



# Microeconomic uncertainties facilitate cooperative alliances and social welfare

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## Abstract

We show that microeconomic chaotic variations of payoffs in the prisoner's dilemma game maintain cooperation over a broad range of defection temptation values where otherwise economic stalemate reigns. Thus, unpredictability at micro scales impedes mutual defection that inflicts social poverty.

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## 1. Introduction

One of the fundamental assumptions of microeconomic theory, originating from the Darwinian concept of “only the fittest survive”, which is that firms always act so as to maximize their profit is in reality often challenged or at least has to be seen in a broader context. In particular, short-term lucrative alliances often inhibit long-term benefits due to partner exploitation and overall infliction of poverty. Thus, firms in different markets have realized long ago that a fearsome competition for the temporarily highest payoff is often less favorable than a cooperative alliance that assures slightly smaller but steady and reliable

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incomes. However, the urge to outperform a competitor and harvest the highest possible profit is often stronger than the tendency to choose the more reasonable cooperative behavior.

A mathematical concept that can be used to describe the above scenario is given by the prisoner’s dilemma game, as argued by Axelrod (1984), where two players or firms, about to engage in a joint enterprise, have to decide simultaneously whether they want to cooperate or defect. The dilemma is given by the fact that although mutual cooperation yields the highest collective payoff, a defector will do better if the opponent cooperates. In the long run this fact inflicts mutual defection that ultimately results in an irreversible economic decline and social stalemate, as succinctly reviewed by Crawford (2002). Although standing firm on mathematical proofs, this unfavorable result is, however, often violated in real life. Thus, scientists search for possible mechanisms that explain the discrepancy between theory predictions and experimental observations, not just in economy (e.g., Möller, 2005), but also in ecology as well as mathematics and physics in general (Doebeli and Hauert, 2005).

Presently, we report a new mechanism that facilitates cooperation among firms that participate in a spatially extended prisoner’s dilemma game where players are arranged on a square grid. The virtue of our approach is to link the intrinsic rules of the game with microeconomic uncertainties that are assumed to be chaotic. In particular, we study effects of additive chaotic variations, introduced to the payoff matrix of the spatial prisoner’s dilemma game, on the evolution of cooperation. The microeconomic chaotic disturbances are modeled by the Behrens-Feichtinger model (e.g., Holyst et al., 1996). We show that an appropriately pronounced microeconomically-based chaos is able to sustain cooperation even for defection temptation values substantially exceeding the one marking cooperation extinction in the absence of explicit payoff variations. Importantly, this is achieved by full anonymity of players and incognito actions (for related literature see e.g., Fudenberg and Maskin, 1986 or Piccione, 2002), as well as without the aid of secondary strategies (e.g., Hauert et al., 2002). Moreover, we show that there exists a well-defined amplitude of chaotic disturbances by which cooperation promotion is best enhanced and thus the overall social welfare, estimated by the average payoff of each participating firm or individual, is maximized.

## 2. The game

We consider an evolutionary prisoner’s dilemma game with players located on vertices of a regular two-dimensional square lattice of size  $n \times n$  (Nowak and May, 1992). For simplicity, but without loss of generality, we assume that each individual interacts only with its four nearest neighbors located to the north, south, east and west, whereby self-interactions are excluded. Each player can decide either to cooperate (C) or to defect (D). Depending on the choice of their strategies, each two players ( $P^i, P^j$ ) receive payoffs summarized succinctly by the payoff matrix(1)

$P^i/P^j$	C	D
C	$1+a\tilde{x}^i / 1+a\tilde{x}^j$	$1+\kappa+a\tilde{x}^i / -\kappa+a\tilde{x}^j$
D	$\kappa+a\tilde{x}^i / 1+\kappa+a\tilde{x}^j$	$0+a\tilde{x}^i / 0+a\tilde{x}^j$

