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The dynamics of human gait

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Abstract
We analyse the dynamics of human gait with simple nonlinear time series analysis methods that are appropriate for undergraduate courses. We show that short continuous recordings of the human locomotory apparatus possess properties typical of deterministic chaotic systems. To facilitate interest and enable the reproduction of presented results, as well as to promote applications of nonlinear time series analysis to other experimental systems, we provide user-friendly programs for each implemented method. Thus, we provide new insights into the dynamics of human locomotion, and make an effort to ease the inclusion of nonlinear time series analysis methods into the curriculum at an early stage of the educational process.

1. Introduction

The theory of deterministic chaos offers a beautiful explanation for the seemingly stochastic behaviour of complex systems. Since the discovery of chaos, research on this topic is flourishing and literature abounds. There exist excellent introductory monographs that explain basic concepts in a simple and informative manner [1–3].

One of the crowning achievements that emerged from the theory of deterministic dynamical systems is the derivation of methods for nonlinear time series analysis. Nonlinear time series analysis methods enable the determination of characteristic quantities, e.g. the number of active degrees of freedom or invariants such as the maximal Lyapunov exponent, of a particular system solely by analysing the time course of one of its variables. From this point of view, nonlinear time series analysis methods are superior to mathematical modelling, since they enable the introduction of basic concepts directly from the experimental data. Especially at the undergraduate level, this approach could additionally inspire students and facilitate interest in the theory of deterministic dynamical systems. While there exist excellent monographs on nonlinear time series analysis [4–6], there is still a shortage of literature showing concrete applications of simple methods to real-life problems.

In this paper, we use basic nonlinear time series analysis methods to analyse the dynamics of human gait. We start the study by introducing the embedding theorem [7],

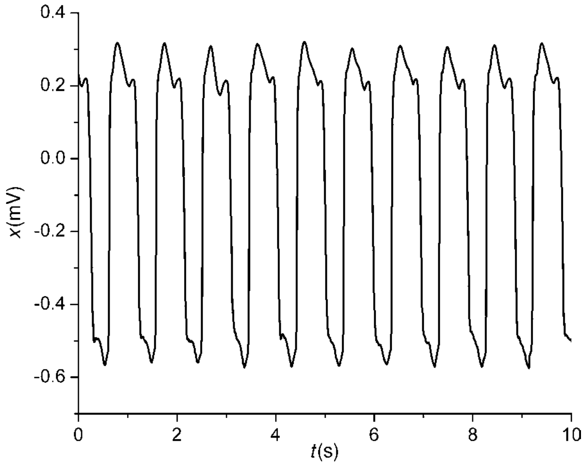

Figure 1. The recording of human gait. Variable x denotes the voltage output of the circuit that was used to drive the two force-sensitive resistors, i.e. the footswitch system [16].

which enables the reconstruction of the phase space from a single observed variable, thereby laying foundations for further analyses. To determine proper embedding parameters, we use the mutual information [8] and false nearest neighbour method [9]. Next, we apply a simple determinism test [10], to verify if the reconstructed phase space possesses a clear deterministic signature. Finally, we calculate the maximal Lyapunov exponent [11, 12] to test the exponential divergence of nearby trajectories. We find that the studied recording of human gait possesses properties typical of deterministic chaotic systems. Thereby, we provide new insights into the dynamics of human locomotion without the use of mathematical modelling.

Moreover, we provide user-friendly programs with a graphical interface for each implemented method, which should make the reproduction of the presented results possible even for individuals with little or no experience with nonlinear time series analysis, and facilitate further applications of the described methods to other experimental systems. Brief usage instructions for the program package are given in the appendix, while a more detailed manual can be found at the download site [13].

