditions of functions.[53] Arrows show the path of tokens' movement.[54] Functions determine the conditions for the tokens to leave a place and locate in the others. In this way, Petri nets provide the opportunity of visualizing sequences

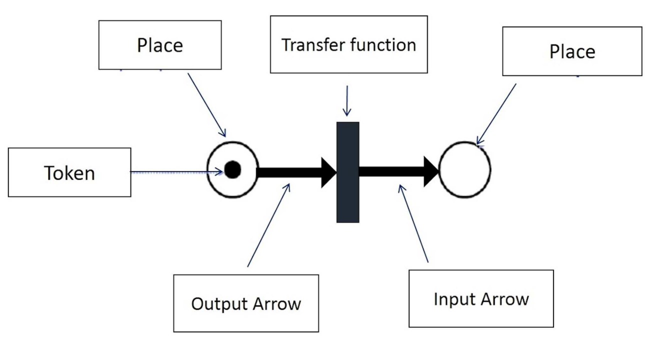

FIG. 1. A simple example of a Petri net. It has four main parts: places, arrows, functions, and tokens. Tokens are located in places and move among them based on the directions of arrows and function rules.

of processes.[55] In neuronal behaviors, tokens can be assumed as the model of many concepts including information.[56] Places can also be considered neurons.[57] Therefore, tokens' movement among places can simulate the circulation of information among neurons.[58]

In this paper, the emergence of synchronization, chimera, and solitary states is studied in the described network. The effects of different factors such as the excitatory and inhibitory tokens and the Petri net timing are investigated. In Sec. II, different parts of the model are explained in detail. The results are presented in Sec. III, and the conclusions are given in Sec. IV.

II. MODEL

In this paper, a two-layer network is considered. The network has a Petri net in its first layer and a ring of Hindmarsh–Rose neurons in the second layer. In Subsections II A–II C, the details about the Petri net layer, the Hindmarsh–Rose layer, and the connections among them are presented, respectively. Each pair of nodes (one Petri net place in the first layer and one Hindmarsh–Rose model in the second layer) represents an agent (a neuron). Each node in the layers is considered to represent an aspect of the neuron behavior.

A. Petri net layer

As mentioned, Petri nets are constructed of four main parts: places, arrows, tokens, and functions (Fig. 1). The considered Petri net has $N = 100$ number of places. A number between 1 and 100 is allocated to each place. It is assumed that each place has just one input arrow and one output arrow. The input of each place is selected randomly. Similarly, each place gives its output arrow to another one randomly. Therefore, a ring of places is formed where the arrangement of places' allocated number is random.

Tokens in Petri nets are located in places and move from one place to another based on the directions of arrows and rules of functions. In this paper, two kinds of tokens are considered: excitatory and inhibitory. The number of excitatory and inhibitory tokens is represented by $Token_{ex}$ and $Token_{in}$, respectively. All functions of the Petri net have the same simple rule. The rule is that tokens must go to the next place after T_{pet} time.

The described Petri net can be presented by the following Bulletins:

(1) N number of places (P) is considered a finite set; i.e.,

$$P = P_1, P_2, P_3, \ldots, P_N.$$

(2) $Token_{ex}$: $K1$ number of excitatory tokens are considered; i.e.,

$$Token_{ex} = Tok_{ex1}, Tok_{ex2}, \ldots, Tok_{exK1}.$$

(3) $Token_{in}$: $K2$ number of inhibitory tokens are considered; i.e.,

$$Token_{in} = Tok_{in1}, Tok_{in2}, \ldots, Tok_{inK2}.$$

(4) A shuffled arrangement of places is considered the path of the excitatory and inhibitory tokens; i.e., $Path =$ shuffle $(P_1, P_2, P_3, \ldots, P_N)$. This set of shuffled arrangement makes a circle that determines the direction of arrows, for instance,

$$P_i \rightarrow P_{i-5} \rightarrow P_1 \rightarrow \cdots \rightarrow P_N \rightarrow \cdots \rightarrow P_s \rightarrow P_i \quad (i, s < N).$$

(5) For each excitatory and/or inhibitory token, a random integer number between 1 and N is allocated as the number of its initial place.

(6) After each T_{pet} time, each token changes its current place to the next. The next place is determined with the Petri net's path (the 4th Bulletin).

B. Hindmarsh–Rose neuron layer

Hindmarsh–Rose equations are used for each node of the second layer. These equations are described as[59]

$$\begin{aligned} \dot{x} &= F(x,y,z) = y - ax^3 + bx^2 - z + I_{ext}, \\ \dot{y} &= G(x,y,z) = c - dx^2 - y, \\ \dot{z} &= H(x,y,z) = r(s(x+1.6) - z). \end{aligned} \tag{1}$$

In these equations, x, y, z represent the membrane voltage variable, the slow recovery current variable, and the membranes' adaption current (the faster one), respectively. In the following simulations, the parameters' values of the equation are set as $a = 1$, $b = 3$, $c = 1$, $d = 5$, $r = 0.006$, $s = 4$, and $I_{ext} = 2.2$. $N = 100$ Hindmarsh–Rose neurons are considered in a ring structure with nonlocal coupling. There are also some time-varying chemical connections that are determined by the Petri net layer.

