Weak synaptic connections may facilitate spiral wave formation under source-sink interactions

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A B S T R A C T

This study explores the interaction between two distinct sites, termed the source and the sink, to analyze the possibility of spiral wave formation. To this aim, a grid of memristive FitzHugh–Nagumo elements is designed to simulate biological excitable media, such as the myocardium. The source, characterized by high excitation levels with a gradual increase in the recovery variable, is primed to generate an excitatory wavefront. In contrast, the sink remains quiescent in excitation yet elevated in the recovery state, thus tending to absorb excitation as it is temporarily unexcitable but soon recoverable. Existing literature on spiral wave formation primarily focuses on rotor formation under source–sink adjacency. This work, on the other hand, examines the potential for re-entrant behavior under spatial separation between the source and the sink. Within the context of eight-nearest neighbor coupling, the results indicate that the time delay for the excitation wave to reach the refractory state of the sink, due to the increased distance, may still facilitate re-entry if the intensity of connections is sufficiently weak. However, beyond a certain threshold of the source–sink distance under weak connections, wave breakage may occur without resulting in re-entry. For the birth of a spiral wave rotor, the tip of the excitatory wavefront must converge with its refractory back, specifically at the outermost contour in the final stage of refractoriness, referred to as the wavetail. The formation of a spiral rotor cannot arise from a wavefront encountering any site earlier in the stage of refractoriness.

1. Introduction

Numerous mechanisms have been identified as potential initiators of cardiac fibrillation. In practice, optical mapping of the epicardial surface has revealed that the transition of a single-core spiral wave into multiple waves can be a fundamental cause of fibrillation [1,2]. The arrhythmias associated with these functional re-entrant waves are a well-documented electrophysiological phenomenon. From a broader perspective, spiral waves represent patterns of self-organized [3] and self-sustained [4–6] activity in spatially extended non-equilibrium systems [7,8]. The intriguing topology of spiral patterns has sparked considerable interest...
in understanding how such unusual yet spontaneous organization is exhibited by excitable ensembles [9], demonstrating a well-coordinated dynamic in both space and time [10,11]. It is recognized that the spiral tip, acting as the generator of re-entry, forms a curved vortex known as the rotor [12]. The rotor in cardiac electrophysiology closely resembles the concept of re-entry. It is characterized by the convergence of the curved excitation wavefront and the refractory wavefront at a phase singularity (PS) site [13,14]. The wavetail comprises cells that have achieved total excitation, or in biological terms, completed the action potential (AP) upstroke and are transitioning back to a resting state during the AP repolarization phase, thus entering a temporary refractory state [12,15].

The refractoriness is a crucial factor as it may lead to wave breakage and the formation of spiral waves [16]. A widely used method to simulate re-entry is the S1-S2 cross-field protocol, which significantly relies on the tissue's refractory period [17]. S1, a series of linear stimuli that generate a plane wave, is followed by a second stimulus, S2, applied when the tissue is partially recovered or refractory. This creates a vulnerable temporal window for applying S2 [18]. The precise placement of S2, applied perpendicularly, is critical, as it can initiate a rotor [19]. The temporal and spatial requirements of the S1-S2 protocol can also be analyzed from the perspective of vortex shedding [20]. Vortex shedding transpires when a wave encounters an obstacle with sharp edges, potentially destabilizing the propagation of the excitation wavefront. This theory may elucidate why spiral rotors anchor to unexcitable obstacles [21–24] or are influenced by the spatial heterogeneity of the medium's refractoriness [18]. It has been evident that the increase in wave curvature occurs as the excitation wavefront reaches a site in the late refractory period. This phenomenon is elucidated by the “source–sink relationship” theory [12]. The highly excited area is regarded as the “source”, while the excitable elements in the late refractory phase are termed the “sink”. Hence, it can be deduced that the simulation of the S1-S2 paired-pulse protocol effectively creates a source region (induced by S2) whose excitation immediately spreads to the sink (formed by S1).

In theoretical studies of excitable media susceptible to the formation of wave patterns, reaction–diffusion models are utilized [25]. In recent investigations of spiral waves, the source–sink relationship has been replicated by carefully choosing initial conditions [26]. Based on the dynamics of model states, the initial assignment of values effectively generates a highly active region as the source and an adjacent area at the edge of refractoriness as the sink. The remainder of the two-dimensional network is often considered silent yet excitable [27]. However, these initial conditions usually assume that the source and sink sites are directly connected, even if the long-range connections are considered [27,28]. Realistically, in an excitable medium, sites with characteristics of sink and source might be spatially separated. Moreover, when examining an excitable medium, the nature of interactions plays a crucial role in determining the collective capacity for wave conduction. In biological media, synaptic connections establish the framework for these interactions, which can be simplified to electrically driven synapses mimicking the diffusion of charged ions [29] or chemical synapses facilitating the nonlinear biochemical transmission [30].