(1)

where  $\kappa \geq 0$  determines the temptation to defect whilst  $a \geq 0$  scales the amplitude of chaotic payoff variations, which are modeled by the rescaled variable  $x \rightarrow \tilde{x}$  of the Behrens-Feichtinger model (e.g., Holyst et al., 1996):

$$x_{m+1} = (1-\gamma)x_m + \frac{\alpha}{1 + \exp[-c(x_m - y_m)]}, \quad (2)$$

$$y_{m+1} = (1-\eta)y_m + \frac{\beta}{1 + \exp[-c(x_m - y_m)]}. \quad (3)$$

Parameter values  $\alpha=0.16$ ,  $\beta=0.9$ ,  $c=105$ ,  $\gamma=0.46$  and  $\eta=0.7$  yield chaotic behavior (see right panel of Fig. 1), whereby we rescale variable  $x \rightarrow \tilde{x}$  prior to introducing it into the payoff matrix so that it satisfies  $\tilde{x} \in [-0.5, 0.5]$  in order to assure symmetrical additions to the payoffs over time. Importantly, in order to assure spatially non-correlated disturbances among players, each player is ascribed with its own Behrens-Feichtinger model, as indicated by the subscripts  $i$  and  $j$  in Eq. (1). Starting from random initial conditions for each of the  $n \times n$  Behrens-Feichtinger models, and uniformly distributed cooperators and defectors on the square lattice, each player can adopt its strategy according to the performance of neighboring players, whereby the probability that a player  $P^i$  will adopt the strategy of one of its randomly chosen nearest neighbors  $P^j$  is determined by the cumulative payoffs  $S^i$  and  $S^j$  of both players according to

$$W(P^i \leftarrow P^j) = \frac{1}{1 + \exp[(S^i - S^j)/K]}, \quad (4)$$

where  $K$  is the uncertainty related to the strategy adoption.  $0 < K \ll 1$  implies that the better performing player is readily adopted, whilst it is not completely impossible to adopt the strategy of a worse

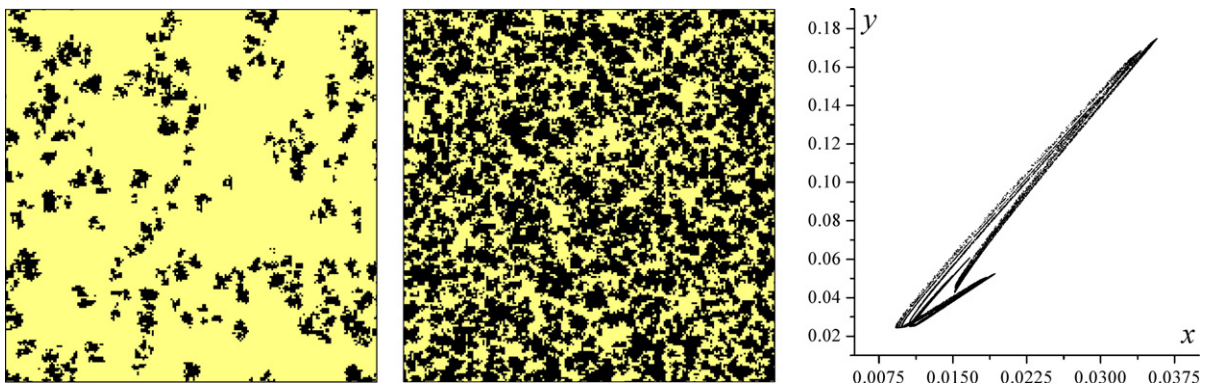


Fig. 1. Characteristic equilibrium spatial distributions of cooperators (black) and defectors (yellow) obtained by  $a=0$  (left panel) and  $a=0.24$  (middle panel) for the defection temptation value  $\kappa=0.006$ . Both panels are depicted on a  $200 \times 200$  spatial grid. Right panel features the phase space portrait of the non-rescaled Behrens-Feichtinger model for the presently applied system parameters. The rescaled variable  $x \rightarrow \tilde{x}$ , modeling microeconomic uncertainties and fluctuations, is introduced additively to the payoffs of each player, thereby boosting the fraction of cooperators from 10% (left panel) to 58% (middle panel). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

performing player. We presently apply  $K=0.1$  but note that deterministic strategy adoption rules, *i.e.* best player is always imitated, yield qualitatively identical results as will be reported below, but often yield step-like transitions which is presently unwanted since it would decrease comparability of results obtained by different  $\kappa$  and  $a$ .