C. The network

A multiplex network is constructed of the described layers. The schematic of the network is shown in Fig. 2. As mentioned, the Hindmarsh–Rose neurons can affect each other with excitatory or inhibitory connections. The number of these excitatory (or inhibitory) connections is shown by the CON variable. The excitatory (or inhibitory) connections of a node in the Hindmarsh–Rose

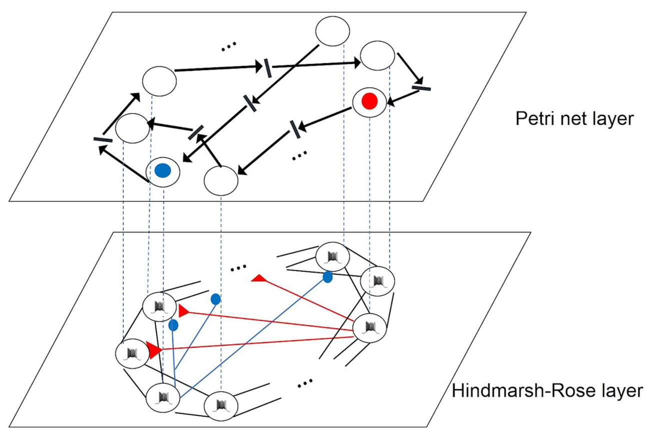

FIG. 2. Schematic of the proposed two-layer network. For each node in the Hindmarsh–Rose layer, there is an equivalent node (a place) in the Petri net layer. Excitatory and inhibitory tokens go from place to place in the Petri net based on the directions of arrows (which can be seen in the Petri layer) and the functions' rule. If an excitatory (inhibitory) token settles in a place in the Petri net, the excitatory (inhibitory) connections in the equivalent Hindmarsh–Rose neuron become activated.

layer are activated if and only if an excitatory (or inhibitory) token exists in the corresponding node in the Petri net layer. In other words, when an excitatory (or inhibitory) token exists in a place in the Petri net layer, the corresponding Hindmarsh–Rose neuron excites (inhibits) the *CON* number of other Hindmarsh–Rose neurons randomly. When there is no token in the place, all of the excitatory (inhibitory) activation contacts of the neurons become off. In the schematic shown in Fig. 2, the inhibitory connections of neuron 1 are activated because it has an inhibitory token in its relevant Petri net place. Similarly, the excitatory connections of neuron 2 are activated because it has an excitatory token.

The equations of coupled neurons can be given as follows:[60,61]

$$\begin{aligned}\dot{x}_i &= F(x_i, y_i, z_i) + \sum_{j=1}^{N} g_{ex}(v_{se} - x_i)\left(\frac{1}{1+e^{(-\lambda(x_j-\theta))}}\right) A1_{ij} \\ &\quad + \sum_{j=1}^{N} g_{in}\,(v_{in} - x_i)\left(\frac{1}{1+e^{(-\lambda(x_j-\theta))}}\right) A2_{ij} \\ &\quad + \sum_{j=1}^{N} g_e\left(x_j - x_i\right) A3_{ij}, \\ \dot{y}_i &= G\left(x_i, y_i, z_i\right), \\ \dot{z}_i &= H\left(x_i, y_i, z_i\right),\end{aligned} \tag{2}$$

where g_e, g_{ex}, and g_{in} represent electrical, excitatory, and inhibitory connection coefficients, respectively. In each time step, $A1$ and $A2$ determine excitatory and inhibitory connections among neurons based on the tokens in the Petri net layer. Thus, $A1_{ij} = 1$ and $A2_{ij} = 1$ if neurons i and j are connected. The Hindmarsh–Rose neurons are electrically connected in a ring network with nonlocal coupling. Therefore, $A3$ is the adjacency matrix of the Hindmarsh–Rose neuron layer. The parameters are set at $v_{se} = 2$, $v_{si} = -1.5, \theta = -0.5$, and $\lambda = 100$.

