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Flights towards defection in economic transactions

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Abstract

We show that Lévy distributed variations of payoffs in the prisoner's dilemma game impede cooperation as the frequency of rare events increases. Lévy flights thus facilitate defection, but also uphold the evolutionary process, arguably maintaining a healthy level of competitiveness amongst the agents.

Keywords: Game theory; Lévy process; Cooperation; Social welfare

JEL classification: C73; D81

1. Introduction

What is best for an individual is often at odds with what is best for society. Short-term lucrative alliances may be extremely beneficial for some, but often inhibit long-term benefits due to partner exploitation and overall infliction of poverty. Social welfare is often swept aside heedlessly in an effort to temporarily outperform a competitor by harvesting the highest possible profit. This urge is innately routed in each individual and can be traced back to the famous Darwinian concept of "only the fittest survive". However, a healthy society depends on individuals keeping in mind the broader picture, hence deciding to act for the common good.

A mathematical concept that can be used to describe the above scenario is given by the prisoner's dilemma game (Axelrod, 1984) where two agents or firms, about to engage in a joint enterprise, have to decide simultaneously whether they want to cooperate or defect. The dilemma is given by the fact that although

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mutual cooperation yields the highest collective payoff, a defector will do better if the opponent cooperates. In the long run this fact inflicts mutual defection that ultimately results in an irreversible economic decline and social stalemate, as succinctly reviewed by Crawford (2002). Remarkably, this unadorned scenario is in reality often influenced by external factors that can have a profound effect on the social welfare, even redirecting the evolutionary process towards cooperativeness (Perc, in press). Indeed, external disturbances have emerged as being potent promoters of cooperation, thus supplementing other mechanisms promoting mutually beneficial alliances not just in economy (*e.g.*, Möller, 2005 or Breitmoser, 2005), but also in ecology as well as mathematics and physics in general (Doebeli and Hauert, 2005).

Presently, we study the impact of an important class of external perturbations on the evolution of cooperation in a spatially extended prisoner's dilemma game. In particular, we focus on Lévy distributed payoff disturbances. Lévy distributions differ from the Gaussian in that rare events occur more frequently, depending on the value of the exponent via which the tails of the Lévy distribution taper off in a power law manner (Devroye, 1986). Lévy stochastic processes are very common in economy (Mantegna and Stanley, 2000), where they account for the statistical description of rare or so-called "big" events (*e.g.* stock market breakdowns, sudden bankruptcies of large enterprises, etc.), which in reality occur far more often as one might have anticipated from the Gaussian distribution. Thus, they represent an important family of stochastic processes that apparently have wide reaching consequences for the welfare of society. We find that Lévy distributed payoff variations are less successful by maintaining cooperation amongst the agents on the spatial grid than their Gaussian counterparts. More precisely, we find that the facilitative effect of stochastic disturbances on the evolution of cooperation decreases steadily as the frequency of rare events increases. Importantly, results are obtained by full anonymity of agents and incognito actions (for related literature see *e.g.*, Fudenberg and Maskin, 1986 or Piccione, 2002), as well as without the aid of secondary strategies (*e.g.*, Hauert et al., 2002). Finally we discuss that, although Lévy flights facilitate defection, they also uphold the evolutionary process if adequately adjusted, and thus can be seen as having a positive effect on the economy by maintaining a healthy level of competitiveness amongst the participating firms or agents.

2. The game

We consider an evolutionary prisoner's dilemma game with agents located on vertices of a regular two-dimensional square lattice of size $n \times n$ (Nowak and May, 1992). We assume that each individual interacts only with its four nearest neighbors located to the north, south, east and west, whereby self-interactions are excluded. Each agent can decide either to cooperate (C) or to defect (D). Depending on the choice of their strategies, each of the two agents (P^i, P^j) receives payoffs summarized succinctly by the payoff matrix

P^i / P^j	C	D
C	$1+\xi_i / 1+\xi_j$	$1+r+\xi_i / -r+\xi_j$
D	$-r+\xi_i / 1+r+\xi_j$	$0+\xi_i / 0+\xi_j$

(1)

where $r \geq 0$ determines the temptation to defect whilst ξ_i and ξ_j are additive noisy additions characterized by an α-stable distribution $S_\alpha(\sigma, \beta, \mu)$ (Devroye, 1986), whereby α defines the characteristic exponent determining the rate at which the tails of distributions taper off. If $1 < \alpha \leq 2$ the mean of the distribution exists and equals μ. Moreover, σ is the scale parameter determining the width of the distribution, which by $\alpha = 2$ is Gaussian with variance σ^2. On the other hand, if $\alpha < 2$ the variance is infinite, whereby the frequency of rare, or "big" events increases as α decreases (Devroye, 1986). Finally, parameter $-1 \leq \beta \leq 1$ determines the skewness of the distribution, which is leftward bound if $\beta < 0$ and otherwise if $\beta > 0$. The following analysis is constrained to the case where $\mu = 0$, preserving the payoff ranking of the prisoner's dilemma over time amongst all interacting agents, and $\beta = 0$, resulting in symmetrical payoff variations with respect to positive and negative additions. Thus, the two main parameters are $\sigma \geq 0$ and $1 < \alpha \leq 2$, determining the effective strength and tail behavior of the distribution of payoff variations, respectively.

