

Cumulative advantage is a double-edge sword for cooperation

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Abstract – The Matthew effect emphasizes the influence of early advantage on shaping long-term development by amplifying it over time, and its implications for public cooperation are yet to be fully understood. In this letter, we propose and study a spatial public goods game driven by cumulative advantage, where players who achieve high payoffs in a given round receive greater allocations in the next. The simulation results show that the Matthew effect leads to an irreversible polarization of individual wealth on the network. Such polarization makes moderate cooperation levels more prevalent, which helps to explain the widespread coexistence of prosocial and antisocial behavior. Meanwhile, heterogeneous networks may restrict the polarization of wealth, but also inhibit the evolution of cooperation, requiring a reconsideration of the commonly held view that heterogeneous networks enhance cooperation.

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Introduction. – Cumulative advantage, often referred to as the Matthew effect, posits that success leads to further success, and advantages tend to accumulate over time [1,2]. This phenomenon typically leads to an unequal distribution of resources and opportunities, and creates the polarisation of wealth, power and influence [3,4]. Moreover, a positive feedback process of self-reinforcement may arise where the early advantages can lead to the accumulation of further benefits, while those struggling may fall further behind. It is also currently widely valued and well researched in various fields such as science [5], education [6], economics [7] and social networks [8]. For example, Merton first investigated the presence of the Matthew effect in the scientific community, leading to unequal distribution of rewards and recognition [9]. By analysing data from academic grant programs, Bol found that recipients just above the grant threshold accumulated more grants than non-recipients with similar review scores [5]. A recent study on complex systems [10] confirmed the existence of positive feedback and the accumulation of

advantages in real-world data, revealing the prevalence of the Matthew effect in various domains such as scientific collaboration [11], socio-technical and biological networks [12,13], citation propagation [14]. This study presents convincing empirical evidence for the widespread existence of the phenomenon.

The issue of unequal resource allocation and social unfairness [15,16], which are important consequences of the Matthew effect, has received significant attention in the field of evolutionary cooperation, as evidenced by studies such as [17,18]. Of growing interest to scholars is the design of effective wealth distribution systems, since theoretical research has demonstrated their potential to address social dilemmas [19–22]. For example, the promotion of the common interest is evident when individuals are allowed to redistribute their contributions in accordance with their previous round earnings [23]. Su *et al.* extend the spatial public goods game to a three-layer weighted network, consisting of the investment layer, the benefit distribution layer, and the strategy dispersion layer. Their results show that the structure of the exchange investment and benefit distribution layers has no

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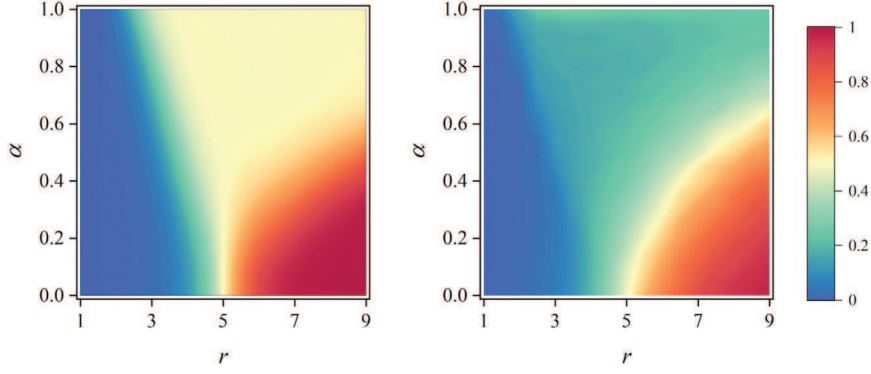


Fig. 1: The stationary fraction of cooperation, F_C , as a function of the synergy factor and the weight factor α . The simulation results depicted in the left panel were obtained from a lattice network with a size of $N = 400 \times 400$, while those in the right panel were obtained from a BA scale-free network with a size of $N = 10000$.

impact on the evolutionary dynamics. However, the success of cooperators is found to be heavily dependent on the correlation between an individual's investment in a given game and the benefits they receive from that same group [24].