2. Results

2.1. The time series

We analyse a short recording of human gait from the publicly accessible MIT Gait Database [14, 15]. The original recording *control1.rit* consists of 90 000 data points that were sampled at $\mathrm{d}t = 0.003$ s, from which only 10 000 were used for our calculations. We found that this amount is sufficient to get representative results in the shortest possible time. Thus, a total of 30 s of human gait were used for the study, from which the first 10 s are shown in figure 1. The studied time series was obtained by using two thin force-sensitive resistors, which were placed inside the subject's right shoe under the heel and toes [16]. Since in the experimental setting both resistors are connected parallel, they essentially act as one large sensor that monitors the distribution of body weight during gait (for a detailed circuit description see [16]). The temporal evolution of one stride can be explained as follows. Each time the heel strikes the ground, the output voltage of the circuit increases rapidly, quickly reaching its overall maximum due to the sudden increase of weight acting upon the force-sensitive resistor placed below the heel. Next, as the weight is transferred from the heel to toes a light voltage descent is recorded because the initial weight impact diminishes. Just before the foot loses contact with the ground the weight is mostly supported by the toes, which again induces a light voltage increase. Finally, a sharp voltage descent occurs when the foot loses contact with the ground, thereby marking the end of one stride. It can be observed that the studied time series indicates a rather regular activity with a predominant frequency ≈1.0 Hz. Next, it is of interest to apply methods of nonlinear time series analysis to obtain a deeper insight into the underlying dynamics that yielded the observed behaviour.

2.2. Phase space reconstruction

Following the succession of tasks we have outlined in the introduction, let us start the study by introducing the embedding theorem [7]. The embedding theorem states that for a large enough embedding dimension m, the delay vectors

$$\mathbf{p}(i) = (x_i, x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+(m-1)\tau}) \tag{1}$$

yield a phase space, i.e. the embedding space, that has exactly the same properties as that formed by the original variables of the system. In equation (1) variables x_i, $x_{i+\tau}$, $x_{i+2\tau}, \ldots,$ $x_{i+(m-1)\tau}$ denote the values of the human gait recording at times $t = i\,\mathrm{d}t$, $t = (i+\tau)\,\mathrm{d}t$, $t = (i+2\tau)\,\mathrm{d}t, \ldots, t = (i+(m-1)\tau)\,\mathrm{d}t$, respectively, whereas τ is the so-called embedding delay. Although the fact that we can reconstruct the whole phase space of a system from a single scalar measurement may appear somewhat mystic, there exists a rather simple intuitive explanation why the reconstruction can be made. The key to understanding lies in the fact that all variables in a deterministic dynamical system are generically connected, i.e. they influence one another. Thus, every subsequent point of a given measurement x_i is the result of an entangled combination of influences from all other system variables. Accordingly, $x_{i+\tau}$ may be viewed as a substitute second system variable, which carries information about the influence of all other variables during time τ. With the same reasoning we can introduce the third ($x_{i+2\tau}$), fourth ($x_{i+3\tau}$), ..., mth ($x_{i+(m-1)\tau}$) substitute variables, and thus obtain the whole m-dimensional phase space where the substitute variables carry the same information as the original system variables, provided m is large enough. At this point, it is crucial to note that the substitute variables carry the same information as the original variables only with respect to the properties associated with the system's dynamics, while they do not possess any particular or additional physical meaning. Thus, the phase space obtained according to equation (1) is not a typical phase space, e.g. consisting of positions and momenta as in classical physics, but nevertheless has the same number of active degrees of freedom, Lyapunov exponents, etc, as the phase space formed by the original variables.