This study investigates the emergence of a spiral rotor from the interaction between source and sink sites, simulated through initial conditions in the memristive FitzHugh–Nagumo model. Our research is grounded in biological principles, aiming to simulate a network that mirrors the fundamental structure responsible for effective signal transmission in excitable tissues, such as the nervous system and cardiac muscle. In the nervous system, interactions primarily occur through electrochemical means, mainly chemical synapses, with the neurotransmitters being released at a local area, the synaptic cleft. However, other forms of biological signal transmission, like field coupling, may exist. The influence of ion exchange across neuronal membranes and the resultant magnetic flux on adjacent cells is an emerging field of study, predominantly supported by recent computational research [31,32]. Nevertheless, since synaptic connections remain fundamental and undeniable, here, the grid of synaptically connected Fitzhugh-Nagumo elements is used for simulating the characters of neuronal ensembles.

The core objective is to find the precise conditions that enable the formation of a rotor, particularly when the source and sink sites are spatially distant. Our research reveals that while the intensity of the connection between these sites plays a crucial role, the pivotal factor for rotor formation is the synchronization of the sink's refractory period with the arrival of the excitation wave. This alignment ensures the necessary dynamical conditions for initiating a re-entry, leading to the development of a stable spiral rotor.

The remainder of this study is structured as follows: Section 2 delves into the mathematical foundation underlying the design of the excitable medium, focusing on the nodal dynamics and the inter-nodal connections. This section also introduces the source and sink sites by assigning specific initial values to designated areas of the plane. Additionally, it presents our three primary scenarios of study, which are characterized by varying spatial distances between the source and sink. In Section 3, the findings related to the impact of nodal connection intensity as the controlling parameter in rotor formation are presented. This section is also involved with a comprehensive analysis of the recovery variable. Finally, Section 4 discusses the principal outcomes of the study and concludes the paper.

2. Methods and models

2.1. Excitable medium design

In the current study, we utilize an improved version of the FitzHugh–Nagumo (FHN) model to simulate the dynamical units of an excitable medium [33]. The FHN model, essentially a reaction–diffusion model, serves as a classic tool for replicating the excitability of myocardial or neuronal networks [34,35]. Traditionally, it consists of two states: one representing the excitable cell’s potential and the other a recovery variable [36]. In the enhanced 3D version, a third component representing magnetic flux has been introduced [37]. This edition incorporates the effects of time-varying electromagnetic fields on the membrane potential resulting from the transmembrane exchange of charged ions, thereby rendering the simulations more realistic [38,39]. The interaction
between magnetic flux and ions is modeled through an elementary circuit element known as a memristor [40] whose effect has contributed to the electrophysiological relevance of numerous biological dynamical models [41]. The mathematics governing the kinetics of the memristive FHN model [42] is as follows:

$$\dot{u}_{ij} = -k_0 u_{ij}(u_{ij} - a)(u_{ij} - 1) - a u_{ij} - k_0 \rho(\varphi_{ij}) u_{ij} + I_{ij}^{syn},$$

$$\dot{v}_{ij} = \left( \varepsilon + \frac{\nu_{ij} \mu_1}{u_{ij} + \mu_2} \right) \left(-v_{ij} - k_0 u_{ij} - a - 1 \right),$$

$$\dot{\varphi}_{ij} = k_1 u_{ij} - k_2 \varphi_{ij}.$$ 

(1)

The $u_{ij}$ variable represents the membrane potential or the excitable state of the FHN unit at nodal index $(i, j)$. It is the primary variable that describes how the excitation of the medium evolves. The $v_{ij}$ state acts as a recovery variable or an inhibitor that modulates the activity of the excitable state represented by $u_{ij}$. It is responsible for the recovery processes that restore the medium to its rest after excitation. The last state, $\varphi_{ij}$, describes the magnetic flux, directly influenced by the excitable state $u_{ij}$.

The parametric configuration of the FHN model offers considerable flexibility in simulations, enabling various temporal responses, including variations in depolarization duration and refractory periods. In our current setup, we have selected the following parameters: $k = 8$, $a = 0.15$, $k_0 = 0.1$, $\varepsilon = 0.008$, $\mu_1 = 0.2$, and $\mu_2 = 0.3$. Additionally, the parameters $k_1 = 0.2$ and $k_2 = 1$ are chosen to define the electromagnetic induction gains. The effect of magnetic flux on the membrane potential equation is incorporated through the memductance function $\rho(\varphi_{ij}) = \frac{d\rho(\varphi_{ij})}{d\varphi_{ij}} = a + 3\beta \varphi_{ij}^2$, where $a = 0.2$ and $\beta = 0.3$ [42].

Note that involving factors such as inhomogeneities in the level of nodal excitability [43] or applying external stimulation [44], whether in the form of different electrical currents or an external magnetic flux, would insert complexity into the wave dynamics. Thus, we deliberately considered a homogeneous lattice with no external stimulation.