Previous studies have established the significant impact of wealth inequality on public cooperation [25,26]. However, the Matthew effect emphasizes the dynamics of wealth allocation over time, indicating the need for further investigation into the co-evolutionary relationship [27,28] between wealth distribution and behavioral decisions. In this study, we introduce a co-evolving spatial public goods game model [29–32] that captures the cumulative advantage by linking wealth accumulation in earlier rounds to the distribution in subsequent rounds. Our results demonstrate a dual effect of cumulative advantage. It is shown that cooperation is strengthened in stronger original dilemmas and weakened in weaker dilemmas. This is attributed to the self-organization and polarization of the allocation of wealth distribution among individuals on the networks. This leads to the prevalence of moderate levels of cooperation, explaining the prevalence of coexistence of pro- and antisocial behaviors in social interactions. Note that many real-world factors contain similar dual effects; for example, the hysteresis effect can act as a roadblock to restoring measles vaccination rates despite a resurgent measles epidemic that poses a significant risk of infection to these deliberately unvaccinated individuals [33]. Furthermore, given the crucial impact of interactive networks on cooperative behavior and the widespread use of heterogeneous networks [34,35], we also investigated the effects on scale-free networks. Our findings indicate that while heterogeneous networks may slightly decrease the overall level of cooperation relative to lattice, they also prevent excessive wealth differentiation.

Model. – The co-evolutionary public goods game is performed on a network, in which players are randomly arranged on the nodes of the network and interact with their k_i neighbors. Each player participates in $G = k_i + 1$

overlapping groups, each of which consists of a focal player and his k_i direct neighbors. In each group of the game, cooperators contribute $c = 1$ into the common pool, while defectors contribute nothing. The total contribution is multiplied by a synergy factor r , and then allocated to the group members. The payoff of player x from each group g can be expressed as

$$P_D^g = \frac{w_x}{\sum_g w} n_C r,$$

$$P_C^g = P_D^g - c,$$

where n_C is the number of cooperators in group g . w_x is the player x 's distribution weight. As a result of the Matthew effect, the distribution weight is positively updated with the last round of returns, *i.e.*,

$$w_x = \begin{cases} w_x + \delta, & \text{if } P_x > \bar{P}, \\ w_x, & \text{if } P_x = \bar{P}, \\ w_x - \delta, & \text{if } P_x < \bar{P}, \end{cases} \quad (1)$$

where $\delta = 0.01$ is a free parameter. $P_x = \sum_g P_x^g$ is the cumulative payoff that player x obtains by interacting with all his neighbors, *i.e.*, as a member of the group $g = 1, \dots, G$ of the public goods game. Similarly, \bar{P} is the average cumulative payoff of his neighbors. Obviously, this evolutionary rule of weight and strategy leads to individuals with high returns in the early stages having a greater advantage in further competition, which is also known as the Matthew effect. Note that w_x takes values in the range $[1 - \alpha, 1 + \alpha]$, to ensure that there are no negative allocation weights.

As individuals tend to seek high payoffs in social interactions, the best response rule [36] is applied to indicate whether player x changes his present strategy to the alternative one with probability $\Gamma = 1/(1 + \exp[(P_x - P'_x)/K])$, where P'_x is the imaginary payoff that player x would receive when he chooses the alternative strategy s'_x , where $K = 0.5$ describes the potential uncertainty in the strategy update process.