Although the implementation of equation (1) is straightforward, we first have to determine proper values for embedding parameters τ and m. The mutual information [8] between x_i and $x_{i+\tau}$ can be used to estimate a proper embedding delay τ. A suitable τ has to fulfil two criteria. First, τ has to be large enough so that the information we get from measuring the value of variable x at time $i+\tau$ is relevant and significantly different from the information we already have by knowing the value at time i. Only then, will it be possible to gather enough information about the system to successfully reconstruct the whole phase space with a reasonable choice of m. Second, τ should not be larger than the typical time in which the system loses memory of its initial state. If τ were to be chosen larger, the embedding space would look more or less random since it would consist of uncorrelated points. The latter condition is particularly important for chaotic systems which are intrinsically unpredictable and hence lose memory of the initial state as time progresses. Since the mutual information between x_i and $x_{i+\tau}$ quantifies the amount of information we have about the state $x_{i+\tau}$ presuming we know x_i [17], Fraser

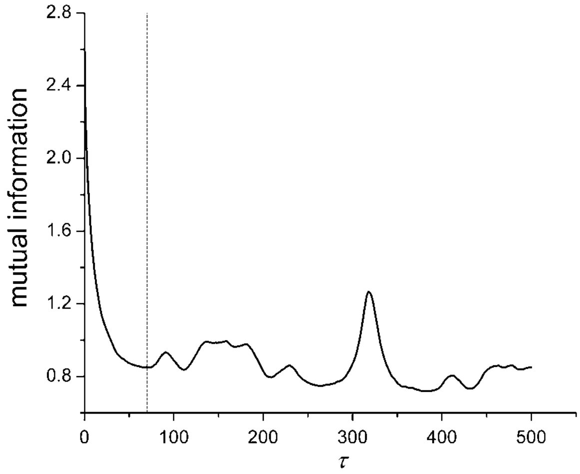

Figure 2. Determination of the proper embedding delay. The mutual information has the first minimum at $\tau = 70$.

and Swinney [8] proposed to use the first minimum of mutual information as the optimal embedding delay.

The algorithm for calculating the mutual information can be summarized as follows. Given a time series of the form $\{x_0, x_1, x_2, \ldots, x_i, \ldots, x_n\}$, one first has to find the minimum ($x_{\min}$) and the maximum ($x_{\max}$) of the sequence. The absolute value of their difference $|x_{\max} - x_{\min}|$ then has to be partitioned into j equally sized intervals, where j is a large enough integer number. Finally, one calculates the expression

$$I(\tau) = -\sum_{h=1}^{j}\sum_{k=1}^{j} P_{h,k}(\tau) \ln \frac{P_{h,k}(\tau)}{P_h P_k}, \tag{2}$$

where P_h and P_k denote the probabilities that the variable assumes a value inside the hth and kth bin, respectively, and $P_{h,k}(\tau)$ is the joint probability that x_i is in bin h and $x_{i+\tau}$ is in bin k. For the human gait recording presented in figure 1, the first minimum of $I(\tau)$ is obtained at $\tau = 70$ time steps (see figure 2). We will use this τ in all future calculations. This result can easily be reproduced with our program *mutual.exe*, which can be downloaded from our web page [13]. The program has only two crucial parameters, which are the number of bins j (in our case 30) and the maximal embedding delay τ (in our case 500). All calculated results are displayed graphically. More precise usage instructions are given in the appendix and on the web page.

Next, we determine a proper embedding dimension m by applying the false nearest neighbour method. The false nearest neighbour method was introduced by Kennel *et al* [9] as an efficient tool for determining the minimal m that is required to fully resolve the deterministic structure of the system in the reconstructed phase space. The method relies on the assumption that the phase space of a deterministic system folds and unfolds smoothly with no sudden irregularities appearing in its structure. By exploiting this assumption, we must come to the conclusion that points that are close in the reconstructed embedding space have to stay sufficiently close also during forward iteration. If a phase space point has a close neighbour that does not fulfil this criterion, it is marked as having a false nearest neighbour. From the geometrical point of view, this occurs whenever two points in the phase space solely appear to be close, whereas under forward iteration they are mapped randomly due to projection effects.

