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Thoughts out of noise

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Abstract

We study the effects of additive Gaussian noise on the behaviour of a simple spatially extended system, which is locally modelled by a nonlinear two-dimensional iterated map describing neuronal dynamics. In particular, we focus on the ability of noise to induce spatially ordered patterns, i.e. the so-called noise-induced pattern formation. For intermediate noise intensities, the spatially extended system exhibits ordered circular waves, thereby clearly manifesting the constructive role of random perturbations. The emergence of observed noise-induced patterns is explained with simple arguments that are obtained by analysing the typical spatial scale of patterns evoked by various diffusion coefficients. Since discrete-time systems are straightforward to implement and require modest computational capabilities, the present study describes one of the most fascinating and visually compelling examples of noise-induced self-organization in nonlinear systems in an accessible way for graduate or even advanced undergraduate students attending a nonlinear dynamics course.

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1. Introduction

Intuitively, randomness is often associated with disorder or some other kind of destructive force that we do not want to affect our every day lives. Surprisingly, however, this intuitive perception of random influences is often in contradiction with the reality. In particular, this is the case when we introduce random perturbations, i.e. noise, to nonlinear systems. Somewhat more than two decades ago, scientists discovered that noise can enhance the response of a nonlinear system to a weak external signal in a resonant manner [1]. The observed phenomenon was termed appropriately as stochastic resonance. Since then, numerous observations of this or similar noise-induced phenomena have been reported in various fields of research. Examples range from improved detection of water currents in the mechanoreceptors of crayfish tails due to thermal fluctuations [2], enhanced sensitivity of shark sensory cells due to a noisy background [3], improved functionality of a cochlear implant due to added noise [4], noise-supported travelling waves in a Belousov–Zhabotinsky light-sensitive chemical medium [5] or even climatic noise-induced periods of warmth during past ice ages [6–8]. This, however, is by no means neither a complete nor the most comprehensive listing of stochastic resonance phenomena. Therefore, the interested reader is advised to seek further information in two comprehensive review articles due to Gammaitoni *et al* [9] and Lindner *et al* [10].

While reports of stochastic resonance phenomena were initially confined to temporal systems, i.e. systems whose solution depends only on time, the last decade witnessed a substantial increase in scientific literature reporting constructive effects of noise also for the so-called spatially extended systems, i.e. systems whose solution, besides being time-dependent, also varies with the position in space. Recently, García-Ojalvo and Sancho wrote an excellent book [11], capturing the basic theory as well as numerous examples of noise-induced phenomena in spatially extended systems. One of the most astonishing and visually compelling examples of the constructive effects of noise in spatially extended systems is the so-called noise-induced pattern formation [12–16]. Thereby, patterns are most often wave-like circular structures that propagate through the media on random support.

Despite the fact that noise-induced phenomena in temporal as well as spatially extended systems have been one of the most flourishing and fascinating topics of research in the field of nonlinear science during the past few years, to this end little effort has been devoted to the inclusion of these phenomena into the curriculum at an early stage of the educational process. The present paper is aimed at introducing a small fraction of possible noise-induced phenomena in spatially extended systems, namely the noise-induced pattern formation, in a practical and concise way. Throughout the paper, we strive towards minimizing both the theoretical and numerical burden required to obtain interesting results, while still providing valuable insights and hopefully motivating students to delve deeper into the presented theory.

Unlike the vast majority of scientific literature devoted to the study of noise-induced phenomena in nonlinear systems we use a discrete-time iterated map, rather than a continuous-time system of differential equations, as a building block for our spatially extended system. These so-called building blocks are arranged in a square grid, whereby each grid element defined by the discrete map is diffusively coupled to its neighbours. Thereby, we obtain a simple discrete spatially extended system, with which we overcome several difficulties associated with its more frequently used continuous-time counterparts. In particular, discrete-time systems are computationally more efficient and allow a much easier numerical implementation of noise terms than continuous-time systems. These two facts, combined with the relatively simple dynamics of the iterated map, designate the employed spatially extended system as an ideal workhorse for the achievement of the designated goal of the present paper.