III. RESULTS

The 4th order Runge–Kutta method is used for numerical simulations. To quantify the coherence of the network for different parameters, the strength of incoherence (*SI*), which is a statistical measurement, is used. To calculate *SI*, first, a transformation is introduced as $w_{l,i} = x_{l,i} - x_{l,i+1}, i = 1, 2, \ldots, N, l = 1, 2, \ldots, d$. Considering $n = N/M$ (dividing N neurons to M groups), *SI* is calculated by the following formula:

$$\begin{aligned}< w_l > &= \frac{1}{N}\sum_{j=1}^{N} w_{l,j}, \\ \sigma_l(m) &= \left\langle \sqrt{\sum_{j=n(m-1)+1}^{nm} (w_{l,i} - < w_l >)^2} \right\rangle_t, \\ SI &= 1 - \frac{\sum_{M=1}^{M} s_m}{M}, \\ s_m &= \theta(\delta_0 - \sigma_l(m)),\end{aligned} \tag{3}$$

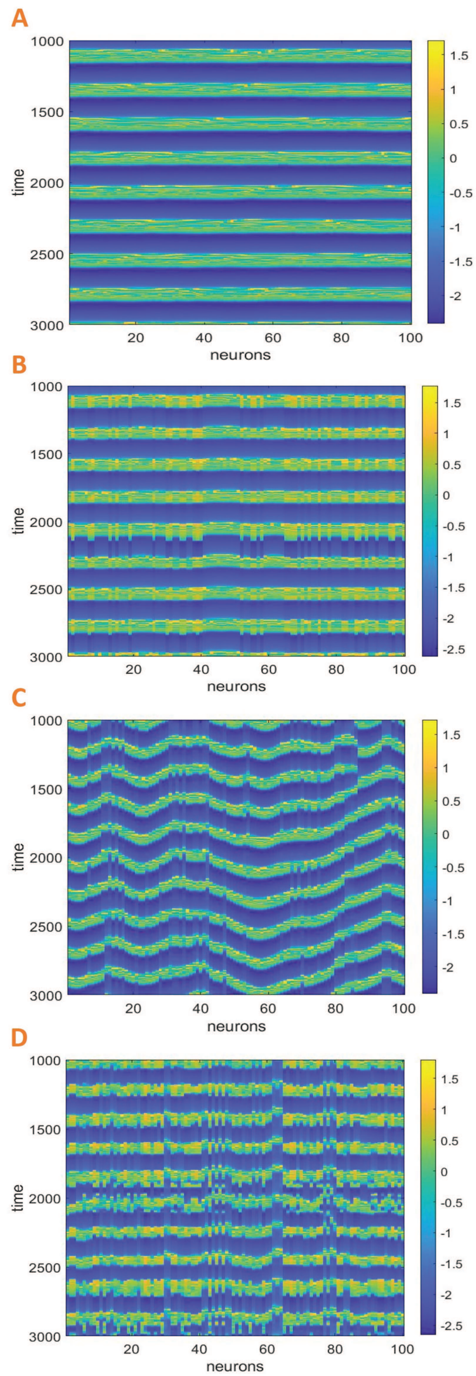

FIG. 3. The effects of inhibitory and excitatory tokens on the network states with considering ($g_e = 0.08$, $CON = 20$, $T_{Pet} = 20$). (a) $Token_{in} = 0$ and $Token_{ex} = 0$: the network is synchronous ($SI = 0$). (b) $Token_{in} = 2$, $Token_{ex} = 0$, $g_{in} = 0.8$, and $g_{ex} = 0$: the synchronization is disturbed with some solitary neurons ($SI = 0.04$). (c) $Token_{in} = 0$, $Token_{ex} = 2$, $g_{in} = 0$, $g_{ex} = 0.8$: the network is in the chimera state ($SI = 0.87$). (d) $Token_{in} = 2$, $Token_{ex} = 2$, $g_{in} = 0.8$, $g_{ex} = 0.8$: the network is asynchronous ($SI = 1$).

where $< \cdots >_t$, δ_0, and $\theta(\cdots)$ denote the average over time, predefined threshold, and the Heaviside step function, respectively. $SI = 0$, $SI = 1$, and $0 < SI < 1$ represent the coherence, incoherence, and chimera state, respectively. In our investigations, we have used $n = 4$ and $\delta_0 = 0.039$.

First, it is investigated how excitatory or inhibitory connections may affect the behavior of the network. To this aim, at first, only the inhibitory tokens and then the excitatory tokens are considered in the Petri net. Next, both excitatory and inhibitory tokens with the same number are considered (Fig. 3). Figure 3(a) shows the spatiotemporal pattern of the network with no token (only electrical connections). It is observed that the network is zero-lag synchronized. When the network has only excitatory tokens [Fig. 3(b)], some neurons tend to escape from the synchronized group. Thus, the synchronous state that was the result of the electrical connections is disturbed and an imperfect synchronization can be seen. With changing the tokens of the network to the inhibitory ones [Fig. 3(c)], more neurons get away from the synchronous group and form the asynchronous clusters. Therefore, the network moves toward a chimera state. If both types of tokens exist in the network, the synchronous clusters disappear and the neurons become asynchronous [Fig. 3(d)].