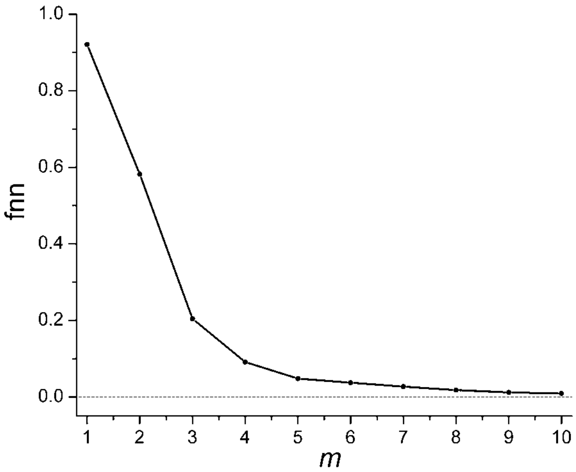

Figure 3. Determination of the minimal required embedding dimension. The fraction of false nearest neighbours (fnn) drops convincingly to zero at $m = 10$.

The random mappings occur because the whole attractor is projected onto a hyperplane that has a smaller dimensionality than the actual phase space and so the distances between points become distorted. As soon as m is chosen sufficiently large, the fraction of points that have a false nearest neighbour (fnn) converges to zero.

In order to calculate the fraction of false nearest neighbours, the following algorithm is used. Given a point $\mathbf{p}(i)$ in the m-dimensional embedding space, one first has to find a neighbour $\mathbf{p}(j)$, so that $\|\mathbf{p}(i) - \mathbf{p}(j)\| < \varepsilon$, where $\|\cdots\|$ is the square norm and ε is a small constant usually not larger than $1/10$ of the standard data deviation. We then calculate the normalized distance R_i between the $(m + 1)$th embedding coordinate of points $\mathbf{p}(i)$ and $\mathbf{p}(j)$ according to the equation:

$$R_i = \frac{|x_{i+m\tau} - x_{j+m\tau}|}{\|\mathbf{p}(i) - \mathbf{p}(j)\|}. \tag{3}$$

If R_i is larger than a given threshold R_{tr}, then $\mathbf{p}(i)$ is marked as having a false nearest neighbour. Equation (3) has to be applied for the whole time series and for various $m = 1, 2, \ldots$ until the fraction of points for which $R_i > R_{\mathrm{tr}}$ is negligible. According to Kennel *et al* [9], $R_{\mathrm{tr}} = 10$ has proven to be a good choice for most data sets. The results obtained with the false nearest neighbour method are presented in figure 3. It can be well observed that the fraction of false nearest neighbours (fnn) drops convincingly to zero ($<1\%$) for $m = 10$. Hence, the underlying system that produced the studied time series has ten active degrees of freedom. In other words, it would be justified to mathematically model the human locomotory apparatus with no more than ten first-order ordinary differential equations.

This result can also be easily reproduced with our program *fnn.exe*, which can be downloaded from our web page [13]. Parameter values that have to be provided are the minimal and the maximal embedding dimensions for which the fraction of false nearest neighbours is to be determined ($m_{\mathrm{min}} = 1$, $m_{\mathrm{max}} = 10$), the embedding delay ($\tau = 70$), the initial ε (0.0005), and the factor for increasing the initial ε (1.41).

Having calculated the optimal τ and m, we are able to successfully reconstruct the phase space of the system. For $\tau = 70$ and $m = 10$, the 3D projection of the 10D attractor obtained according to equation (1) is presented in figure 4. As argued above (see text below equation (1)), note that besides variable x_i, which indirectly measures the distribution of weight