The presently used iterated map was introduced by Rulkov [17] in order to shed light on the behaviour of coupled neurons. Thus, in a very simple way, the spatially extended system studied below describes the dynamics of the brain, which was also the motivation for the title of the present paper. Moreover, since neurons are known to be noisy analogue units, which only if coupled can carry out highly complex and advanced operations with cognition and reliability [18], it is evident that neural tissue combines features of being both noisy and spatially extended. The present study is therefore motivated also from the biological point of view.

2. The iterated map

This section is devoted to the description of some basic properties of the iterated map [17] that we are going to use in the next section as the building block for the spatially extended system, as briefly outlined in the introduction. The map takes the form

$$u_{n+1} = \alpha \big/ \left(1 + u_n^2\right) + v_n, \tag{1}$$

$$v_{n+1} = v_n - \beta u_n - \gamma, \tag{2}$$

where the neuron cell membrane voltage u_n and the variation of ion concentration near the neuron membrane v_n are considered as dimensionless variables, n is the discrete time index, while α, β and γ are system parameters. The main system parameter is α, while β and γ essentially act as time scaling parameters for the variable v_n. By choosing $\beta = \gamma = 0.001 \ll 1$, we thus achieve that v_n changes slowly in comparison with u_n. For $\alpha < 2.0$, the map is governed by a single excitable steady state (u^*, v^*) that can be derived analytically by setting $u_n = u_{n+1}$ and $v_n = v_{n+1}$ in equations (1) and (2), respectively. Thereby, we obtain $u^* = -1$ and $v^* = -1 - (\alpha/2)$. By setting $\alpha = 1.99$, the system thus occupies the excitable steady state $(u^*, v^*) = (-1, -1.995)$, which are also the initial conditions we will use in all subsequent calculations. Additionally, we note that for $\alpha > 2.0$ the excitable steady state loses its stability via a Hopf bifurcation. However, the behaviour of the map for $\alpha > 2.0$ is presently not of particular importance, so the interested reader is, regarding this detail, referred to the original article [17]. In summary, throughout the paper we will focus on the excitable steady state of the iterated map $(u^*, v^*) = (-1, -1.995)$, which sets in for the parameter values $\alpha = 1.99$ and $\beta = \gamma = 0.001$.

Before we venture into the introduction and analysis of the spatially extended system we first examine the properties of the excitable steady state $(u^*, v^*) = (-1, -1.995)$ in some detail. Excitability is a fascinating property of several real-life systems. Examples include neurons, cardiac tissue, optical devices or chemical media [10]. A steady state is excitable if a small external perturbation introduced to the system evokes a large excursion of the trajectory in the phase space before the latter re-settles into the steady state. Since we are presently studying noise-induced phenomena, the obvious candidate to deliver this small perturbation is noise. Thus, in order to enable a visual perception of the excitability described, we introduce an additional term of the form $\sigma\xi_n$ to equation (1), whereby ξ_n are Gaussian distributed random numbers with zero mean and unit variance [19], whereas σ is a parameter that determines the standard deviation of noise. The updated equation (1) thus takes the form

$$u_{n+1} = \alpha \big/ \left(1 + u_n^2\right) + v_n + \sigma\xi_n, \tag{3}$$

whilst equation (2) remains the same. Since equations (2) and (3) constitute a discrete-time system the numerical implementation of the noise-driven model is straightforward. Importantly, this is not the case when dealing with noisy continuous-time systems that yield stochastic differential equations as model equations [20]. After setting the initial conditions $(u_0, v_0, \xi_0) = (-1, -1.995, \text{some random number})$, the two-dimensional map can be normally iterated, whereby at each n ξ_n should be replaced by a new random number. Temporal as well as phase space plots for three different σ are presented in figure 1. In the top row, σ is too small to evoke large amplitude excitations. For intermediate σ, the system starts to produce large amplitude spikes that manifest as noise-induced limit cycles in the phase space, as can be seen in the middle row of figure 1. If σ is further enlarged, the number of spikes in a given time interval increases, which is understandable since larger σ constitute stronger perturbations of the excitable steady state. Nevertheless, larger σ also increase the overall disorder, which results in an overridden spiking phase of the system in the temporal domain, as well as a blurred limit cycle in the phase space, as presented in the bottom row of figure 1. Hence, there exists an optimal σ for which the noise-induced response of the iterated map is well pronounced but still largely ordered, as is the case for $\sigma = 0.015$. This phenomenon