In the next step, the network behavior is investigated by considering different values for the Petri net time (T_{pet}). It was mentioned that the time-varying links can be interpreted as the changes among neurons' synapses, which is known as neural plasticity. Considering this presumption, a higher value of T_{pet} might be interpreted as a lower level of plasticity and vice versa (a lower value of T_{pet} may represent a higher plasticity level). Figures 4(a) and 4(b) show

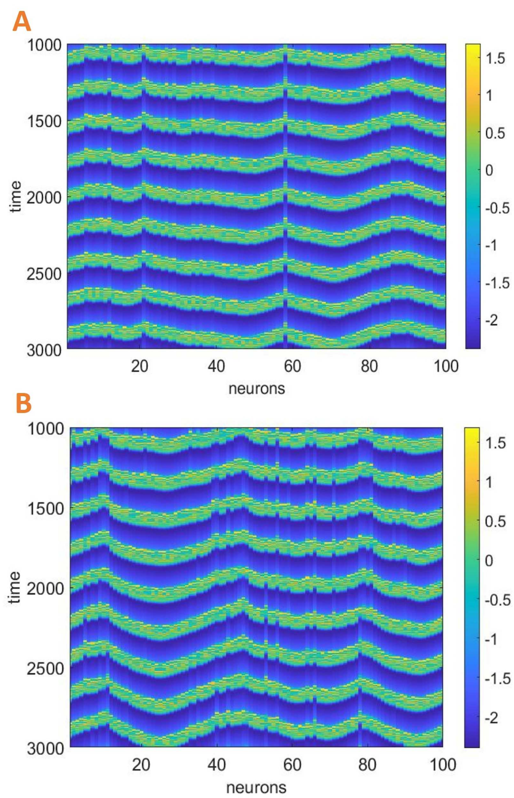

FIG. 4. The effects of the Petri net timing (T_{Pet}) on the network's state ($g_e = 0.04$, $g_{in} = 0.2$, $g_{ex} = 0$, $CON = 1$, $Token_{in} = 1$). (a) $T_{pet} = 0.1$ ($SI = 0.64$). (b) $T_{pet} = 20$ ($SI = 0.89$). Increasing T_{pet} causes an increment in the number of neurons that tend to escape from the synchronization.

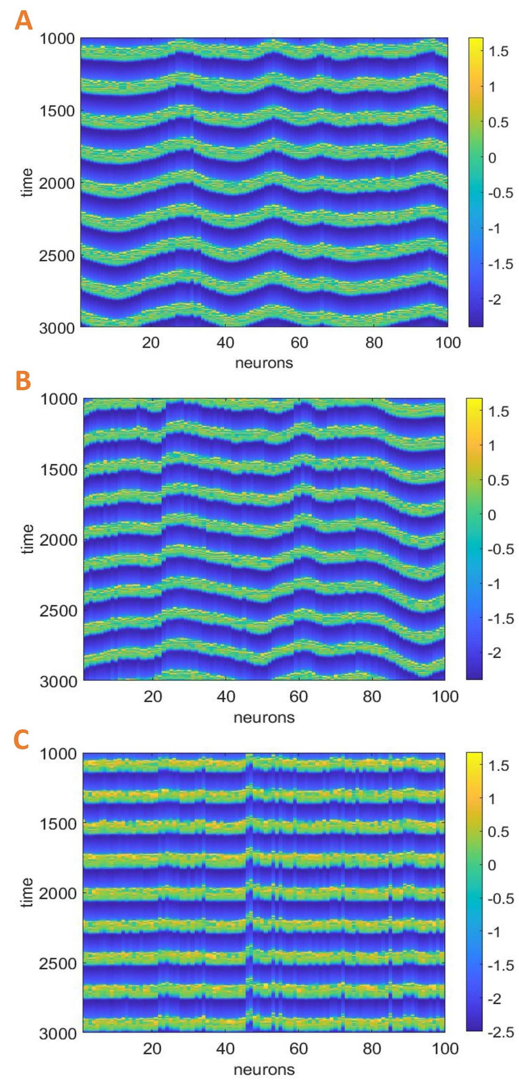

FIG. 5. The effect of the number of chemical connections on the behavior of the network ($g_e = 0.04$, $g_{in} = 0.12$, $g_{ex} = 0.12$, $Token_{in} = 5$, $T_{pet} = 20$). (a) $CON = 10$, $Token_{ex} = 0$ ($SI = 0.32$). (b) $CON = 50$, $Token_{ex} = 0$ ($SI = 0.67$). (c) $CON = 50$, $Token_{ex} = 5$ ($SI = 0.24$). Increasing the number of connections results in synchronization disturbance. Adding excitatory tokens leads to a chimera state.

spatiotemporal patterns of the network under two different values of T_{pet}. It is observed that the lower value of T_{pet} [Fig. 4(a)] results in phase synchronization with a few neurons tending to escape from this synchronous state. For a higher value of T_{pet} [Fig. 4(b)], the number of the escaping neurons is increased and the chimera state is formed. One possible reason for this might be that the lower value of T_{pet} causes the tokens to rotate among the places faster. Thus, in a lower period, all nodes could have tokens at least for one time. Therefore, it might make the network more homogenous, and consequently, the trend of escaping neurons is decreased.