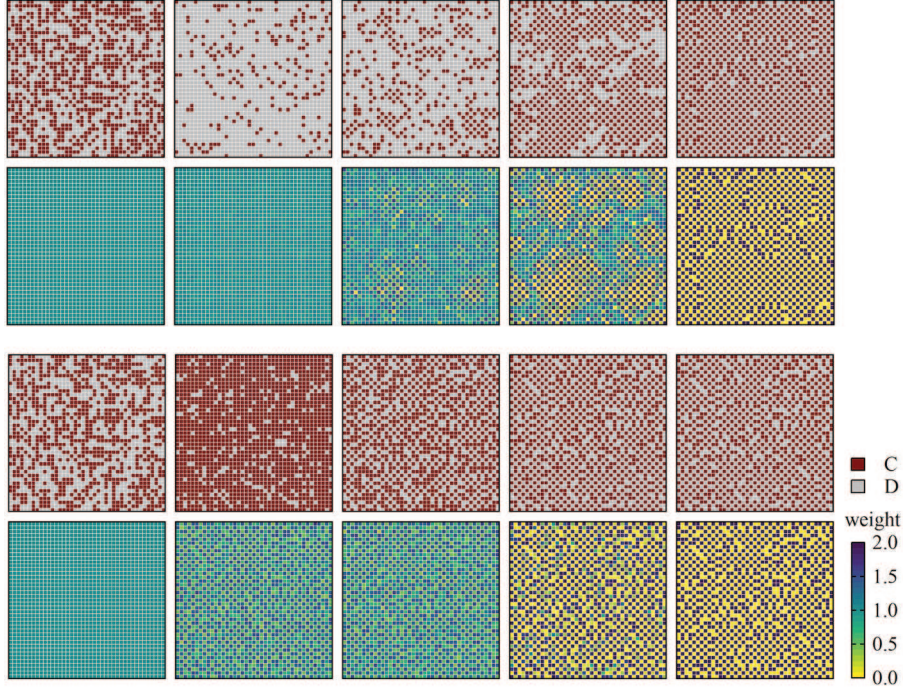


Fig. 2: The spatiotemporal dynamics of strategy and distribution weights on the lattice. Specifically, the first (last) two rows show successive snapshots from a random strategy distribution and homogeneous distribution weights for the case when $r = 4$ ($r = 6$). A very small network size of $N = 40 \times 40$ was adopted for the clear observation of spatial characteristics. The results in the first two rows are obtained at $t = 0, 8, 100, 150, 50000$, while the last two rows are obtained at $t = 0, 10, 40, 1000, 50000$. The other parameter is $\alpha = 1$.

At the start of the simulation, individuals located on the network nodes have equal allocation weights, which are initially set to 1, and adopt a random strategy of either cooperation or defection. In each subsequent Monte Carlo step, each player has an equal chance to update their strategy and allocation weight based on the coevolutionary rules described above. The quantitative cooperation frequency presented in this paper is determined by averaging the last 5000 steps out of 50000 steps. The network employed in the spatial game is a lattice with von Neumann neighbors and periodic boundaries and a BA scale-free network [37] with average degree $\bar{k} = 4$, respectively. Note that the quantitative results of the scale-free network were obtained by averaging the outcomes of 20 independent simulations, with a new network being regenerated for each simulation.

Result. — Figure 1 depicts the stable fraction of cooperators as a function of the synergy factor, r , and α , in the lattice (left panel) and BA scale-free network (right panel). The left panel reveals the potential dividing line at $r = 5$, separating the heat map into two halves. On the left half, the cooperation rate gradually increases with the increase of α , whereas on the right half, the cooperation rate decreases significantly with increasing α . This highlights the dual impact of heterogeneous allocation driven by the Matthew effect, which promotes moderate cooperation in the system, particularly when

the value of α is high. In the scale-free network, the right panel depicts an expansion of the region with a medium cooperation rate as α increases. Notably, this rate is slightly lower than that of the lattice network, likely due to differences in network topology and evolutionary dynamics.

Next, we elucidate the mechanisms underlying the Matthew effect in terms of its facilitation of cooperation when r is small and its inhibition when r is large. The continuous spatial distribution of strategies and corresponding weights are depicted in fig. 2, which can be further investigated through the animations in the Supplementary Movies `Dynamics_in_the_lattice_when_r=4.avi` and `Dynamics_in_the_lattice_when_r=6.avi`. The upper panels present the results for $r = 4$, while the lower panels show those for $r = 6$. In both cases, the strategies and weights are self-organized into cross-like structures. It is noteworthy that the formation of this patch structure is a common occurrence in spatial snowdrift games or extortion game, with the Nash equilibrium dictating the interdependence of cooperation and defection [38,39]. Obviously, the phenomenon of spatial self-organization in this study is driven by the intrinsic evolution of allocation weights. In the lower r case, which witnessed a rapid elimination of cooperators in the early stages of the evolutionary system, only a few high-weighted cooperators survive and hold a visible allocation advantage. However, while this advantage serves as a motivation for their

