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Social diversity promotes cooperation in spatial multigames

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Abstract – Social diversity is omnipresent in the modern world. Here we introduce this diversity into spatial multigames and study its impact on the evolution of cooperation. Multigames are characterized by two or more different social dilemmas being contested among players in the population. When a fraction of players plays the prisoner's dilemma game while the remainder plays the snowdrift game cooperation becomes a difficult proposition. We show that social diversity, determined by the payoff scaling factors from the uniform, exponential or power-law distribution, significantly promotes cooperation. In particular, the stronger the social diversity, the more widespread cooperative behavior becomes. Monte Carlo simulations on the square lattice reveal that a power-law distribution of social diversity is in fact optimal for socially favorable states, thus resonating with findings previously reported for single social dilemmas. We also show that the same promotion mechanism works in time-varying environments, thus further generalizing the important role of social diversity for cooperation in social dilemmas.

Cooperative phenomenon exists widely in the real world, ranging from animal to human societies [1]. However, how to understand why selfish players are willing to donate to the collective income at individual cost remains unclear. This confusion is normally investigated using the evolutionary game theory [2–6] as a classical theoretical framework. Furthermore, the prisoner's dilemma game and the snowdrift game as the exemplifications for addressing the cooperative phenomenon have received substantial attention [7–29]. For example, in a standard prisoner's dilemma game played by two players, each should simultaneously decide whether to cooperate or to defect. Since a defecting player will acquire the maximum payoff if encountering a cooperative player, the emergence of cooperation faces an enormous challenge. In fact, the entire population resorts to defection within the prisoner's dilemma game in well-mixed populations [4]. Therefore, much research has been devoted to exploring mechanisms that can bring about the promotion of the cooperative strategy among selfish players.

In order to resolve social dilemmas, where selfish players pursuing short-term individual benefits might lead to the tragedy of the commons [30], an impressive amount of research has been carried out in this field over the past years [31–41]. In ref. [36], Nowak has discussed five classic mechanisms for the promotion of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, as well as group selection. In ref. [37], Nowak and May have first introduced the prisoner's dilemma game on the square lattice. They have found that cooperators form compact clusters and so escape the exploitation of the defecting players, thus facilitating cooperative behavior. Following this significant breakthrough, network reciprocity attracted considerable attention [42–47]. In ref. [42], Santos and Pacheco have discovered that scale-free networks promote the emergence of cooperation both in the snowdrift game and the prisoner's dilemma game far beyond the boundaries imposed by regular lattices. Moreover, coevolutionary rules where strategies of players and other properties simultaneously evolve have been investigated [48–54], further enriching the mechanisms for raising the degree of cooperation. In addition, several other approaches have been considered that may favorably influence the evolution of cooperation, like payoff noise [55–59], strategic complexity [60–67], inhomogeneous activity of players [68,69], populations of mobile individuals [70], as well as multilayer networks [71–76].

Recently, evolutionary multigames [77–79] or mixed games [39] have attracted more attention and brought pivotal progression in evolutionary game theory. Cressman *et al.* have initially analyzed a two-decision two-player model where agents may adopt different strategies in different situations, and they showed that the eventual state of the game could be represented by the dynamics of the separate game [80]. Wang *et al.* have studied evolutionary multigames in structured populations inspired by different individuals with a different perception under the same social dilemma. Research revealed that the level of cooperation could be enhanced because of the application of different payoff matrices [81].

Our main motivation in this letter is rooted in the fact that while diversity can boost cooperation [82–84], we are likely to perceive diversity differently, and moreover, will likely be unaware of its utilization in different contexts. We expect that the introduction of social diversity in the spatial multigames can enlarge the range of impact of the evolutionary multigames on the promotion of cooperation. In this paper, we study the role of the adoption of social diversity in the spatial multigame environment in the advance of the cooperative strategy. The term multigame environment is fulfilled by individuals adopting distinct magnitude of the sucker's payoff, where a portion of the players plays the snowdrift game while the remaining portion of the players plays the traditional prisoner's dilemma game. It is worth mentioning that the mean payoff matrices turn to the weak prisoner's dilemma game on account of the equal distribution of positive and negative magnitude of the sucker's payoff. Meanwhile, we investigate the social diversity of individuals determined by random variables which are obtained from the uniform, exponential, or power-law distribution and thus generating several disparate kinds of diversity. The random variables might increase or decrease the value of payoffs, and they depict various social states of game participants. Our outcomes show that regardless of the distribution types of random variables, social diversity can improve considerably the levels of cooperation in the spatial evolutionary multigames. Particularly, the power-law distribution of the scaling factor causes the greatest facilitation of cooperative behavior among the entire range of parameters. Moreover, for the purpose of testing the robustness of our primary research, we also consider the facilitative impact of social diversity on cooperation in a time-varying multigame environment, where we observe similarly positive evolutionary outcomes.