The term $I_{ij}^{syn}$ represents the synaptic input to each FHN unit and is designed to mimic the behavior of chemical synaptic connections. Chemical synapses typically exhibit threshold-like responses and tend to saturate when synaptic terminals are highly excited. Thus, a sigmoid function is used in the $I_{ij}^{syn}$ formulation. For each FHN unit, the synaptic input considers interactions with its immediate spatial context, precisely, the eight nearest neighbors, and is as follows:

$$I_{ij}^{syn} = -g_s (u_{ij} - V_{rec}) \times \left( \Gamma (\lambda, \theta, u_{(i+1j)}) + \Gamma (\lambda, \theta, u_{(i-1j)}) + \Gamma (\lambda, \theta, u_{(ij+1)}) + \Gamma (\lambda, \theta, u_{(ij-1)}) + \right) \left( \frac{1}{2} \times \Gamma (\lambda, \theta, u_{(i+1j+1)}) + \Gamma (\lambda, \theta, u_{(i+1j-1)}) + \Gamma (\lambda, \theta, u_{(i-1j+1)}) + \Gamma (\lambda, \theta, u_{(i-1j-1)}) \right),$$

(2)

where the sigmoid function is defined as:

$$\Gamma (\lambda, \theta, u) = \frac{1}{1 + e^{-4u - \theta}},$$

(3)

In the parametric configuration of the chemical synapse, $V_{rec}$ is the voltage level at which the inward and outward currents are in equilibrium. The choice $V_{rec} = 2$ ensures that $(V_{rec} - u_{ij}) > 0$ indicating that the synapse is excitatory. The activation threshold $\theta = 0.25$ is a proper value considering that the FHN voltages are bounded within the range $0 \leq u_{ij} < 1$. This means that any voltage higher than a quarter of the full range is sufficient to activate the synaptic terminal, contributing to the excitation of the network. The steepness of the synaptic activation curve is controlled by the parameter $\lambda$. Research suggests that a relatively high sigmoidal slope is required to simulate spiral waves successfully [30]. Consequently, we set $\lambda = 35$. At last, the $g_s$ signifies the intensity of synaptic connection, which is treated as a control parameter in the present work.

2.2. Initial conditions replicating the sink and source sites

The medium is structured as a $200 \times 200$ lattice comprised of FHN units. In theoretical studies, there are various methods of generating spiral waves. Assigning random initial conditions can lead to the emergence of multiple spiral seeds [45]. Noise can also facilitate the formation of spiral waves as another stochastic factor [32,46]. Random sites with sink and source characteristics may be generated in scenarios with random initial conditions or noise. However, due to the randomness factor, it is not feasibly controllable for the source–sink distance variable. Thus, we employ deterministic initial conditions in the present work to form specific zones with source and sink characteristics.

The majority of the lattice is initially in a quiescent state, with $u_0 = v_0 = \varphi_0 = 0$. We specify distinct sets of initial conditions for the source–sink configuration, as shown in Table 1. The source and sink sets are further explained as follows:

- **Source** identifies a specific rectangular area of the medium with a high excitatory state $u_0 = 2$ and a low recovery variable $v_0 = 0$. The magnetic flux $\varphi_0$ is set to 0. The elevated value of $u_0$ in this region signifies a highly excited state, making it an excitation source. The lack of recovery variable activity $v_0$ indicates that this region can readily propagate excitation to its surroundings.

- **Sink**, another rectangular area of the medium, is configured with a non-excited state $u_0 = 0$ and a significant level of the recovery variable $v_0 = 0.8$. The magnetic flux $\varphi_0$ is slightly elevated to 0.2. The lack of an excitatory state $u_0$ combined with a relatively high recovery state $v_0$ characterizes this region as a sink. It is in a recovery state, making it less likely to propagate excitation and more inclined to absorb it.
It is noted in our experiment that assigning initial values of magnetic flux does not affect the characteristics of the source and sink, allowing for the possibility of setting it to zero. Moreover, the spatial indices depend on the parameter $d$, which determines the source–sink vertical separation. The unit of distance is a single node of the network. In this study, we consider three scenarios based on the values of $d$, considering that our coupling radius is 1 (the eight nearest neighbors). Any distance $d \geq 2$ results in indirect interactions between source and sink units. The three prominent cases in our study are:

- $d = 0$, where the sink and source are adjacent and interact directly.
- $d = 4$, where the sink and source are separated, leading to indirect interactions. The intervening space comprises four rows of initially inactive excitable elements. The synaptic current, originating from the source's excitable input, is directly applied only to its immediate neighboring row due to the 8-nearest neighbor coupling configuration. Consequently, the stimulation of the subsequent three rows is delayed due to the lack of direct synaptic coupling.
- $d = 8$, where the sink and source are further apart, leading to more indirect interactions. The separation includes eight rows of initially silent excitable elements. In this configuration, the propagation of the stimulation current from the source is further delayed across the additional rows due to the doubled number of rows in the gap between the source and sink compared to the case when $d = 4$.