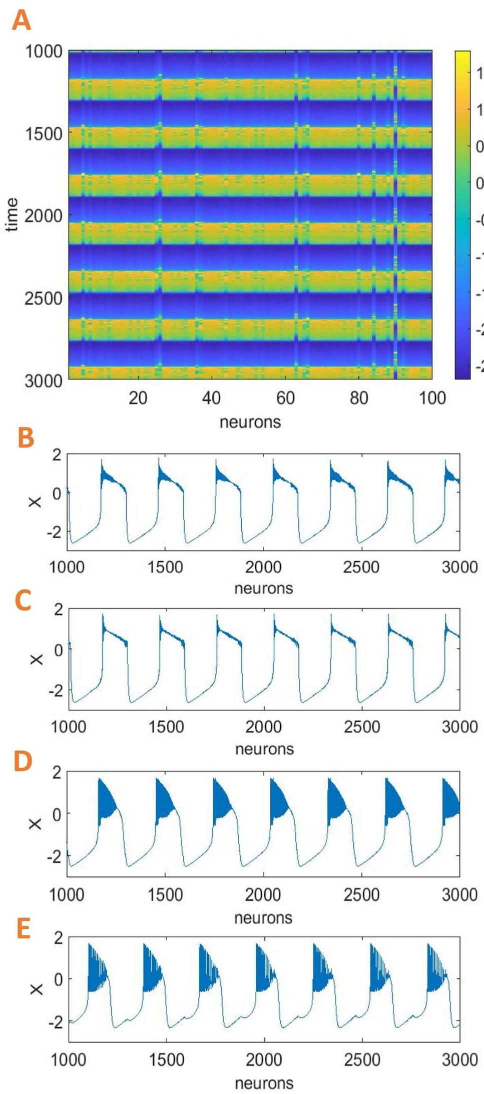

FIG. 6. Different time series of the neurons when $g_e = 0.04$, $g_{ex} = 0.12$, $Token_{in} = 0$, $Token_{ex} = 40$, $T_{pet} = 20$, $CON = 20$ ($SI = 0.08$). (a) The spatiotemporal pattern of the network shows the solitary state. Parts B, C, D, and E are the time series of neurons $i = 6$, 39, 88, and 90, respectively.

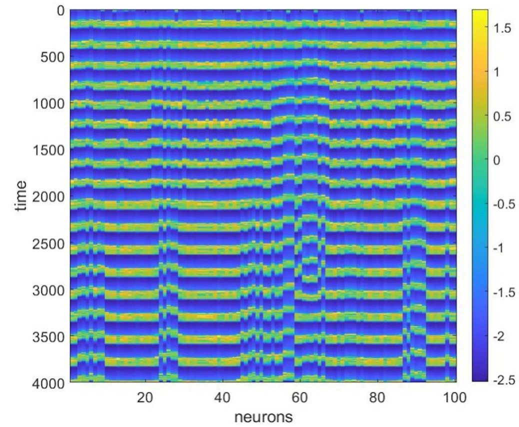

FIG. 7. The Petri net timing is set at $T_{Pet} = 2000$. The chimera state is preserved when a sudden change happens in the network's chemical links ($SI = 0.48$). The parameters are set at $g_e = 0.08$, $g_{ex} = 0.4$, $g_{ex} = 0.4$, $Token_{in} = 2$, $Token_{ex} = 2$, and $CON = 40$.

The effect of different numbers of excitatory and inhibitory connections (CON), which are activated when a neuron has a token, is represented in Fig. 5. When there are a few inhibitory connections, the network is phase synchronized. Comparing Figs. 5(a) and 5(b) reveals that for a fixed number of inhibitory tokens, increasing the number of connections causes some neurons to leave the synchronous group. Besides, Fig. 5(c) shows that adding the excitatory tokens with the same number as the inhibitory tokens and similar strength results in the increment of asynchronous neurons and changes the network's state to the chimera.

Figure 6 demonstrates the time series of different neurons when the network is in a solitary state. In this case, only excitatory neurons with $Token_{ex} = 40$ and $CON = 20$ are considered. The timing of the Petri net is set at $T_{Pet} = 0.2$, which means that each token rotates almost among ten places during a burst duration. There are some solitary neurons with different bursting behaviors in the network.

With considering a large value for the timing of the Petri net, there will be a sudden change in the structure of the coupled neurons. To investigate the effect of this change, $T_P et = 2000$ is selected. The pattern of the network for $g_e = 0.08$, $g_{ex} = 0.4$, $g_{ex} = 0.4$, $Token_{in} = 2$, $Token_{ex} = 2$, and $CON = 40$ is illustrated in Fig. 7. The network in this case exhibits a chimera state. It is observed that with changing the network's links, the chimera state is preserved. However, the location of the synchronous and asynchronous neurons is changed.