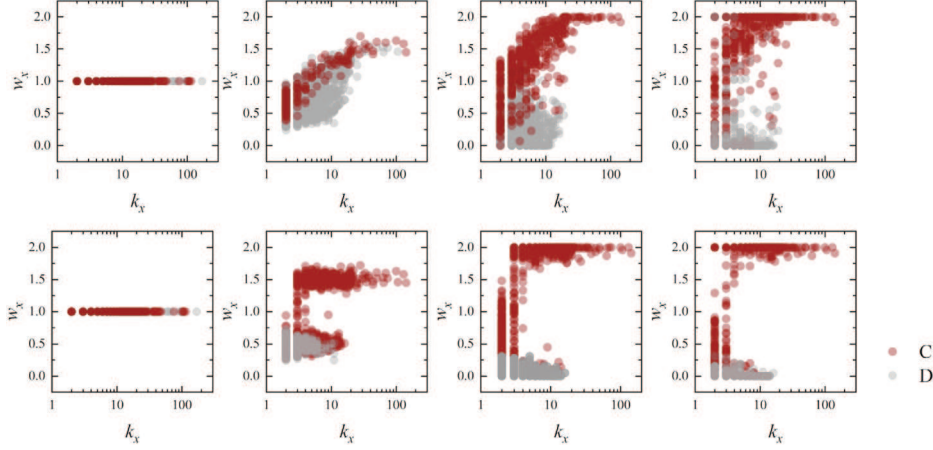


Fig. 3: Strategy distribution on scale-free networks. Wine represents the cooperator and gray represents the defector. The allocation weights evolve over time and are polarized by the Matthew effect. Meanwhile, the cooperators occupy more of the large degree nodes and with high allocation weights, while the defectors are the opposite. Compared to the results of the lattice, it can be observed that there are still a few individuals who have moderate weights in the stable period of evolution, which means that it somewhat prevents the problem of extreme polarization. The top (second) row shows results for $r = 3$ ($r = 9$), while the columns correspond to $t = 0, 50, 100$, and 50000 . The other parameters are $\alpha = 1$ and $N = 10000$.

cooperation, it appears exploitative and detrimental to their direct opponents. This phenomenon expands and solidifies over time, resulting in the stable distribution of weights in the cross-like view. On the other hand, when r is higher, cooperators experience an initial expansion but eventually reach the similar balance with defectors. The emergence and expansion of this unique structure is rooted in the establishment of weight differences, which reduces or eliminates the risk and temptation for cooperation and defection, respectively. As a result, high-weighted individuals opt to cooperate while low-weighted individuals defect, leading to the development of a special reliance between the two. It is crucial to note that our strategy update approach determines the spontaneous emergence and reinforcement of highly weighted cooperators, regardless of whether the early victor is cooperators or defectors.

The evolutionary dynamics of cooperation on a scale-free network are presented in the animations in the Supplementary Movies `Dynamics_in_the_scalefree_network_when_r=3.mp4` and `Dynamics_in_the_scalefree_network_when_r=9.mp4`, where the node size in wine red and gray represents the distribution weights of cooperators and defectors. Over time, it can be observed that the node size co-evolves with the strategy. Our results align with observations on the lattice, where early disadvantages (advantages) tend to be reinforced in subsequent evolution, end with the strong choosing to cooperate and the weak choosing to defect. However, the small network size adopted in the animation does not fully reveal the intrinsic mechanism of cooperation evolution. To further illustrate this, fig. 3 plots the strategy and weight distribution at key moments for $r = 3$ (first row) and $r = 9$ (second row). Starting with the first row, in the early stage

of evolution (the second column), defectors with varying weights quickly dominate nodes with different degrees. It becomes apparent that the small number of cooperators have a higher average weight than the defectors. Over time, node weights become polarized, with defectors losing their dominance over large-degree nodes and cooperators no longer occupying small-degree nodes. Given the scale-free network's power-law distribution, cooperators with larger degrees only hold a small proportion in the stable state. In the second row, the primary distinction from the first row is mainly reflected in the temporary dominance achieved by the cooperators in the early evolution, but this does not affect the similar characteristics in the further stable case. The observed tendency for cooperators to occupy high-degree nodes and defectors to occupy low-degree nodes in this heterogeneous network is attributed to the higher likelihood of obtaining high returns and competitive advantage for players in high-degree nodes, thus reducing the risk of cooperation. Although the low proportion of high-degree nodes in scale-free networks results in a relatively low cooperation rate, the heterogeneous topology also leads to the richness in node weights (wealth allocation), which to some extent reduces social polarisation and inequity.

Conclusions. — In this study, we examine the impact of heterogeneous allocation driven by cumulative advantage on public cooperation on network. We introduce the Matthew effect into social interactions by implementing a co-evolutionary individual weight that links winning in one round of interaction to the allocation advantage in the next. It may create a bifurcation in the distribution of social wealth, which is strongly associated with class entrenchment. This situation presents a significant

challenge that cannot be resolved without effective policy intervention.

Interestingly, in a poor social environment, significant distributional inequalities driven by cumulative advantage can enhance overall wealth and cooperation, but as the social environment improves, it hinders the spread of pro-social behavior. Moreover, the structure of social interactions is a crucial factor in the evolution of public cooperation and equity, and while heterogeneous social networks may not always be conducive to cooperation, they can aid in promoting social equity.

This study reveals that the coexistence of pro-social and anti-social behavior in society can be attributed to the double-edged sword effect of the Matthew effect. This implies that balancing social cooperation and general equity is a critical challenge for policymakers. Nevertheless, some important mechanisms proposed in previous studies, such as network rewiring [40] and migration [41], may potentially help to break the self-reinforcing process of cumulative advantage and mitigate the risk of polarization, in addition to helping to improve public cooperation.

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Data availability statement: No new data were created or analysed in this study.

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