In the continuation of this letter we first describe the evolutionary multigame on the square lattice and the introduction of social diversity that is realized by scaling factors from the three different distributions. Next we present the main results from Monte Carlo simulations, and finally conclude with a discussion and possible directions for future research.

We study evolutionary multigames with individuals located on the square lattice with periodic boundary conditions. Every participant interacts only with its $k = 4$ nearest neighbors. Initially, each player situated on the square lattice could choose cooperation (C) or defection (D) with equal probability. Furthermore, we calculate the payoffs of players in pairwise games according to the standard form [37]. If both participants choose cooperation, they will gain the reward R. Moreover, if one defector competes with one cooperator, the defector gains the temptation T while the cooperator gains the sucker's payoff S. If both participants choose defection, both of them gain the punishment P. Inspired by the previous work concerning the multigames [81], we employ diverse S values to characterize that the identical social dilemma can be perceived variously by various individuals. Especially, half of the entire randomly selected population applies $S = -\Theta$, whereas the other half applies $S = +\Theta$, where $0 < \Theta < 1$. In other words, we have a part of the players which plays the snowdrift game while the other part of the players plays the traditional prisoner's dilemma. We use the same distribution of negative and positive S values among the population, thus entire payoff matrices turn to $S = 0$ the weak prisoner's dilemma.

In the former literature as regards the evolutionary multigames, players utilize the payoff matrices where most of the elements are the same [79,81]. Nevertheless, because diversity is ubiquitous in the reality, we introduce social diversity into the spatial multigame environment. Motivated by the preceding work with respect to the prisoner's dilemma game [82], we rescale the payoffs as

$$\Psi' = \Psi(1 + \xi), \tag{1}$$

where Ψ is either T, R, P or S, $\xi = \min(\xi_i, \xi_j)$, and ξ_i or ξ_j is a scaling factor acquired randomly from a prescribed distribution for each game participant just once at the beginning of the simulation. In this paper, we primarily consider three different distributions of the scaling factor, namely the uniform, the exponential, and the power-law distribution, which are described by the following expressions:

$$\xi = a * (-2\chi + 1), \tag{2}$$

$$\xi = a * (-\ln\chi - 1), \tag{3}$$

$$\xi = a * (\chi^{-1/2} - 2). \tag{4}$$

Here χ is a uniformly distributed random number from the unit interval, and $\int_0^1 \xi(\chi)\mathrm{d}\chi = 0$ in all cases, thus yielding the mean of ξ in the whole population equal to zero. The coefficient a ($0 \le a \le 1$) determines the amplitude of fluctuations of social diversity, such that the greater the value of a, the greater the amplitude of fluctuations. Specially, $a = 1$ is the maximum value that still assures $(1 + \xi_i) \ge 0$ for any i, while $a = 0$ returns the original setup without social diversity.

The spatial distributions of the three different social diversity for $a = 1$ are exhibited in fig. 1. We can see clearly that the mildest dispersion of the scaling factor is guaranteed by the uniform distribution, whereas the power-law

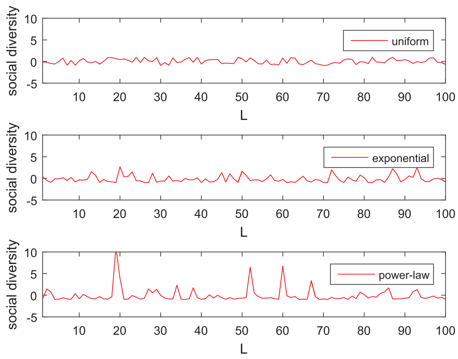

Fig. 1: (Colour online) Spatial distribution of scaling factors drawn from a uniform (top), exponential (middle), and power-law (bottom) distribution, as obtained for $a = 1$. Each panel shows a characteristic 1D cross-section of the square lattice.

distribution offers the greatest segregation of social diversity. The exponential distribution of ξ results in a dispersion of the scaling factor which is between the uniform and power-law distribution.