IV. DISCUSSION AND CONCLUSION

Neuronal population models have been used frequently for studying collective behaviors. The usual networks that are only composed of coupled dynamical models may not reveal all aspects of the neurons' communications. In other words, even though neuronal networks are useful for understanding collective behaviors, further models may represent other neurons' aspects in a better way. The Petri nets can demonstrate the circulation of information among neurons simply and properly.[56] Because of this ability of the Petri nets, we tried to link these models and the dynamical population neurons. Considering Petri nets besides coupled dynamical neuronal models can expand the capacity of neuronal networks. Therefore, in this paper, a two-layer framework was proposed to consider different aspects of neuronal interactions. In one layer, a ring of coupled Hindmarsh–Rose neurons was considered to assess different collective behaviors such as synchronization, solitary, and chimera state. The other layer was a Petri net to represent the circulation of information. It was assumed that the movement of the tokens in the Petri net's places defines which neurons are connected chemically. To this aim, two kinds of excitatory and inhibitory tokens were considered, and a token in a place led to the existence of the chemical connections to the corresponding neuron. Thus, the chemical connections were time-varying. We examined the effects of different factors on the neurons' behavior. First, the existence of the excitatory and inhibitory tokens was investigated. For the situations of just excitatory and just inhibitory tokens, the network showed imperfect synchronization and chimera, respectively. With considering both tokens, asynchronization was seen in the network. It was also observed that decreasing the timing of the Petri net causes less number of neurons to escape from the phase synchronous state. Besides, increasing the number of inhibitory connections (*CON*) led to disturbing the neurons' synchronization, while adding excitatory connections led the network toward the chimera state.

We hope that the introduced platform can be used in future research for modeling different aspects of neuronal dynamics.

AUTHORS' CONTRIBUTIONS

All authors contributed equally to this work.