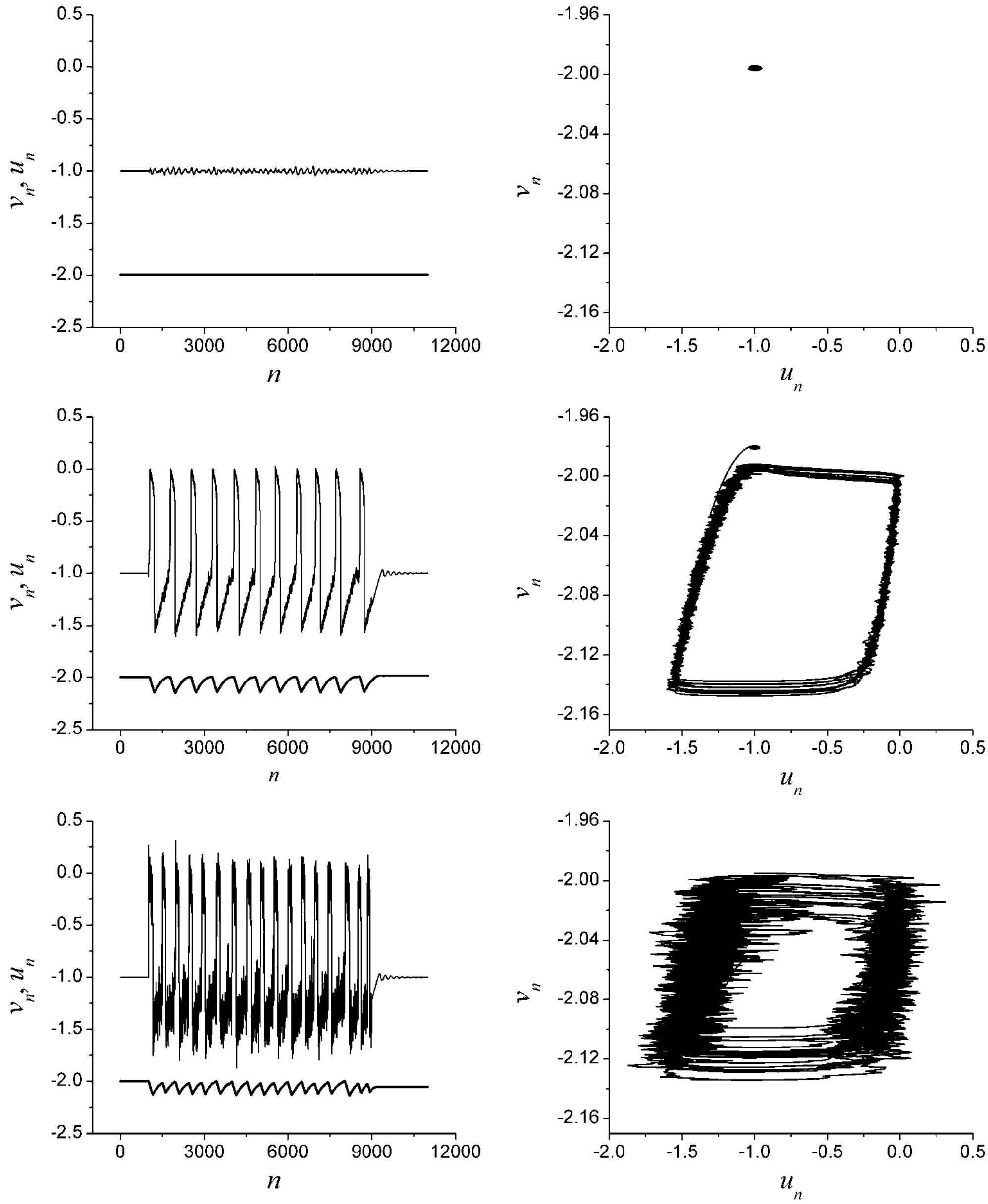

Figure 1. Noise-induced oscillations for small (top row, $\sigma = 0.0015$), intermediate (middle row, $\sigma = 0.015$) and large (bottom row, $\sigma = 0.09$) levels of random perturbations, as well as the corresponding phase space plots. In the temporal plots the thin line corresponds to the time course of u_n, while the thick line shows v_n. Note that the random perturbations start when $n > 1000$ and stop when $n < 9000$ to emphasize the impact of added noise on the behaviour of the iterated map.