As the nodal distance between the source and sink sites increases, the formation of a spiral rotor is challenged. According to the source–sink relationship theory, the excitation wavefront emitted from the source must reach the sink site during a critical period late in the refractory phase. Initially, the sink is transitioning from unexcitability to excitability. Increasing the distance extends the time for the excitation wavefront to reach the sink. As the dynamic evolution of sink elements progresses, the window of latent refractoriness concludes with the damping of the $v$ state. Thus, we aim to identify the conditions under which the extension of time for the initial interaction between source and sink can still lead to re-entry and rotor formation.

Utilizing the Euler forward method, we conducted simulations on a grid of three-variable FitzHugh–Nagumo (FHN) models with no-flux boundary conditions. We determined that a time step of 0.05 is adequate based on the dynamic evolution of the FHN elements. The total simulation time varies depending on the employed coupling strength, as weaker coupling results in slower propagation of waves. For each scenario analyzed, from Figs. 1 to 3, we have specified appropriate run times in the figure captions. These run times are critical for observing various phenomena: the interaction between the excitation wavefront emitted from the source and the sink, the onset of re-entry, and the development of a fully evolved spiral wave pattern in the medium.

3. Results

3.1. Adjacent source–sink with $d = 0$

In the initial case presented, the source and sink are in direct contact, as delineated by the absence of spatial separation between them. This configuration is depicted in Fig. 1(a), where the boundary dividing the source and sink is located at $y = 100$, indicated by a dashed line on the plane. Initially, as illustrated in Fig. 1(b), the coupling intensity is established at $g_c = 0.045$. At $t = 25$, an initial excitation wave, corresponding to the rectangular shape of the source, is emitted outward. This wave crosses the dashed boundary line and enters the sink region. However, there is no observed symmetry breaking in the wavefront that could potentially lead to re-entrant phenomena. Additionally, the values of the excitation state $v$ are considerably high within the confines of the initial wave, suggesting a state of high excitation within the lattice. Over time, subsequent snapshots at $t = 75, 125, 175$ and 225 illustrate the progression of the excitation within the lattice, without any indication of deactivation or refractory states being achieved. Note that the trapped excitation within the initial wavefront is due to the high coupling strength rather than the adjacency of the source and sink sites. Applying the same coupling intensity to cases with a sink-source distance of $d = 4$ and $d = 8$ yields identical wave dynamics.

Upon reducing the coupling intensity to $g_c = 0.035$, the dynamics of the initial wave emitted from the source undergo a significant change, as shown in Fig. 1(c). Initially, the upper section of the excitation wave is dissipated shortly after it enters into the sink region at $t = 25$. The remaining segment of the wave, now in direct contact with the sink's refractory state, exhibits characteristics conducive to re-entry around the division line, as evident at $t = 75$. Consequently, a stable rotor is formed, leading to the development of a spiral wave pattern within the lattice, as observed at $t = 175$ and 225.

Therefore, the results presented in Fig. 1 suggest that the interaction between source and sink can indeed facilitate re-entry, with the intensity of nodal connections within the excitable medium serving as a critical determinant. The sink possesses an intrinsic property that sets its excitability to a state of late refractoriness. This property must be preserved until the excitation wave reaches the sink, coinciding with the moment the refractory period concludes. It is inferred that with a strong coupling intensity of $g_c = 0.045$, the progression of the excitation within the lattice, without any indication of deactivation or refractory states being achieved. Note that wave, suggesting a state of high excitation within the lattice. Over time, subsequent snapshots at $t = 75, 125, 175$ and 225 illustrate the progression of the excitation within the lattice, without any indication of deactivation or refractory states being achieved. Note that the trapped excitation within the initial wavefront is due to the high coupling strength rather than the adjacency of the source and sink sites. Applying the same coupling intensity to cases with a sink-source distance of $d = 4$ and $d = 8$ yields identical wave dynamics.

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Fig. 1. The potential of spiral wave rotor formation in the excitable medium with adjacent sink-source sites ($d = 0$): (a) The source–sink configuration with the division line at $y = 100$ represented by a dashed line. (b) The plane of the medium’s excitation state $u$. The evolution of the initial excitation wave from the source when the coupling intensity is $g_c = 0.045$. The wave’s integrity is preserved, as no breakage or subsequent re-entry occurs. The snapshots are taken at $t = \{25, 75, 125, 175\}$. (c) The elimination of the upper compartment of the wave upon reaching the sink beyond the division line when the coupling intensity $g_c = 0.035$. The subsequent free end undergoes re-entry, evolving into a stable spiral wave rotor over time. The snapshots are taken at $t = \{25, 75, 175, 225\}$.

this characteristic of the sink is prematurely dissipated, as the highly excited state of the source site accelerates the excitation of the sink’s elements. Higher coupling intensity correlates with faster propagation speeds of the excitation wave. Therefore, when the coupling strength is reduced to $g_c = 0.035$, the terminal phase of sink refractoriness can synchronize with the arrival of the excitation wave, as it is not prematurely extinguished by a strong connection to the excited elements of the source.

3.2. Separated source–sink with distance $d = 4$

The introduction of spatial separation between the hypothesized source and sink sites introduces a significant challenge to the potential for re-entry, as the initial excitation wave emitted from the source must reach the sink within a specific, temporally vulnerable window. Fig. 2(a) illustrates a new configuration with a distance of four nodes ($d = 4$) between the source and sink, where the dashed line marks the lower boundary of the sink at $y = 104$.