All players will get corresponding payoffs after each instance of the game, and they simultaneously accumulate payoffs by interacting with their closest four neighbors. Moreover, all individuals simultaneously renew their strategies after every complete iteration cycle of game. A player i randomly chooses one neighbor j, then player i imitates the strategy S_j from player j with the probability decided by the difference of their overall payoffs [85]:

$$W(s_i \leftarrow s_j) = \frac{1}{1 + \exp[(\Pi_i - \Pi_j)/K]}, \tag{5}$$

where $K = 0.1$ determines the uncertainty in the strategy adoption process. We simulate the evolutionary multigames according to the classic Monte Carlo simulation method, and simulations are primarily performed on $N = 100 \times 100$ square lattices. Near phase transition points we have further increased the system size to avoid accidental extinctions and to ensure suitable accuracy. The fraction of cooperators f_c over the entire population is used to measure the level of cooperation in the system. We acquire the fraction of cooperators in the stationary state by calculating the average value over the last 2000 full Monte Carlo steps after sufficiently long transients are discarded. To further improve accuracy, the final results are averaged over 20 independent realizations.

Then, we investigate the effect of social diversity on the promotion of cooperation by simulation experiments of the above-mentioned spatial multigames. According to the previous literature [48,86,87], we assume that cooperative players can coexist with defecting players in spatial populations due to network reciprocity. Besides, the multigame setting captured by adopting disparate values of the sucker's payoff can improve the level of cooperation. What is more, the introduction of social diversity in multigame environment can further influence the cooperative behavior.

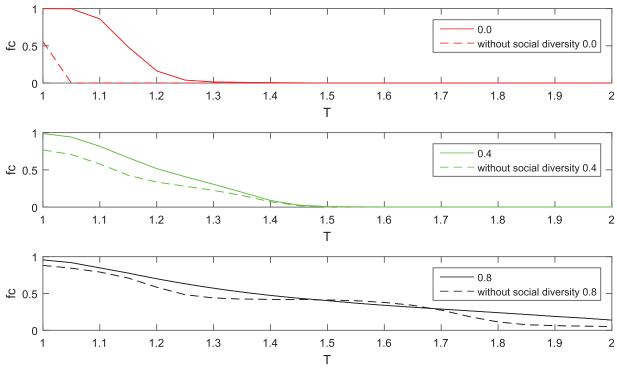

Fig. 2: (Colour online) Fraction of cooperators f_c on the square lattice in dependence on the temptation to defect T, as obtained with and without uniformly distributed social diversity for three different values of Θ (see legend). Other parameter values are $a = 1$ and $K = 0.1$.

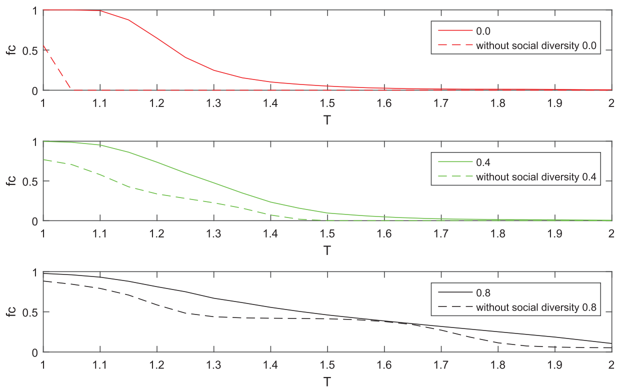

Fig. 3: (Colour online) Fraction of cooperators f_c on the square lattice in dependence on the temptation to defect T, as obtained with and without exponentially distributed social diversity for three different values of Θ (see legend). Other parameter values are $a = 1$ and $K = 0.1$.

In fig. 2, fig. 3 and fig. 4, we show that cooperator density f_c changes with temptation T for three various distributions of the scaling factor and three different values of Θ when $a = 1$, $K = 0.1$. We can observe that the greater the value of temptation T, the smaller the value of cooperator density f_c irrespective of the distribution types of social diversity and the value of the sucker's payoff. Moreover, it can be seen that the introduction of social diversity in the spatial multigame environment can promote the degree of cooperation significantly regardless of distribution cases of social diversity in most of the range of parameters. Obviously, we can find that the fraction of cooperative individuals f_c increases as the value of Θ increases for all three diverse distributions of the scaling factor. Meanwhile, as shown in fig. 5, we show that the fraction of cooperative individuals f_c varies with temptation T for four cases, while $\Theta = 0.4$, $a = 1$ and $K = 0.1$. The revealed results suggest that the power-law distribution produces the greatest promoter of cooperative behavior and the uniform distribution causes the worst promoter of cooperative behavior

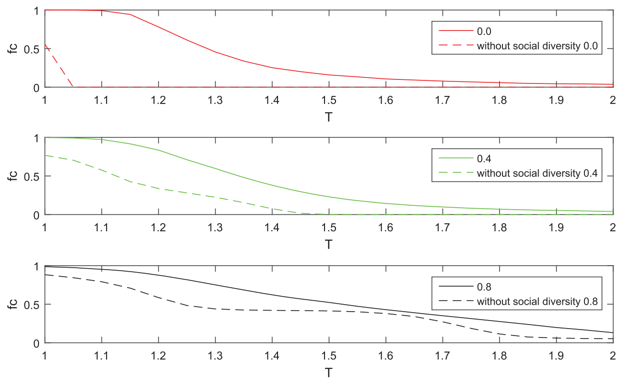

Fig. 4: (Colour online) Fraction of cooperators f_c on the square lattice in dependence on the temptation to defect T, as obtained with and without power-law distributed social diversity for three different values of Θ (see legend). Other parameter values are $a = 1$ and $K = 0.1$.