is known as the solely noise-induced (since there are no other external deterministic signals introduced to the system) stochastic resonance or the so-called coherence resonance, which was introduced by Pikovsky and Kurths in [21]. To learn more about attainable temporal stochastic resonances in the presently studied iterated map, as well as possible ways of how to quantify the order of the noise-induced temporal behaviour (besides the presently introduced visual assessment), we refer the reader to the recent article by Jiang [22]. In what follows, we will expand the map introduced above into a discrete spatially extended system and investigate how the noise-induced spikes in the temporal domain propagate through space.

3. The spatial expansion

In this section, we will describe the spatially extended system and subsequently subject it to noise. As announced, we will witness the birth of propagating circular wave-like patterns brought about by the random perturbations. At the end, we will provide a fairly simple explanation for the emergence of observed noise-induced patterns, thereby concluding the present paper.

In order to expand our iterated map from the temporal also to the spatial domain, we first introduce a discrete grid, which will act as a bearing frame for our spatially extended system. Particularly, let the grid be planar and square, occupying 128 units in each direction, whereby i counts the units in the horizontal and j in the vertical directions. Moreover, we assume that the temporal dynamics of each unit of the grid, denoted by a given pair of indices (i, j), is determined by a single iterated map given by equations (2) and (3), whereby all units are diffusively coupled with its neighbours. Following the description above, we end up with a discrete spatially extended system of the form

$$u_{n+1}^{(i,j)} = \alpha \big/ \left(1 + \left(u_n^{(i,j)}\right)^2\right) + v_n^{(i,j)} + \sigma \xi_n^{(i,j)} + D\nabla^2 u_n^{(i,j)}, \tag{4}$$

$$v_{n+1}^{(i,j)} = v_n^{(i,j)} - \beta u_n^{(i,j)} - \gamma, \tag{5}$$

whereby now both variables as well as the Gaussian noise term have obtained an additional subscript (i, j) to indicate that their values are no longer uniquely determined by the time index n, but vary also in space. In general, the numerical implementation of the spatially extended system is identical to the temporal case above, only that for each time step n, instead of a single map, 128×128 must be iterated. Importantly, note that $\xi_n^{(i,j)}$ also obtains a subscript to emphasize that for each space unit (i, j) a new random number must be chosen. Formally, the introduced spatially extended system is subjected to the so-called spatiotemporal random perturbations. Again, we emphasize that procedures remain simple strictly because we are studying a discrete system with unit-sized space elements, whilst otherwise the introduction of spatiotemporal noise to continuous-time spatially extended systems with non-unit-sized space elements requires a slightly more sophisticated approach, as nicely described in [11].

Prior to inspecting the results pertaining to the introduced spatially extended system, we briefly turn our attention to the newly added diffusive term in equation (4) given by $D\nabla^2 u_n^{(i,j)}$. As usual, D is the diffusion coefficient that determines the rate of diffusive spread among neighbouring space units, whilst the Laplacian is simply the short notation of the second derivative that can be implemented numerically according to the equation [19]

$$\nabla^2 u_n^{(i,j)} = u_n^{(i+1,j)} + u_n^{(i-1,j)} + u_n^{(i,j+1)} + u_n^{(i,j-1)} - 4u_n^{(i,j)}, \tag{6}$$

whereby we have already taken into account the fact that the area of each unit of space, indexed by the pair (i, j) equals unity. When incorporated into equation (4), equation (6) can be directly implemented in numerical fashion. For our calculations below, we choose periodic boundary conditions [19], thus setting $u_n^{(i<1,j)} = u_n^{(i=128,j)}$, $u_n^{(i>128,j)} = u_n^{(i=1,j)}$, $u_n^{(i,j<1)} = u_n^{(i,j=128)}$, and $u_n^{(i,j>128)} = u_n^{(i,j=1)}$.