Initially, we maintain the previously successful coupling intensity of $g_c = 0.035$, which facilitated re-entry in the $d = 0$ scenario. The results, depicted in Fig. 2(b), indicate that the initial wave successfully traverses the dashed line, entering the sink boundary without disruption, attributed to the sink’s refractoriness and its failure to propagate the excitation. Snapshots taken at $t = 25$, 75, 125, and 225 demonstrate the preservation of the initial excitation wave’s morphology, as it continues to evolve outward. This suggests that the added spatial distance results in the premature loss of the sink’s refractoriness before the arrival of the initial wave at this specific coupling intensity. To counteract the premature dampening of the sink’s refractory qualities, the coupling intensity is further reduced to $g_c = 0.02$. The outcomes, presented in Fig. 2(c), affirm that decreasing the coupling strength proves effective. Initially, at $t = 25$, the excitation wave is observed below the dashed sink boundary line, with its upper portion remaining intact. By $t = 75$, as the initial wave surpasses the dashed line, the sink’s delayed excitation leads to the elimination of the wave’s upper section, creating a free end. Subsequent snapshots at $t = 275$ and 500 illustrate that this free end undergoes re-entry, evolving in
Fig. 2. The potential of spiral wave rotor formation in the excitable medium with separated sink-source distance by $d = 4$: (a) The source-sink configuration with the sink lower boundary at $y = 104$ represented by a dashed line. (b) The plane of the medium's excitation state $u$. The evolution of the initial excitation wave from the source when the coupling intensity is $g_c = 0.035$. No wave breakage or re-entry occurs. The snapshots are taken at $t = \{25, 75, 125, 225\}$. (c) The elimination of the upper compartment of the wave upon passing the sink's boundary when the coupling intensity is $g_c = 0.02$. The subsequent free end undergoes re-entry, evolving into a spiral wave rotor over time. The snapshots are taken at $t = \{25, 75, 275, 500\}$. Weak connections, although give rise to re-entry, they weaken the wavefronts and slow down the wave propagation speed. Yet, the rotor formed in the sink site is not spatially stable and slightly meanders in a closed loop.

3.3. Separated source–sink with distance $d = 8$

In the final scenario, the distance between the source and sink is extended to $d = 8$, with the sink's lower boundary delineated by a dashed line as shown in Fig. 3(a). Initially, the coupling intensity is set at $g_c = 0.02$, the same value that previously facilitated the formation of a stable rotor in the $d = 4$ scenario, with outcomes illustrated in Fig. 3(b). At $t = 25$, the integrity of the initial wave remains intact, positioned below the sink's border. By $t = 125$, the wave has approached the sink, but as predicted, the coupling strength is too high for the wavefront to disintegrate upon entry into the sink region. Consequently, without any indication of re-entry, the wave progresses until it reaches the lattice boundaries, as observed in snapshots at $t = 225$ and 425. It is noteworthy that, in comparison to the initial excitation waves depicted in Figs. 1(b) and 2(b), the wavefront strength appears diminished in Fig. 3(b).

To explore the possibility of re-entry, the coupling strength is reduced to $g_c = 0.01715$, as shown in Fig. 3(c), with snapshots captured at $t = 25, 125, 275,$ and 500. At this specific intensity, the interaction between the excitation wave and the sink disrupts the wave's upper portion, separating it from the remainder of the wave and leaving it with a free end. However, this detached upper
Fig. 3. The potential of spiral wave rotor formation in the excitable medium with separated sink-source distance by \((d = 8)\): (a) The source-sink configuration with the sink lower boundary at \(y = 108\) represented by a dashed line. (b) The plane of the medium’s excitation state \(u\). The evolution of the initial excitation wave from the source when the coupling intensity is \(g_c = 0.02\) with no subsequent wave breakage or re-entry. The snapshots are taken at \(t = \{25, 125, 225, 425\}\). (c) Reducing the coupling intensity to \(g_c = 0.01715\) disrupts the integrity of the initial excitation wave as it passes the sink boundary. However, the disconnected upper compartment preserves, and none of the subsequent free ends undergo re-entry. The snapshots are taken at \(t = \{25, 125, 275, 500\}\). (d) Further decrease in coupling strength to \(g_c = 0.0171\) leads to the total elimination of the upper compartment, yet not followed by re-entry. The snapshots are taken at \(t = \{25, 125, 275, 500\}\). (e) Weakening the connections is not effective for re-entry until \(g_c = 0.0152\) the initial excitation wave is so weak that dies in the locality of the source, without even approaching the sink site. The snapshots are taken at \(t = \{25, 75, 125, 175\}\).
portion remains intact as it exits the sink area, indicating that the sink no longer possesses the capacity to eliminate it. This results in a non-promising pattern where the fragmented initial wave maintains its morphology until it reaches the plane’s boundaries. A further reduction in coupling intensity to $g_e = 0.0171$, as depicted in Fig. 3(d), facilitates the elimination of the initial wave’s upper portion as it crosses the dashed line, demonstrated in the initial two snapshots at $t = 25$ and 125. This minor adjustment in coupling intensity, in the order of $10^{-5}$ effectively preserves the sink’s refractoriness until the arrival of the initial wave. Although the elimination of the upper portion is necessary for symmetry breaking and the creation of a free end, re-entry necessitates the interaction between the tip of the fragmented excitation wave and the excitable elements of the sink, which have just concluded their refractory phase and are ready to be excited again. In the last two snapshots of Fig. 3(d), taken at $t = 275$ and 500, it is evident that the free end of the fragmented wave, altered by the spatial configuration of the source, fails to re-enter, preventing the formation of a spiral wave rotor. Further decreasing the strength of nodal connections, as attempted in Fig. 3(e) with $g_e = 0.0152$, results in the initial wave from the source dissipating near its origin without even crossing the sink border, as shown in snapshots at $t = 25$, 75, 125, and 175, due to insufficient nodal connectivity for excitation propagation.