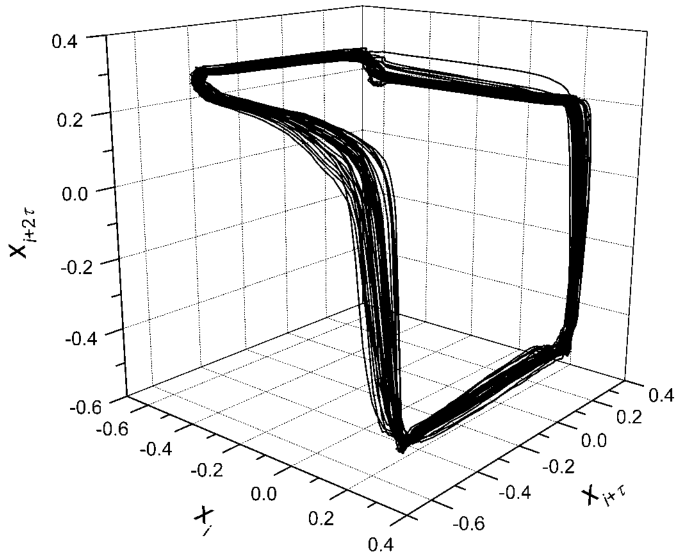

Figure 4. Three-dimensional projection of the reconstructed phase space obtained with optimal embedding parameters: $\tau = 70$ and $m = 10$. Since the substitute variables have no particular physical meaning, the physical units in the axis labels were omitted. This result can be reproduced with the program *embedd.exe*.

acting upon the force-sensitive switches placed below the foot, other substitute variables, i.e. $x_{i+\tau}, x_{i+2\tau}, \ldots, x_{i+9\tau}$, have no particular or additional physical meaning. Nevertheless, in terms of properties associated with the system's dynamics, the substitute variables carry the same information as the original ones. In this sense, the correct phase space reconstruction is a key step that enables further analyses of the dynamics associated with the experimentally obtained gait recording. In the continuation, we will apply the determinism test and calculate the maximal Lyapunov exponent, to establish whether the human gait recording possesses properties typical of deterministic chaotic systems.

2.3. Determinism test

We must bear in mind that deterministic chaos is only one possible source of irregular behaviour in real-life systems. The other, in fact more probable, source is the ever-present noise, which even if combined with linear dynamics can also produce irregularly appearing behaviour. Therefore, a determinism test [10, 18, 19] is of crucial importance since it enables us to distinguish between deterministic chaos and irregular random behaviour, which often resembles chaos.

We apply a simple yet effective determinism test, originally proposed by Kaplan and Glass [10], that measures average directional vectors in a coarse-grained embedding space. The idea is that neighbouring trajectories in a small portion of the embedding space should all point in the same direction, thus assuring uniqueness of solutions in the phase space, which is the hallmark of determinism. To perform the test, the embedding space has to be coarse grained into equally sized boxes. The average directional vector pertaining to a particular box is obtained as follows. Each pass p of the trajectory through the kth box generates a unit vector $\mathbf{e}_p$, whose direction is determined by the phase space point where the trajectory first enters the box and the phase space point where the trajectory leaves the box. In fact, this is the average direction of the trajectory through the box during a particular pass. The average directional vector $\mathbf{V}_k$ of the kth box is then simply

$$\mathbf{V}_k = \frac{1}{a}\sum_{p=1}^{a}\mathbf{e}_p, \tag{4}$$

where a is the number of all passes through the kth box. Completing this task for all occupied boxes gives us a directional approximation for the vector field of the system. If the time series originates from a deterministic system, and the coarse-grained partitioning is fine enough, the obtained directional vector field should consist solely of vectors that have unit length (remember that each $\mathbf{e}_p$ is also a unit vector). This follows directly from the fact that we demand uniqueness of solutions in the phase space. If solutions in the phase space are to be unique, then the unit vectors $\mathbf{e}_p$ inside each box may not cross, since that would violate the uniqueness condition at each crossing. Note that each crossing decreases the size of the average vector $\mathbf{V}_k$. For example, if the crossing of two unit vectors inside the kth box occured at right angles, then the size of $\mathbf{V}_k$ would be, according to Pythagoras, $\sqrt{2}/2 \approx 0.707 < 1$. Hence, if the system is deterministic, the average length of all directional vectors κ will be 1, while for a completely random system $\kappa \approx 0$. The determinism factor for the embedding space reconstructed with $\tau = 70$ and $m = 10$ is $\kappa \approx 0.95$. We thereby show that the studied human gait recording indeed originates from a deterministic process, which is certainly an astonishing result. Furthermore, this result justifies further analyses with nonlinear time series methods and guarantees relevancy of future results, since it refutes the possibility of noise being a key ingredient in the underlying dynamics. This result can be reproduced with the program *determinism.exe*, which can be downloaded from our web page [13].