Finally, we have all in place to study the effects of random spatiotemporal perturbations on the behaviour of the studied spatially extended system. The results in figure 2 show snapshots

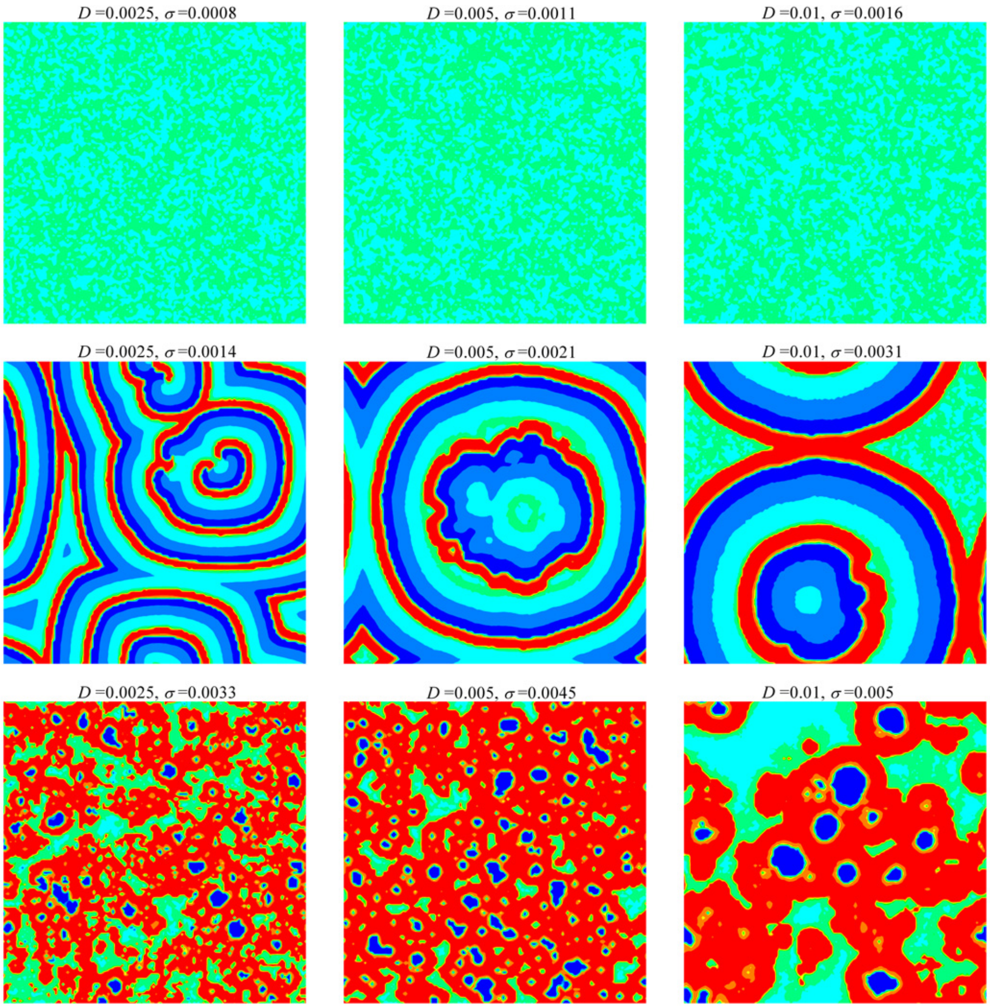

Figure 2. Noise-induced pattern formation in the studied spatially extended system. All figures depict values of $u_n^{(i,j)}$ on a 128×128 square grid. The colour maps were obtained with a linear colour profile, blue marking -1.6 and red 0.0 values of $u_n^{(i,j)}$. Note that each picture is a snapshot of the spatial grid at a given n. To see the temporal evolution of noise-induced patterns, see the multimedia enhancement 'thoughts.exe' in the online version of this paper (stacks.iop.org/EJP/27/451).