The findings from Fig. 3 underscore that as the sink becomes more distant from the source, the dynamic evolution of sink refractoriness is not sufficiently delayed by weak nodal connections, suggesting that distant excitable sites with the properties of a sink and a source are unlikely to interact specifically. Thus, assessing the sink-source interaction theory indicates that such interactions are also contingent upon perfect timing, as suggested by the SI-S2 protocol.

It is important to note from Figs. 1 to 3 the spatiotemporal activity of the excitation state $u$ is recorded. $u$ state highlights the fast dynamics of the FHN model. Conversely, the recovery state $v$ represents slow dynamics, and the degrees of refractoriness and the evolution of the excitation wave can be better analyzed by recording the spatiotemporal pattern of the $v$ state. This analysis is crucial for determining the condition of the sink as the initial wave reaches it, elucidating why re-entry occurs in some instances while failing in others.

### 3.4. Recovery variable analysis

The $v$ slow variable plays a pivotal role in distinguishing the excitation wavefront (denoted as $E_f$) and the following tail is the refractory wave back (denoted as $R_b$). $E_f$ aligns with the initial wave in the $u$ plane, exhibiting high values in both $u$ and $v$ states, while $R_b$ is characterized by inactive $u$ state and high yet decreasing values of $v$ state. The approximate range of $v \in (0.05, 0.5)$ signifies a state of neither excitability nor activity, thus representing the late phase of refractoriness that can only be detected in the $v$ plane. The in-time interaction between $E_f$ and $R_b$, when $E_f$ has been broken, is a sufficient condition for rotor development.

In Fig. 4(a), the experiment with a source-sink distance of $d = 4$ and a coupling intensity of $g_e = 0.02$ is replicated to further elucidate the dynamics between the $u$ and $v$ states. The plots corresponding to the $u$ plane at $t = 150$ and 250 are accompanied by $v$ plane plots, revealing a wider range of $v$ values spanning approximately $v = 0$ to $v = 2$. The highest values of $v$, depicted in green, correlate with the excitation wavefront observed in the $u$ plane, thereby designated as the “wavefront”. This wavefront is followed by contours of lower $v$ values, the counterparts of which in the $u$ plane are inactive, indicating a silent refractoriness facilitated by the slow dynamics of the $v$ state. The last distinguishable contour is designated as the “wavetail”.

Fig. 4(b) presents the temporal activity of the node at coordinates (110, 100) to clarify the interplay between the $u$ and $v$ states. As previously noted, the $u$ state exhibits rapid dynamics, characterized by swift upstroke and damping. In contrast, the simultaneous evolution of the $v$ state is markedly slower in attaining its peak, with the activation and partial inactivation of the $v$ state corresponding to the full activation phase of the $u$ state. This period, specified by two dashed lines, corresponds to the wavefront as identified in Fig. 4(a). The subsequent inactivation phase of the $v$ state, which aligns with the complete quiescence of the $u$ state, is referred to as the wavetail. It is not until the $v$ value decreases to approximately 0.05 that this node becomes capable of being excited once again.

Note that refractory back $R_b$ is situated within the excitation wavefront $E_f$, presenting as a tail, not a front. Furthermore, the green color of the excitation wavefront $E_f$ is encased by a light blue contour on the outside, termed a refractory front. However, this concept is not informative as we explore the quality of re-entry. Here is a brief explanation: As the excitation reaches a zone immediately following the upstroke of excitation, that zone undergoes a period of refractoriness. Concurrently, the excited response aligns with a preliminary increase in the $v$ state, followed by an increase in the $u$ state. Conversely, the refractory back $R_b$ is a contour where the $v$ is decreasing while the $u$ state has already been dampened.