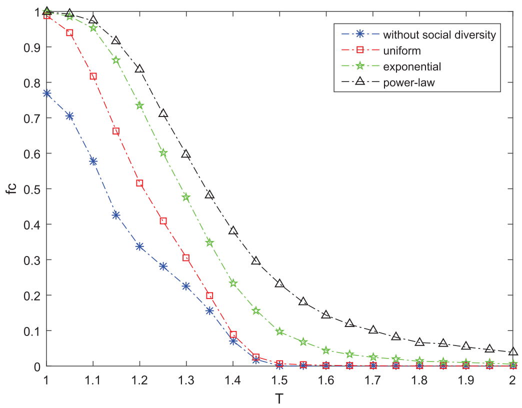

Fig. 5: (Colour online) Fraction of cooperators f_c on the square lattice in dependence on the temptation to defect T, as obtained without social diversity, as well as with uniformly, exponentially, and power-law distributed social diversity (see legend). It can be observed that power-law distributed social diversity is most successful in ensuring cooperation in the studied multigame. Other parameter values are $\Theta = 0.4$, $a = 1$ and $K = 0.1$.

across the entire range of T. We can see that the critical value of the cooperative behavior extinction improves from $T = 1.50$ without social diversity to $T = 1.55$ (the uniform distribution of social diversity), $T = 2$ (the exponential distribution of social diversity), and cooperative players exist in the whole range of T for the power-law distribution.

Our consideration for the promotion of cooperation mentioned above is due to the introduction of the heterogeneous fitness of individuals, which facilitates robust cooperative clusters around those individuals that possess the large values of ξ. In fact, when cooperative individuals occupy the main sites of the lattice (analogous to the hubs of a scale-free or similar network), they begin spreading their strategy in the form of compact clusters. That is to say, cooperative individuals may prevail on the whole network due to clusters of cooperators, where they

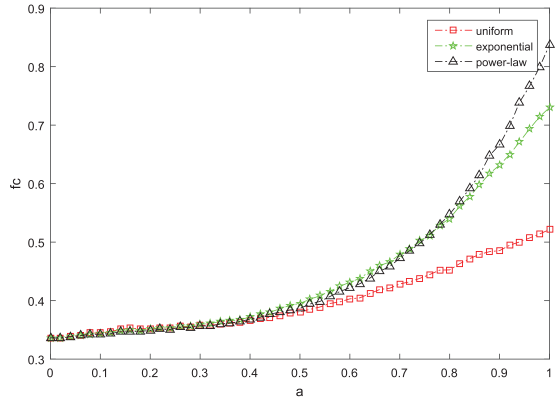

Fig. 6: (Colour online) Fraction of cooperators f_c on the square lattice in dependence on the amplitude of social diversity a, as obtained for the uniform, exponential, and power-law distributed social diversity (see legend). As in fig. 5, it can be observed that power-law distributed social diversity is most successful in ensuring cooperation. Other parameter values are $\Theta = 0.4$, $T = 1.2$ and $K = 0.1$.

cooperate with each other. Nevertheless, the defective individuals are in the absence of this feature and so they fail to make use of social diversity to diffuse their strategies. A similar case is that the evolutionary game in scale-free graphs, and the individuals with the maximum connection can control the results of the evolutionary game. Once cooperators occupy the hubs of the network, they can spread cooperative behavior rapidly, while defectors cannot reach this effect. Besides, the power-law distribution of social diversity presents the strongest inhomogeneous of the scaling factor, as exhibited in fig. 1. Those participants with low ranking of the scaling factor may follow the high-ranking participants and resist the exploitation of defectors.

We present the effects of amplitude a of social diversity on the promotion of cooperation for different distributions of the scaling factor in fig. 6. Clearly, the level of cooperative behavior might be markedly elevated as the value of a increases in the three different situations. In accordance with the expressions of the scaling factor ξ, it can be seen that the amplitude a affects the fluctuation of the scaling factor. Practically, the greater value of the amplitude a gives rise to the greater fluctuation for the scaling factor as well as the greater part of low-ranking players.