(This figure is in colour only in the electronic version)

of the spatial grid at given times n for various σ and D. Independent of D, the results in figure 2 show great conceptual similarity with the results presented in figure 1. In particular, small σ at each particular D are unable to excite the system strongly enough to evoke any particularly outstanding spatial structures, as shown in the top row of figure 2. On the other hand, for intermediate values of σ at each particular D, noise-induced patterns emerge in space that propagate through the medium on random support, as can be seen in the middle row of figure 2. These circular waves are beautiful examples of noise-induced pattern formation in a spatially extended system. At larger σ, ordered spatial patterns give way to strong noisy perturbations, which indent the circular waves in a random fashion, yielding disordered looking spatial portraits, as presented in the bottom row of figure 2. Similarly as in the strictly temporal case, this scenario is characteristic for the solely noise-induced spatial stochastic resonance, or as introduced in [23–25], the so-called spatial coherence resonance. For the interested reader, [23–25] also provide detailed information on how, besides visual inspection, noise-induced order can be accurately quantified in the spatial domain, which is presently beyond the scope of interest.

Since figure 2 consists solely of snapshots of the spatial grid, we also provide a user-friendly program 'thoughts.exe' (Windows® executable) as a multimedia enhancement in the online version of this paper. The program iterates and displays the spatially extended system in real time, whereby σ and D can be set arbitrarily. Thus, interested readers are able to see for themselves how the spatial patterns emerge for various values of the parameters, and so experience an even better perception of the phenomenon described. Since the executable was developed in the Windows® environment, we also supply the source code ('source.zip') to provide the possibility of transferring the program to computers with different operating systems. Moreover, we advise potential users to read all help messages that can be accessed via the 'help' command in the main window, prior to using the program.

To shed some light on the observed noise-induced pattern formation, we study snapshots presented in the middle row of figure 2 in more detail. What should be noted is the fact that the typical width of the patterns increases with the increasing diffusion coefficient D in a rather settled way. Moreover, the standard deviation of noise σ that is required to induce the most ordered spatial patterns in space also increases with D. Bearing these facts in mind, we propose the following explanation for the observed phenomenon. We argue that as soon as a particular unit of the spatial grid gets excited due to noisy perturbations (see the explanation pertaining to figure 1) it acts as a circular front initiator. This is due to the diffusive coupling amongst neighbouring space units, whilst the circular shape is assumed simply because after local initialization all directions for spreading are equally probable. When embarking on neighbouring grid units the local excitation can, depending on σ, cause new excitations or die out. In particular, if σ is large enough, neighbouring sites have a large probability of also becoming excited, which eventually nucleates a wave that propagates through the medium. Since larger D constitute faster diffusive spread, and thus local excitations can propagate further through space in a given amount of time, it is understandable that the characteristic width of spatial waves induced by increasing D increases. However, since for larger D local excitations tend to die out more quickly, and larger coherent structures in space also require a higher rate of local excitations to propagate through the spatial grid, it is evident that larger σ are required to produce sustained waves. This, in turn explains the increasing σ that are required to induce the most ordered spatial patterns in space at larger D, as shown in the middle row of figure 2.

Remarkably, the above arguments can be made quantitative by noting that once a grid unit gets excited it remains in this condition only for a limited amount of time. The latter, so-called excursion time n_e, is given by the width of spikes depicted in the temporal plots of the middle and bottom rows of figure 1. Thus, the excursion time n_e, combined with the diffusive spread rate, which is in each particular space direction proportional to $\sqrt{D}$ (note that D alone constitutes the rate with which a given area, rather than a particular space direction, spreads in time), defines a typical width of the noise-induced waves, which we will denote as w. Our reasoning thus predicts the dependence $w = \sqrt{\tau D}$, whereby $\tau \propto n_e =$ constant. The left panel of figure 3 shows the estimation of the typical width of a particular circular wave front, whilst the right panel shows the dependence of w on D. It is evident that values obtained are in excellent agreement with the predicted square root function, thereby validating our explanation above. Nevertheless, an open question remains as to how the constant τ