The different outcomes observed in Figs. 2(c) and 3(d) regarding the potential re-entrant behavior following the successful elimination of the upper compartment in the $u$ plane can be attributed to the specific interactions between the excitatory wavefront $E_f$ and the different contours of the refractory waveback. To enhance the visualization of the refractory wave back contours, all $v$ values greater than 0.4 are mapped to $v = 0.4$. Thus, as a threshold is applied to the $v$ plane, we call it $v_{th}$ plane. In such a plane, the $E_f$ is presented in dark red. The most refractory zones are depicted in cyan, symbolizing $R_b$, which is in the early phase of refractoriness, whereas the wave tail represents the contour at the edge of concluding refractoriness. For the generation of a spiral rotor, it is crucial that the excitatory wavefront $E_f$ encounters the refractory wave tail.

Initially, the sink conditions are set with $v_0 = 0.8$, indicating that at the commencement of the simulation, it is in the early refractory phase, thus possessing the characteristics of $R_b$ rather than those of the wave tail. As time progresses, the $v$ values of the sink decrease, transitioning towards the late phase of refractoriness. Consequently, when $E_f$ encounters the sink under conditions of late refractoriness, it leads to the wavefront meeting the wave tail, thereby creating a singularity point, known as a “Phase Singularity (PS)'', or a spiral rotor.
Fig. 4. The analysis of fast excitation state $u$ and slow recovery state $v$: (a) The respective spatiotemporal activity in the $u$ plane (upper row) is depicted in the $v$ plane (lower row). The excitatory wavefront in the $u$ plane corresponds to a wavefront high in the recovery variable in the $v$ plane. This excitatory front $E_f$ is followed by contours of the refractory back $R_b$, each in different phases of refractoriness, approaching its final stage at the wavetail. (b) The excited response of an arbitrary node, as an instance nodal index $(110,100)$ in the medium, hit by the excitatory wavefront. The rapid upstroke of the excitation state $u$ is followed by a slow, smooth increase in the $v$ variable. From this upstroke until the $u$ activity is almost damped corresponds to the wavefront. As the activation state $u$ becomes silent, the recovery variable $v$ has reached halfway, keeping the FitzHugh–Nagumo element at a stage of refractoriness; this period corresponds to the wavetail.

In Fig. 5, the visualization of sink and source sites is enhanced by not rendering their respective $v$ values as entirely silent, providing clearer insights into the dynamics leading to successful and unsuccessful re-entrant behavior, as evidenced in the comparison between Fig. 2(c) (successful case at $d = 4$) and 3(d) (failed case at $d = 8$).

In the successful scenario depicted in Fig. 5(a1-d1), at Fig. 5(a1), the source is in a state of early refractoriness $R_b$ following the outward propagation of the initial wave, leaving remnants of the eliminated upper compartment at the sink’s entrance. By Fig. 5(b1), the free end of the initial wave, or the tip of $E_f$, reaches the sink, whose $v$ values have further diminished, signifying a transition to the late phase of refractoriness. Consequently, the area immediately in front of the wavefront tip, specifically a node at the sink’s edge, becomes excitable. The wavefront tip, upon entering this node, effectively re-enters itself by rotating around the sink’s edge, as illustrated in Fig. 5(c1). The resultant evolved spiral pattern in Fig. 5(d1) clearly shows the PS site, characterized as a non-excited area encircled by the excitatory wavefront and the recently concluded refractory wave tail.

Conversely, Fig. 5(a2-d2) portray the sequence leading to the failed case in Fig. 3(d). The delayed elimination of the initial wave’s upper compartment, as shown in Fig. 5(a2), leads the wavefront tip to encounter the refractory site created by the upper compartment rather than directly engaging with the sink nodes, as depicted in Fig. 5(b2). This delay allows further damping of the $v$ state values at the sink site, resulting in a loss of proper excitability, as demonstrated in Fig. 5(c2). As a result, the sink now presents low $v$ values, rendering it as excitable as the silent nodes surrounding both sites. This prevents the wavefront tip from forming a curved path towards the refractory tail, leading it instead to propagate outward across the silent plane, as evidenced in Fig. 5(d2).

4. Discussion

In cardiac electrophysiology, the depolarized region that conducts excitation is termed the source, while the cardiomyocytes in a resting state, positioned ahead of the excitation wavefront, are defined as the sink [47]. Thus, within this framework, the cardiac tissue sites exhibiting source and sink characteristics are typically considered adjacent. In such scenarios, the concept of source–sink mismatch is extensively studied [48]. A well-matched system is characterized by the source generating an impulse that the sink can fully absorb, facilitating the expected propagation of the excitation wave. However, if the sink area is insufficiently large or lacks the necessary excitability to manage the incoming excitation, this leads to incomplete or halted wave propagation [12]. This mismatch state has been replicated in various studies, for instance, by creating regions capable of rapid excitation wave propagation resulting from source activity [49] or by introducing local obstacles that make a mismatch in wave conduction at different sites within the medium [50].
Fig. 5. Convergence of the wavefront and the wavetail shown in thresholded $v$ ($v_{th}$) plane: Any value $v \geq 0.4$ is mapped to $v = 0.4$, to better distinguish contours of refractoriness following the wavefront. The left snapshots correspond to Fig. 2(b) ($d = 4$ and $g_s = 0.02$) where the spiral wave rotor has successfully been formed. The right snapshots correspond to Fig. 3(d) ($d = 8$, $g_s = 0.0171$) where the formation of the rotor fails despite wave breakage. In the successful case on the left (a1-d1), the timely elimination of the wave’s upper compartment in the sink and the wavefront tip’s meeting with the final stage of refractoriness of the sink (shown at (b1)), compel the wave tip to converge inwardly, maintaining contact with an unexcitable node that becomes immediately excitable. The excitation revolves around an unexcited locus, known as a phase singularity (PS). In the failed case on the right (a2-d2), the upper compartment of the wave is eliminated too late, leading to the wavefront tip’s meeting with the sink at a time when it has excessively lost its refractoriness due to significant dampening of the $v$ state (shown at (c2)). Consequently, sink nodes become as excitable as the rest of the plane nodes, preventing re-entry.