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

REFERENCES

[1] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," Phys. Rep. **424**, 175–308 (2006).
[2] U. Ozturk, N. Marwan, O. Korup, H. Saito, A. Agarwal, M. J. Grossman, M. Zaiki, and J. Kurths, "Complex networks for tracking extreme rainfall during typhoons," Chaos **28**, 075301 (2018).
[3] A. Agarwal, N. Marwan, R. Maheswaran, U. Ozturk, J. Kurths, and B. Merz, "Optimal design of hydrometric station networks based on complex network analysis," Hydrol. Earth Syst. Sci. **24**, 2235–2251 (2020).
[4] A. Agarwal, L. Caesar, N. Marwan, R. Maheswaran, B. Merz, and J. Kurths, "Network-based identification and characterization of teleconnections on different scales," Sci. Rep. **9**, 8808 (2019).
[5] R. Pastor-Satorras, C. Castellano, P. Van Mieghem, and A. Vespignani, "Epidemic processes in complex networks," Rev. Mod. Phys. **87**, 925 (2015).
[6] Z. Wang, C. Y. Xia, S. Meloni, C. S. Zhou, and Y. Moreno, "Impact of social punishment on cooperative behavior in complex networks," Sci. Rep. **3**, 3055 (2013).
[7] J. Ma and J. Tang, "A review for dynamics in neuron and neuronal network," Nonlinear Dyn. **3**, 1569–1578 (2017).
[8] K. Usha and P. Subha, "Collective dynamics and energy aspects of star-coupled Hindmarsh–Rose neuron model with electrical, chemical and field couplings," Nonlinear Dyn. **96**, 2115–2124 (2019).
[9] E. Nikitin, A. Y. Malyshev, P. Balaban, and M. Volgushev, "Physiological aspects of the use of the Hodgkin–Huxley model of action potential generation for neurons in invertebrates and vertebrates," Neurosci. Behav. Physiol. **47**, 751–757 (2017).
[10] B. K. Bera, S. Majhi, D. Ghosh, and M. Perc, "Chimera states: Effects of different coupling topologies," Europhys. Lett. **118**, 10001 (2017).
[11] M. Vaiana and S. F. Muldoon, "Multilayer brain networks," J. Nonlinear Sci. **30**, 2147–2168 (2020).
[12] Y. Zhuang and O. Yağan, "Information propagation in clustered multilayer networks," IEEE Trans. Netw. Sci. Eng. **3**, 211–224 (2016).
[13] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. Szolnoki, "Statistical physics of human cooperation," Phys. Rep. **687**, 1–51 (2017).
[14] F. Nazarimehr, S. Panahi, M. Jalili, M. Perc, S. Jafari, and B. Ferčec, "Multivariable coupling and synchronization in complex networks," Appl. Math. Comput. **372**, 124996 (2020).
[15] S. Majhi, M. Perc, and D. Ghosh, "Chimera states in a multilayer network of coupled and uncoupled neurons," Chaos **27**, 073109 (2017).
[16] J. Ma, L. Huang, J. Tang, H.-P. Ying, and W.-Y. Jin, "Spiral wave death, breakup induced by ion channel poisoning on regular Hodgkin–Huxley neuronal networks," Commun. Nonlinear Sci. Numer. Simul. **17**, 4281–4293 (2012).
[17] L. Lu, Y. Jia, M. Ge, Y. Xu, and A. Li, "Inverse stochastic resonance in Hodgkin–Huxley neural system driven by Gaussian and non-Gaussian colored noises," Nonlinear Dyn. **100**, 877–889 (2020).
[18] S. Malik and A. Mir, "Synchronization of Hindmarsh Rose neurons," Neural Netw. **123**, 372–380 (2020).
[19] A. Bahramianl, A. Nouril, F. Towhidkhah, and S. Jafari, "Introducing neural-network based model and pretraining method to design central pattem generator (CPG)," in *2019 26th National and 4th International Iranian Conference on Biomedical Engineering (ICBME)* (IEEE, 2019), pp. 15–20.
[20] S. Mostaghimi, F. Nazarimehr, S. Jafari, and J. Ma, "Chemical and electrical synapse-modulated dynamical properties of coupled neurons under magnetic flow," Appl. Math. Comput. **348**, 42–56 (2019).
[21] N. Zandi-Mehran, S. Jafari, S. M. R. H. Golpayegani, F. Nazarimehr, and M. Perc, "Different synaptic connections evoke different firing patterns in neurons subject to an electromagnetic field," Nonlinear Dyn. **100**, 1809–1824 (2020).
[22] J. Ma, L. Mi, P. Zhou, Y. Xu, and T. Hayat, "Phase synchronization between two neurons induced by coupling of electromagnetic field," Appl. Math. Comput. **307**, 321–328 (2017).
[23] S. Krishnagopal, J. Lehnert, W. Poel, A. Zakharova, and E. Schöll, "Synchronization patterns: From network motifs to hierarchical networks," Philos. Trans. R. Soc. London A **375**, 20160216 (2017).
[24] A. Agarwal, N. Marwan, M. Rathinasamy, B. Merz, and J. Kurths, "Multi-scale event synchronization analysis for unravelling climate processes: A wavelet-based approach," Nonlinear Process. Geophys. **24**, 599–611 (2017).
[25] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," Phys. Rep. **469**, 93–153 (2008).
[26] R. D. Traub and R. Wong, "Cellular mechanism of neuronal synchronization in epilepsy," Science **216**, 745–747 (1982).
[27] P. Sauseng and W. Klimesch, "What does phase information of oscillatory brain activity tell us about cognitive processes?," Neurosci. Biobehav. Rev. **32**, 1001–1013 (2008).
[28] B. K. Bera, D. Ghosh, and M. Lakshmanan, "Chimera states in bursting neurons," Phys. Rev. E **93**, 012205 (2016).
[29] D. Dudkowski, Y. Maistrenko, and T. Kapitaniak, "Occurrence and stability of chimera states in coupled externally excited oscillators," Chaos **26**, 116306 (2016).