The described spatial prisoner's dilemma game is iterated forward in time using a synchronous update scheme, letting all individuals interact pairwise with all their nearest neighbors and then simultaneously update their strategy according to Eq. (4). After each synchronous game iteration the profile of microeconomic chaotic variations imposed to the payoffs of each player varies as dictated by the discrete numerical integration procedure of Eqs. (2) and (3).

### 3. Results and discussion

With the presently applied game settings only 10% of cooperators are able to survive if the temptation to defect equals  $\kappa=0.006$  and  $a=0$  in Eq. (1). Remarkably, the addition of weak chaotic variations to the payoffs, determined by  $a=0.24$ , is able to boost the fraction of cooperators to nearly 60% by the same  $\kappa$ . Fig. 1 captures this phenomenon. Note that with or without the addition of chaotic variations cooperators survive by forming clusters so as to protect themselves against being exploited by defectors. Cooperators located in the interior of such clusters enjoy the benefits of mutual cooperation and are therefore able to survive despite the constant exploitation by defectors along the cluster boundaries.

To quantify the ability of chaotic variations to facilitate and maintain cooperation in the studied spatial prisoner's dilemma game more precisely, we calculate the fraction of cooperators over a broad range of  $\kappa$  in the absence and by a fixed magnitude  $a$  of chaotic variations. Results presented in Fig. 2 show that microeconomic chaotic variations promote cooperation even for defection temptation values that are an order of magnitude larger than the threshold for cooperation extinction by  $a=0$ .

Above results clearly show that additive chaotic payoff variations promote cooperative alliances

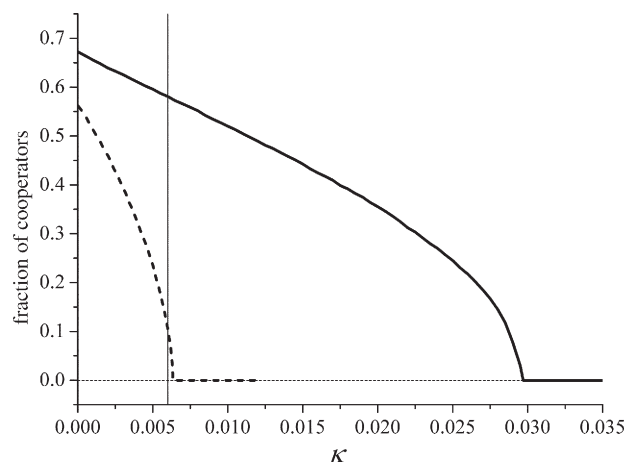


Fig. 2. Promotion of cooperation by microeconomic chaotic uncertainties. In the absence of chaotic payoff variations cooperators die out at  $\kappa=0.0063$  (dashed line) whereas by  $a=0.24$  they persist up to  $\kappa=0.029$  (solid line). Thin vertical line denotes  $\kappa=0.006$  used in Fig. 1.

among firms participating in the spatial prisoner's dilemma game. It remains of interest to determine the optimal  $a$ , by which cooperation is enhanced best. In order to do so we calculate the fraction of cooperators in dependence on  $\kappa$  and, as presented in the left panel of Fig. 3. Evidently, there exist an optimal amount of microeconomic uncertainties by which firms are most prone to engage in cooperative alliances. For the presently applied chaotic microeconomic disturbances we find the optimal value to equal  $a=0.24$  (which we also used for the results in Figs. 1 and 2) irrespective of the temptation to defect, as indicated by the vertical line in the left panel of Fig. 3.

In accordance with the payoff ranking of the prisoner's dilemma game, the social welfare is maximal when the fraction of cooperative firms on the square lattice is the highest, as evidenced in the right panel of Fig. 3. In particular, this means that if the fraction of cooperators is the highest, the summarized average income of all  $n \times n$  firms on the square lattice is maximal. If all firms would cooperate, then, in accordance with Eq. (1) and the applied synchronous iteration scheme, the average income of each firm would be exactly 8 in dimensionless units. On the other hand, if all firms would choose to defect their average income would be 0. Thus, the social welfare is directly proportional to the fraction of cooperators, as can be seen at a glance by comparing the left and right panel of Fig. 3. Importantly, it is crucial to note that some firms in the socially-optimal cooperative scenario earn less as if they would choose to defect. However, such payoff boosts can only be of short lasting nature, since once all cooperators vanish and there is no one left to exploit, economic stalemate and poverty start to reign.