In the context of our work, the mismatch state between source and sink is analyzed from the standpoint of excitability. Specifically, the mismatch between the source and sink is attributed to the degree of the sink’s refractoriness. Here is a detailed analysis of the importance of mismatch caused by refractoriness property.

In our study, we observed that within the recovery variable plane, the contours of waves – each characterized by varying levels of the recovery variable – were of great importance. The highest values of $v$, correlating with non-zero $u$, defined the wavefront. Immediately after the wavefront traversed a site within the medium, the nodes became non-excited. According to the FitzHugh–Nagumo model’s excitable response, a value of $v = 0.4$ corresponds with $u = 0$. Therefore, any value $v > 0.4$ was considered part of the wavefront. However, this wavefront was followed by contours with decreasing lower $v$ values. The last contour before quiescence was identified as the wavetail. The wavetail represented the final stage of refractoriness, and any $v$ value between this phase and $v = 0.4$ was considered in the early stage of refractoriness.

The sink had to be in a refractory stage akin to the wavetail to facilitate two critical steps in the formation of a spiral wave rotor. First, the excitation wave approaching the sink’s boundaries had to be eliminated by the sink’s refractoriness. Thus, adequate refractoriness was essential at the precise moment the excitation wave, emitted from the source, reached the sink. Second, as the sink was in the final stage of refractoriness, it lost its refractory capacity immediately following wave breakage. Consequently, the free end of the broken excitation wave formed a curved trajectory, aided by the newly excitable sink nodes positioned directly in its path.

The results indicated that with increasing source–sink distance, the intensity of nodal coupling required reduction to sustain the re-entrant process without interruption. Although weakened connections resulted in diminished wavefronts, they could delay the loss of the sinks’ refractoriness. For distances up to 8 nodes, the time it took for the excitation wave to reach the sink with appropriate refractoriness could be adjusted by reducing coupling intensity. At a distance of 8 nodes, diminishing the intensity could lead to eliminating a wave segment, creating a free end yet failing to induce re-entry. It was revealed that the absence of re-entry, despite wave breakage, was due to the excitatory wavefront converging with refractory back contours in an early phase of refractoriness rather than a late phase. The added spatial distance introduced a time delay to the wave breakage that could not be compensated, ultimately leading to the death of the sink’s refractoriness.
5. Conclusion

This study constructed a 200 × 200 grid of excitable elements, each governed by the memristive FitzHugh–Nagumo model and interconnected via eight nearest-neighbor chemical synapses. Initially, the medium had two distinct sites: a highly excited ‘source’ ready to propagate excitation and a ‘sink’ in the late refractory phase, poised to absorb excitation—this configuration allowed for exploring rotor formation, a concept postulated within spiral wave theory. Our investigation focused on the potential for spiral wave rotor emergence when the source and sink are not adjacent—a novel concept. We examined the impact of varying distances between the source and sink, specifically at intervals of 4 and 8 nodes, with these gaps consisting of initially inactive FitzHugh–Nagumo elements.

Our findings indicated that increased source–sink distance posed challenges to spiral rotor formation. Re-entry occurred only at lower intensities of eight nearest-neighbor coupling as the distance expanded. Further analysis of the medium on the recovery-variable plane revealed that a reduction in coupling intensity slowed the sink’s dynamic evolution, maintaining it in an optimal refractory state. This synchronization allowed the late refractory phase of the sink to align precisely with the arrival of the excitation wave that had traversed the gap between the source and sink. It was observed that the sink’s late refractory phase initially contributed to the breakage of the excitation wave. Subsequently, as the sink fully recovered and became excitable, the free end of the excitation wave traced a curved path, leading to re-entry.

CRediT authorship contribution statement

Dorsa Nezhad Hajian: Writing – original draft, Software, Conceptualization. Fatemeh Parastesh: Writing – original draft, Validation, Methodology. Karthikeyan Rajagopal: Writing – original draft, Validation, Methodology, Investigation. Sajad Jafari: Writing – review & editing, Supervision, Methodology. Matjaž Perc: Writing – review & editing, Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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