[30] D. Dudkowski, J. Wojewoda, K. Czołczyński, and T. Kapitaniak, "Transient chimera-like states for forced oscillators," Chaos **30**, 011102 (2020).
[31] T. Chouzouris, I. Omelchenko, A. Zakharova, J. Hlinka, P. Jiruska, and E. Schöll, "Chimera states in brain networks: Empirical neural vs modular fractal connectivity," Chaos **28**, 045112 (2018).
[32] D. C. Gooding, K. E. Luh, and K. A. Tallent, "Evidence of schizophrenia patients' reduced perceptual biases in response to emotion chimera," Schizophr. Bull. **27**, 709–716 (2001).
[33] N. Frolov, V. Maksimenko, S. Majhi, S. Rakshit, D. Ghosh, and A. Hramov, "Chimera-like behavior in a heterogeneous Kuramoto model: The interplay between attractive and repulsive coupling," Chaos **30**, 081102 (2020).
[34] S. Majhi, M. Perc, and D. Ghosh, "Chimera states in uncoupled neurons induced by a multilayer structure," Sci. Rep. **6**, 39033 (2016).
[35] M. Mikhaylenko, L. Ramlow, S. Jalan, and A. Zakharova, "Weak multiplexing in neural networks: Switching between chimera and solitary states," Chaos **29**, 023122 (2019).
[36] E. Rybalova, V. Anishchenko, G. Strelkova, and A. Zakharova, "Solitary states and solitary state chimera in neural networks," Chaos **29**, 071106 (2019).
[37] J. Patrycja, S. Brezetsky, R. Levchenko, D. Dudkowski, T. Kapitaniak, and Y. Maistrenko, "Solitary states for coupled oscillators with inertia," Chaos **28**, 011103 (2018).
[38] F. Hellmann, P. Schultz, P. Jaros, R. Levchenko, T. Kapitaniak, J. Kurths, and Y. Maistrenko, "Network-induced multistability through lossy coupling and exotic solitary states," Nat. Commun. **11**, 592 (2020).
[39] S. Majhi, T. Kapitaniak, and D. Ghosh, "Solitary states in multiplex networks owing to competing interactions," Chaos **29**, 013108 (2019).
[40] Z. Hagos, T. Stankovski, J. Newman, T. Pereira, P. V. McClintock, and A. Stefanovska, "Synchronization transitions caused by time-varying coupling functions," Philos. Trans. R. Soc. **377**, 20190275 (2019).
[41] T. Stankovski, "Time-varying coupling functions: Dynamical inference and cause of synchronization transitions," Phys. Rev. E **95**, 022206 (2017).
[42] S. Rakshit, B. K. Bera, D. Ghosh, and S. Sinha, "Emergence of synchronization and regularity in firing patterns in time-varying neural hypernetworks," Phys. Rev. E **97**, 052304 (2018).
[43] S. Rakshit, S. Majhi, B. K. Bera, S. Sinha, and D. Ghosh, "Time-varying multiplex network: Intralayer and interlayer synchronization," Phys. Rev. E **96**, 062308 (2017).
[44] I. V. Belykh, V. N. Belykh, and M. Hasler, "Blinking model and synchronization in small-world networks with a time-varying coupling," Physica D **195**, 188–206 (2004).
[45] A. Destexhe and E. Marder, "Plasticity in single neuron and circuit computations," Nature **431**, 789–795 (2004).
[46] F. Parastesh, C.-Y. Chen, H. Azarnoush, S. Jafari, and B. Hatef, "Synchronization patterns in a blinking multilayer neuronal network," Eur. Phys. J. Spec. Top. **228**, 2465–2474 (2019).
[47] Z. Wang, S. Baruni, F. Parastesh, S. Jafari, D. Ghosh, M. Perc, and I. Hussain, "Chimeras in an adaptive neuronal network with burst-timing-dependent plasticity," Neurocomputing **406**, 117–126 (2020).
[48] T. Murata, "Petri nets: Properties, analysis and applications," Proc. IEEE **77**, 541–580 (1989).
[49] J. Ashraf, J. Ahmad, A. Ali, and Z. Ul-Haq, "Analyzing the behavior of neuronal pathways in Alzheimer's disease using Petri net modeling approach," Front. Neuroinf. **12**, 26 (2018).
[50] V. P. Metta, K. Krithivasan, and D. Garg, "Modeling spiking neural systems using timed Petri nets," in *2009 World Congress on Nature & Biologically Inspired Computing (NaBIC)* (IEEE, 2009), pp. 25–30.
[51] M. Herajy and M. Heiner, "Modeling and simulation of multi-scale environmental systems with generalized hybrid Petri nets," Front. Environ. Sci. **3**, 53 (2015).
[52] F. Liu and M. Heiner, "Modeling membrane systems using colored stochastic Petri nets," Nat. Comput. **12**, 617–629 (2013).
[53] M. Herajy and M. Heiner, "Adaptive and bio-semantics of continuous Petri nets: Choosing the appropriate interpretation," Fundam. Inform. **160**, 53–80 (2018).
[54] F. Liu, M. Heiner, and D. Gilbert, "Fuzzy Petri nets for modelling of uncertain biological systems," Brief. Bioinform. **21**, 198–210 (2020).
[55] A. Sackmann, M. Heiner, and I. Koch, "Application of Petri net based analysis techniques to signal transduction pathways," BMC Bioinf. **7**, 482 (2006).
[56] M. Heiner, G. Ventre, and D. Wikarski, "A Petri net based methodology to integrate qualitative and quantitative analysis," Inf. Softw. Technol. **36**, 435–441 (1994).
[57] M. K. Habib and R. W. Newcomb, "Neuron type processor modeling using a timed Petri net," IEEE Trans. Neural Netw. **1**, 282–289 (1990).
[58] R. Kim, J. Gangolly, and P. Elsas, "A framework for analytics and simulation of accounting information systems: A Petri net modeling primer," Int. J. Acc. Inf. **27**, 30–54 (2017).
[59] J. González-Miranda, "Complex bifurcation structures in the Hindmarsh–Rose neuron model," Int. J. Bifurcat. Chaos **17**, 3071–3083 (2007).
[60] I. Belykh, R. Reimbayev, and K. Zhao, "Synergistic effect of repulsive inhibition in synchronization of excitatory networks," Phys. Rev. E **91**, 062919 (2015).
[61] D. Alaçam and A. Shilnikov, "Making a swim central pattern generator out of latent parabolic bursters," Int. J. Bifurcat. Chaos **25**, 1540003 (2015).