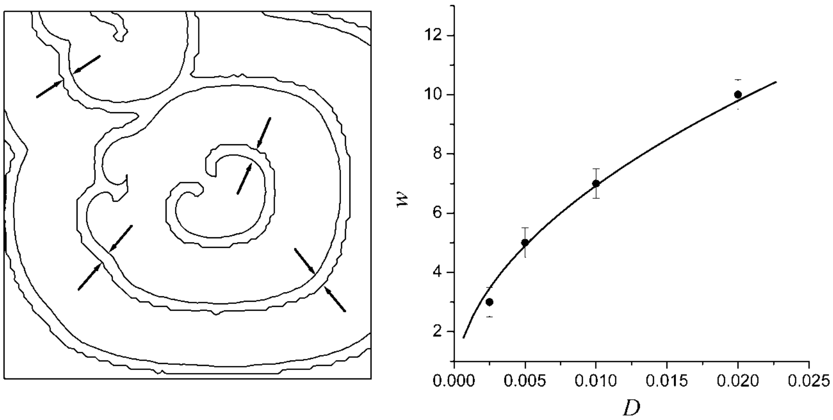

Figure 3. Explanation of the emergence of noise-induced patterns. Left panel features an 82×82 excerpt of the leftmost layer of the middle row of figure 2, whereby lines indicate the position of the wave front at values between -0.2 and 0.0 (red stripes in figure 2). Arrows at several places indicate the typical width of the wave front w, which can be seen to be rather constant. The right panel features the typical width of the wave front w obtained by various diffusion coefficients D. Dots with estimated error margins denote the evaluated typical width of the waves (via a procedure depicted in the left panel), whilst the curve shows the predicted $w = \sqrt{\tau D}$ dependence for $\tau = 4800$.

is explicitly linked to n_e. Since a particular excited grid unit acts as a front initiator only when the variable $u_n^{(i,j)}$ is above a certain threshold value (not during the whole n_e), and also because other constants determining the exact rate of diffusive spread are not known, the task of explicitly linking τ and n_e is left for future studies. The main point is that the square root function $w = \sqrt{\tau D}$ fits to the numerically obtained w with a *constant* τ, which reflects a fixed excursion time n_e that is characteristic for excitable systems [21], thus explaining the noise-induced pattern formation in the spatially extended system studied.

4. Summary

In the present paper, we study effects of spatiotemporal random perturbations on the behaviour of a spatially extended system, which is locally modelled by an iterated map. First, the temporal behaviour of the map is studied in dependence on different levels of additive Gaussian noise, whereby we discover that there exists an optimal σ for which the noise-induced spikes are optimally ordered in time. Second, we introduce a spatial grid that becomes the bearing frame for our spatially extended system. By subjecting the spatially extended system to spatiotemporal random perturbations, we find, similarly as by studying solely the temporal behaviour of the map, that there also exists an optimal σ for which noise-induced spatial patterns are optimally ordered in space. We explain the reported noise-induced pattern formation by studying the typical width of emergent circular waves in dependence on various D. As anticipated, w increases with the square root of D, thereby confirming the proposed explanation for the observed phenomenon.

Throughout the study, emphasis on clear-cut guidance and reproducibility of the results presented is given, which hopefully makes the work interesting also for beginners in the field of noise-induced phenomena in nonlinear systems, to whom this paper may represent the first contact with the presented theory. Readers who have been inspired by what they read are advised to seek further information about possible effects of noise on the dynamics of spatially extended systems in the book by García-Ojalvo and Sancho [11], which is currently one of the most comprehensive sources related to the material presented above. Of note, other interesting noise-induced phenomena in spatially extended systems include the spatiotemporal stochastic resonance [26], structure formation by coloured spatiotemporal noise [27], noise-sustained coherence of spacetime clusters and self-organized criticality [28], persistency of noise-induced spatial periodicity [29], noise-enhanced and induced excitability [30, 31], noise-induced propagation of harmonic signals [32], noise-sustained and controlled synchronization [33] as well as spatial decoherence due to small-world connectivity [34].

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