2.4. Maximal Lyapunov exponent

It remains of interest to verify if the human locomotory apparatus possesses properties typical of chaotic systems. For this purpose, we have to test the divergence of nearby trajectories in the phase space, which is constituted by the maximal Lyapunov exponent ($\Lambda_{\max}$) of the system. If $\Lambda_{\max} > 0$, two initially nearby trajectories diverge exponentially fast as time progresses, thereby constituting extreme sensitivity to changes in initial conditions, which is the hallmark of chaos [1–3].

In order to calculate the maximal Lyapunov exponent of the reconstructed phase space, we use the algorithm developed independently by Rosenstein *et al* [11] and Kantz [12]. The algorithm tests the exponential divergence of nearby trajectories directly, and thus allows a robust estimation of the maximal Lyapunov exponent. To estimate the exponent, we first have to find all neighbours $\mathbf{p}(k)$ that are closer to a particular reference point $\mathbf{p}(i)$ than ε. We thereby obtain a set of starting points for nearby trajectories in the embedding space. Next, we have to calculate the average distance of all trajectories to the reference trajectory as a function of the relative time Δn according to the equation

$$D_i(\Delta n) = \frac{1}{b}\sum_{s=1}^{b}|x_{k+(m-1)\tau+\Delta n} - x_{i+(m-1)\tau+\Delta n}|, \tag{5}$$

where s counts the number of different points $\mathbf{p}(k)$ (there are a total of b) that fulfil $\|\mathbf{p}(k) - \mathbf{p}(i)\| < \varepsilon$. Finally, the average of the logarithm of $D_i(\Delta n)$, obtained for several different reference points $\mathbf{p}(i)$, is the effective expansion rate

$$S(\Delta n) = \frac{1}{c}\sum_{i=1}^{c}\ln[D_i(\Delta n)], \tag{6}$$

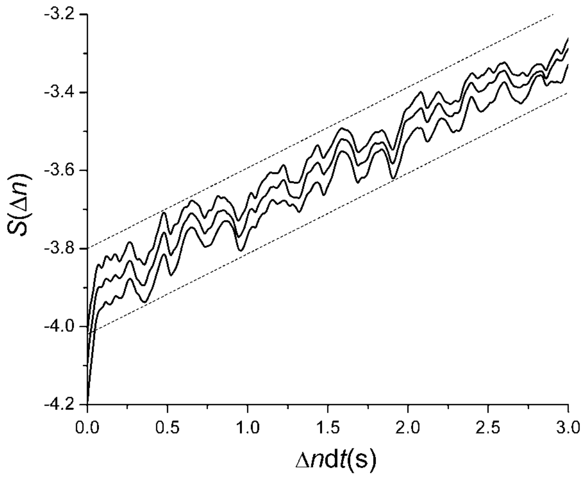

Figure 5. Calculation of the maximal Lyapunov exponent. The slope of linear lines that indicate the predominant slope of the effective expansion rate $S(\Delta n)$ in dependence on $\Delta n\,\mathrm{d}t$ is a robust estimate for the maximal Lyapunov exponent, which equals $\Lambda_{\max} = (0.21 \pm 0.02)$ s^{-1}. This result can be reproduced with the program *lyapmaxk.exe*.