Since the complexity of the studied spatial prisoner's dilemma game is too high to enable an analytical treatment, we explain the reported phenomenon in a descriptive manner. We argue that cooperators are able to cope with microeconomic uncertainties better than defectors due to their cooperative behavior. In particular, if two neighboring cooperators receive an unexpected positive and negative income respectively, mutual cooperation always decreases the relative difference between their cumulative payoffs, thus keeping the unlucky cooperator competitive despite of its temporary bad luck. On the other hand, the defecting strategy increases stratification among players or keeps it the same at best (if all are defectors). Although in this sense defection threatens cooperators and defectors equally in that their strategy is

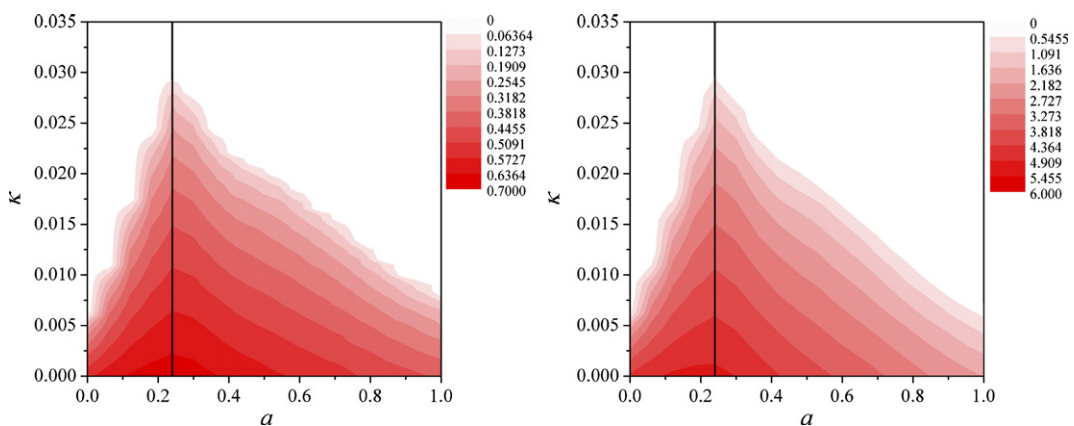


Fig. 3. Color-coded fraction of cooperators (left panel) and average payoff of each participating firm (right panel) in dependence on the temptation to defect  $\kappa$  and the amplitude of microeconomic chaotic uncertainties  $a$ . Evidently, there exists a well-defined amplitude of chaotic disturbances, marked by the vertical line in both graphs, at which cooperation promotion is best enhanced and thus the overall social welfare is maximized.

rendered unsuccessful and thus unlikely to be adopted, clustered cooperators have an edge since they help each other out. Thus, while moments of welfare are equally cherished by both strategies, clustered cooperators join their forces even in times of despair to make the best out of a bad situation. We argue that cooperators are therefore more successful in mastering unpredictable crisis situations than defectors, which ultimately results in overall social stability and welfare.

Given the harsh environment competitors have to face in an open economy, it is reasonable to assume that unpredictable factors at micro scales diminish the willingness of individuals or firms to engage into high risk alliances. In particular, an unfavorable evolution of unpredictable factors might present just the edge that renders the whole business fatal if the high risk doesn't pay off. This fact forces individuals to engage into more safe, although slightly less profitable, enterprises. Thus, due to the unpredictability at micro scales the overall economy at midi and macro scales is more prone to cooperative actions, which, although it might decrease the profit of an individual, in turn facilitates social welfare and presents and escape hatch out of economic stalemate. The potentially irritating message is that although unpredictability of economic behavior at micro scales is unwanted and appears destructive, we show that these uncertainties may actually make the economy at midi and macro scales stable and prosperous due to the effect of fear from unpredictable risks arising at the level of individual firms.

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