Finally, we further examine the robustness of our primary conclusions in a time-varying multigame setting. In reality, individuals often perceive the same social dilemma differently over time. With respect to our model, the same players employ different values of the sucker's payoff in every game. That is to say, players adopt $S = +\Theta$ or $S = -\Theta$ with equal probability in every game instead of possessing a constant value of the sucker's payoff as discussed above. Surely, the overall mean of the payoff matrices turns to $S = 0$, and thus to the weak prisoner's dilemma game. According to the above-presented results

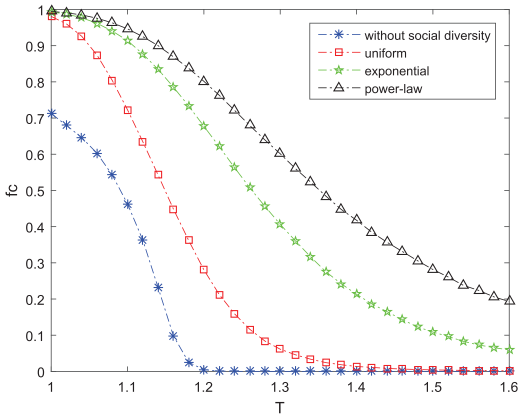

Fig. 7: (Colour online) Fraction of cooperators f_c on the square lattice in dependence on the temptation to defect T, as obtained without social diversity, as well as with uniformly, exponentially, and power-law distributed social diversity (see legend), this time in a time-varying multigame (compare with fig. 5). As above, it can be observed that power-law distributed social diversity is most successful in ensuring cooperation, even in a time-varying multigame, thus further generalizing the important role of social diversity for cooperation in social dilemmas. Other parameter values are $\Theta = 0.4$, $a = 1$ and $K = 0.1$.

obtained for a classical multigame, the introduction of social diversity in the spatial multigame can enhance the level of cooperative behavior. Figure 7 displays how the fraction of cooperators f_c changes with T at $\Theta = 0.4$, $a = 1$, and $K = 0.1$ in a time-varying spatial multigame. One can observe that in the time-varying multigame environment, cooperation is extensively promoted for all the three distributions of the scaling factors if compared with the time-varying multigame without social diversity. Moreover, as before, cooperation is promoted best if the scaling factors that determine social diversity come from a power-law distribution.

To sum up, we have investigated the effects of social diversity in spatial multigames on the evolution of cooperation. In particular, we have primarily considered multigames where one half of the population plays the snowdrift game while the other half plays the traditional prisoner's dilemma game. The social diversity was introduced by means of the uniform, exponential, or the power-law distribution of the scaling factors that multiply the payoff matrix. According to the outcomes of Monte Carlo simulations, we conclude that, regardless of the distribution type of the scaling factors, social diversity promotes cooperation in multigame environments. Specifically, the power-law distribution induces the greatest facilitation of the cooperative behavior while the uniform distribution yields the lowest facilitation of the cooperative behavior, and this holds across the entire range of the temptation to defect. The enhancement of cooperative behavior is due to the introduction of various social states of competing individuals, which facilitates the emergence of robust cooperative clusters around the individuals that possess the large values of the scaling factor. To be specific, once cooperators occupy important locations of the square lattice, they can propagate their cooperative strategy to the cluster-forming feature. We have also shown, expectedly, that the larger the value of Θ, the higher the concentration of cooperators on the square lattice. What is more, with the purpose of verifying the robustness of our primary results summarized above, we have also studied the stimulative impact of social diversity on cooperation in time-varying evolutionary multigames, where we have observed the same results as in static evolutionary multigames.

Our results thus reaffirm the significance of social diversity for the successful evolution of cooperation, and we also hope they provide further insights into the resolution of social dilemmas among selfish players, in particular where multigames are at play. Indeed, it is easy to imagine that payoff matrices are composed of mixtures of different games at different times, or that different players adopt different matrices when playing, as representative of the natural environment and different circumstances. In this respect, it is easy to imagine further research along similar lines, for example by considering other evolutionary games constituting the multigame environment, where the previous research on universal scaling parameters of the social dilemma strength will be of value [88,89]. Further, it would of course be very interesting to test these theoretical predictions in human experiments and in practical applications [90–94]. A simple descriptive example of a multigame is how the owner of a cheap car can have a very different risk perception on a highway crossing compared to the owner of a new expensive car. Recent research has already revealed that this is an important consideration with far-reaching consequences for the outcome of evolutionary games [39,77–79,81,95,96], and with this letter we hope to add further to this inspiring avenue of research.

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