Starting from uniformly distributed cooperators and defectors on the square lattice, the spatial prisoner's dilemma game is iterated forward in time using a synchronous update scheme (*e.g.*, Hauert, 2002). After every iteration cycle of the game all agents update their strategy simultaneously according to the best-takes-over strategy adoption rule and reset their payoffs to zero.

3. Results and discussion

With the presently applied game settings cooperators survive as long as $r < 1/3$, as shown in the right panel of Fig. 1. Remarkably, the addition of stochastic disturbances to the payoffs, determined by $\sigma = 0.45$ and $\alpha = 2$ (Gaussian noise), is able to boost the fraction of cooperators to 50% even by $r = 0.35$. However, by setting $\alpha = 1.7$ (Lévy noise), thus increasing the fraction of rare events, the fraction of cooperators able to survive on the spatial grid drops to 20%. The two rightmost panels of Fig. 1 capture this phenomenon. Note that irrespective of the type of noise cooperators survive by forming clusters so as to protect themselves against being exploited by defectors. Cooperators located in the interior of such clusters enjoy the benefits of mutual cooperation and are therefore able to survive despite the constant exploitation by defectors along the cluster boundaries.

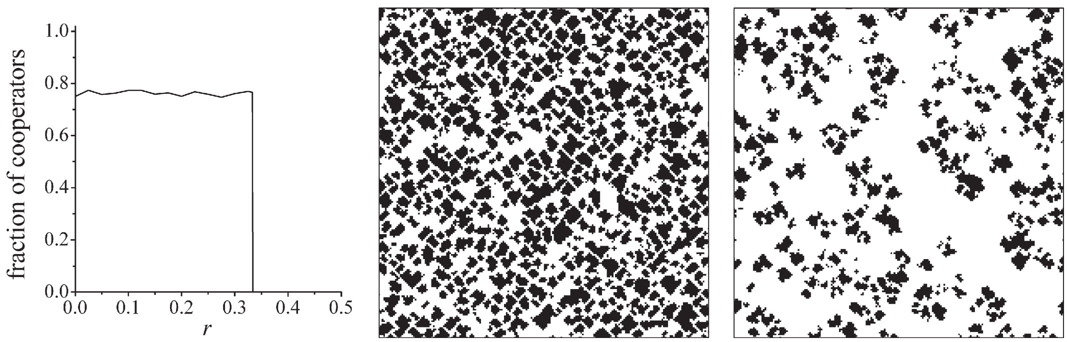

Fig. 1. Fraction of cooperators in dependence on r for the deterministic version of the game ($\sigma = 0$). The two rightmost panels feature characteristic equilibrium spatial distributions of cooperators (black) and defectors (white) obtained by $\alpha = 2$ (middle panel, 50% of cooperators) and $\alpha = 1.7$ (right panel, 20% of cooperators) for the defection temptation value $r = 0.35$ and $\sigma = 0.45$. Both panels are depicted on a 200×200 spatial grid.

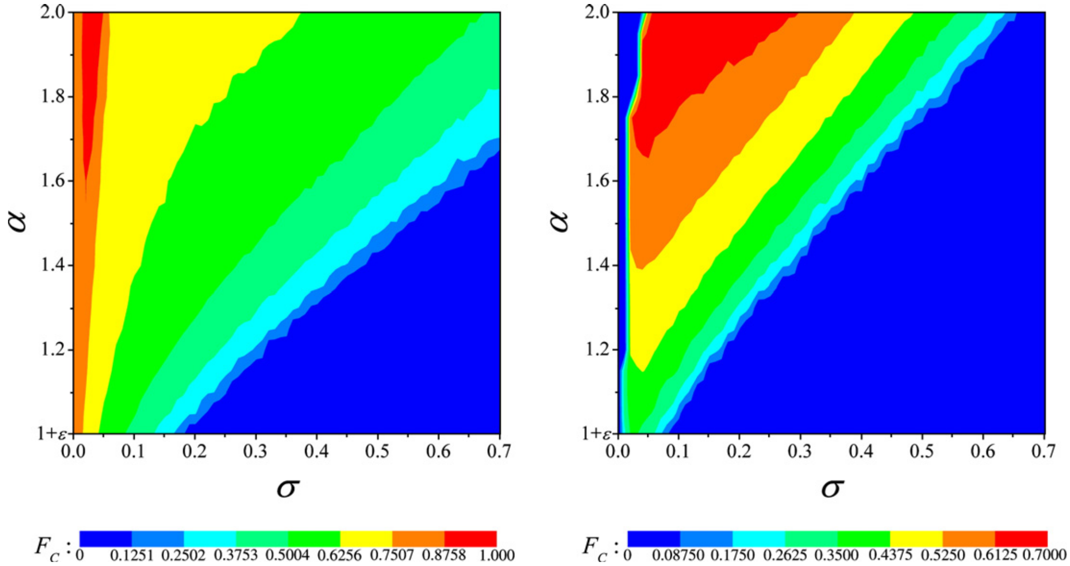

Fig. 2. Color-coded fraction of cooperators in dependence on σ and α for $r=0.3$ (left panel) and $r=0.35$ (right panel). In both cases $\varepsilon=0.001$ (see the y axis). Note that the span of values for color profiles in the left and right panel is not equal.