where c is the number of different $\mathbf{p}(i)$ used for the calculation. If there exists a steady linear increase of $S(\Delta n)$ in dependence on Δn, the slope of the fitted line is a robust estimate for the maximal Lyapunov exponent of the system. To obtain an accurate result, the whole algorithm has to be repeated for a few hundred different reference points $\mathbf{p}(i)$ (e.g. $c \geqslant 500$) and various ε. In particular, ε should be chosen as small as possible, but still large enough so that on average each reference point has at least a few neighbours (e.g. $b \geqslant 10$). The results obtained for $\varepsilon = 0.07$–0.15 are presented in figure 5. The function $S(\Delta n)$ shows a rather robust linear increase for all ε. Thus, the predominant slope of $S(\Delta n)$, indicated by the two linear lines, is a good estimate for the maximal Lyapunov exponent of the system. We find that the latter equals $\Lambda_{\max} = (0.21 \pm 0.02)$ s^{-1}, from which we finally conclude that the studied short continuous recording of the human gait possesses properties typical of deterministic chaotic systems.

3. Discussion

We systematically analyse a short continuous recording of human gait in order to obtain a deeper insight into the dynamics of the locomotory system. In particular, the mutual information [8] and false nearest neighbour [9] method are explained in detail, and used to obtain the best possible attractor reconstruction [7]. For the reconstructed attractor, a determinism test [10] is performed and the largest Lyapunov exponent [11, 12] is calculated. We find that the studied gait recording possesses properties typical of deterministic chaotic systems.

Recent studies of the human locomotory apparatus indicate that fluctuations of stride-to-stride intervals derived from long-term continuous gait recordings are scale-free, i.e. resemble random behaviour, nevertheless possessing long-range correlations typical of fractal systems [20]. By considering this fact together with the results from our current study, we find that the state of research of human gait is very similar as for human electrocardiographic recordings. There, it was also found that inter-beat interval fluctuations derived from long-term electrocardiographic recordings do not possess clear deterministic signature [21], while short continuous densely sampled electrocardiographic recordings, may possess properties typical of deterministic systems [22]. Thus, our recent findings, combined with the results from similar studies, lead us to the conclusion that several vital functions of the human body are deterministic on short time scales, whereas over long times, stochastic environmental influences affect the functioning, making it indistinguishable from randomness. Our findings thus provide new insights into the dynamics of the human locomotory system, as well as complementing existing studies, to give an overall insight into the dynamics that governs the functioning of the human body.

Furthermore, to enable the reproduction of the presented results, and to facilitate applications of nonlinear time series analysis methods to other experimental systems, we developed user-friendly programs for each implemented method [13]. Thereby, we make an effort to facilitate the inclusion of nonlinear time series analysis methods into the curriculum at an early stage of the educational process. Particularly for undergraduate students, methods of nonlinear time series analysis present an excellent alternative to mathematical modelling, which is predominantly used for analysing irregular behaviour in complex systems, since they enable the introduction of basic concepts directly from the experimental data, thus guaranteeing a better link between real-life phenomena and the theory.

Appendix. The program package

The whole program package that can be downloaded from our web page [13] consists of five programs (*embedd.exe*, *mutual.exe*, *fnn.exe*, *determinism.exe* and *lyapmaxk.exe*) and an input file *gait.dat*, which contains the studied gait recording. All programs have a graphical interface and display results in forms of graphs and drawings. In order to run the programs, Windows® environment is required and the input file has to be in the working directory. After downloading, the content of the *gaitpackage.zip* file should be extracted into an arbitrary (preferably empty) directory. Thereafter, the programs are ready to run via a double-click on the appropriate icon. First, a parameter window will appear, which allows the insertion of proper parameter values (by default they are set equal to those used in this paper). Next, the OK button needs to be pressed to execute the program. Finally, a progress bar will appear, which indicates how fast the program is running and when it will eventually finish. After completion, the results are displayed graphically in a maximized window. Before using the programs, we strongly advise the reader to read the manual pertaining to the programs on our web page [13] where more detailed usage instructions can be found.

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