To quantify the ability of stochastic disturbances to facilitate and maintain cooperation in the studied spatial prisoner's dilemma game more precisely, we calculate the fraction of cooperators over a broad range of σ and α for two different r. Results presented in Fig. 2 show that Gaussian distributed stochastic disturbances ($\alpha=2$) are most successful in promoting cooperation. The facilitative effect on the cooperative strategy decreases steadily as the frequency of rare events increases (α decreases). Moreover, cooperation is always enhanced best for an intermediate value of σ, manifesting a resonant dependence on the width of the distribution. In particular, the resonant dependence on σ suggests that moderate

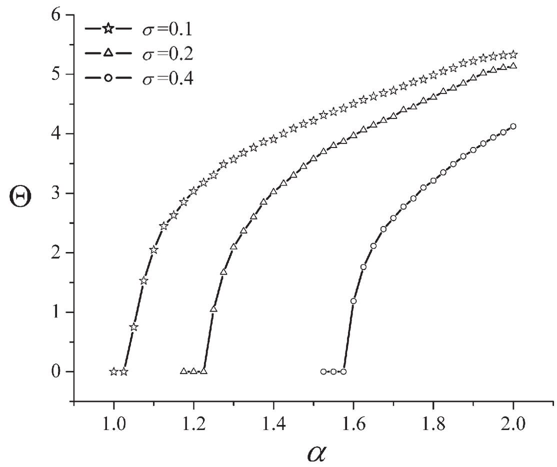

Fig. 3. Average payoff of each participating firm Θ in dependence on α and three different values of σ by $r=0.35$.

uncertainties at micro scales promote cooperative alliances at intermediate and macro scales, thus representing a viable escape hatch out of economic stalemate. However, as uncertainty levels rise (σ increases) defection again starts to reign. We attribute this to the diluted economic environment that emerges due to the strong stochastic component. In such an environment it is virtually impossible to uphold the focus on long-term benefits of mutual cooperation. Thus, defection becomes widespread and social poverty inevitable.

Above results clearly show that additive Lévy distributed stochastic payoff variations promote defection among firms or agents participating in the spatial prisoner's dilemma game. It remains of interest to inspect the social welfare, given by the average income Θ of all $n \times n$ firms on the square lattice, in dependence on α more precisely. If all firms would cooperate, then, in accordance with Eq. (1) and the applied synchronous iteration scheme, the average income of each firm would be exactly 8 in dimensionless units. On the other hand, if all firms would choose to defect $\Theta = 0$. Fig. 3 clearly shows that, by a given σ, the social welfare increases steadily as the frequency of rare events decreases. Importantly, it is crucial to note that some firms in the socially-optimal cooperative scenario earn less as if they would choose to defect. However, such payoff boosts can only be of short lasting nature, since once all cooperators vanish and there is no one left to exploit, economic stalemate and poverty start to reign.

Given the harsh environment competitors have to face in an open economy, it is reasonable to assume that unpredictable factors at micro scales diminish the willingness of individuals or firms to engage into high-risk alliances. In particular, an unfavorable evolvement of unpredictable factors might present just the edge that renders the whole business fatal if the high risk doesn't pay off. Thus, cooperation, being the slightly less profitable yet safe alternative, is favored. However, if the uncertainty levels rise, either via increasing σ (the central width of the distribution increases) or decreasing α (the frequency of big events increases), unpredictability becomes the dominant factor. In such a diluted economic environment common sense usually breaks down and everybody is heedlessly trying to get the most out of bad situations, thus choosing defection as the last recourse.

Importantly, we would like to note that although, in the framework of the evolutionary prisoner's dilemma game, widespread cooperators are the undisputed winners assuring social welfare (see Fig. 3), this might not be the case in general. In particular, in the case where only cooperators remain, implying *equal status for all* involved, the driving force leading to progress and evolution in society might be lost. Individuals become satisfied with what they have, and thus stop striving towards higher goals. Such a society begins to fall behind in comparison to those who still manage to maintain a certain level of competitiveness among their ranks, which is usually brought about by defecting individuals trying to outperform their partners. Thus, we argue that rare events predicted by Lévy distributions obtained when $\alpha < 2$ might be crucial in preventing the economy to settle onto a temporarily socially desirable, yet steady state (see the left panel of Fig. 2 at $\sigma = 0.05$), eventually leading to a global decline in comparison to other societies due to the inability of further evolution. The latter discussion appears very reasonable when considering former socialist countries, where equal status for all was often the main governmental credo.

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