Chaos promotes cooperation
in the spatial prisoner's dilemma game

M. Perc

Department of Physics, Faculty of Education, University of Maribor
Koroška cesta 160, SI-2000 Maribor, Slovenia

Abstract. – We introduce chaotic variations, modelled by a spatially extended Lorenz system, to the payoffs of the spatial prisoner's dilemma game and study their effects on the evolution of cooperation. We show that chaotic variations of appropriate amplitude promote cooperation over a wide range of payoff parameters at which defection is the only strategy in a sterile environment. An appropriately pronounced chaotic environment can assure permanent domination of cooperation by full anonymity of players and without the aid of secondary strategies, thus designating chaotic payoff variations as a general and stand-alone mechanism for cooperation in the spatial prisoner's dilemma game.

The prisoner's dilemma (PD) game [1] is considered a paradigm for studying the evolution of cooperative behaviour among egoistic individuals. Originally, the game consists of two players who have to decide simultaneously whether they want to cooperate or defect. The dilemma is given by the fact that although mutual cooperation yields the highest collective payoff, a defector will do better if the opponent cooperates. Since players are aware of this fact they both decide to defect whereby none of them gets a profit. This unfavourable equilibrium state is, however, often violated in real life [2–5]. Accordingly, several mechanisms have been proposed to explain the emergence of cooperation in various types of games. Examples include spatial extensions [6–12], direct and indirect reciprocity [13–15], as well as voluntary participation [16–18].

Besides intrinsic mechanisms that facilitate cooperative behaviour, extrinsic factors, affecting either the players [19, 20] or the game itself [21–24], were recently also acknowledged as being important agonists in the evolution of cooperation, influencing the overall population gain and equilibrium selection, as well as the nature of phase transitions from one equilibrium towards the other. Indeed, maintenance of different personality types favours the evolution of cooperation in the PD game [20], whilst additive noise introduced to the classical replicator dynamics, supplemented by adaptive learning rates, can enhance the average payoff of the population in a coherence-resonance–like manner [24]. Moreover, noise introduced to the strategy adoption process or the payoffs of spatially distributed players can induce phase transitions that fall under the directed percolation universality class [21, 22], as well as maintain cooperation at the highest possible level with respect to intrinsic game parameters [23, 25].

Presently, we report a new mechanism for the promotion of cooperation in finite-size populations on square grids. The virtue of our approach is to link the intrinsic rules of the game with

the environment that is assumed to be chaotic. In particular, we study the effects of additive chaotic variations, introduced to the payoff matrix of the spatial PD game, on the evolution of cooperation. The chaotic environment is modelled by a simple spatial extension of the classical Lorenz system [26]. We show that appropriately pronounced chaotic disturbances are able to sustain dominance of cooperators even for defection temptation values substantially exceeding the one marking cooperation extinction in the absence of explicit payoff variations. We argue that real-life environmental variations are often chaotic by nature, and not simply random [25], so that the presented approach is indeed plausible and appropriate for modelling the evolution of cooperation in the real world. Examples of environmental chaotic influences might include the weather, exposure to sunlight, or variations in food supplies. Also, we suggest that such influences present a viable mechanism for cooperation promotion in various environments, ranging from human and animal societies to economic cycles or even biochemical systems [27,28].

For the following calculations, we consider an evolutionary PD game with players located on vertices of a regular two-dimensional square lattice of size $n \times n$ with periodic boundary conditions. Moreover, we assume that each individual interacts only with its four nearest neighbours located to the north, south, east and west, whereby self-interactions are excluded. Each player can decide either to cooperate (C) or to defect (D). Depending on the choice of their strategies, each two players (P_i, P_j) receive payoffs summarized succinctly by the payoff matrix

$$
\begin{array}{c|cc}
P_i/P_j & C & D \\
\hline
C & 1 + ax_i/1 + ax_j & 1 + \kappa + ax_i/-\kappa + ax_j \\
D & -\kappa + ax_i/1 + \kappa + ax_j & 0 + ax_i/0 + ax_j
\end{array}
\tag{1}
$$

where $\kappa \geq 0$ determines the temptation to defect whilst $a \geq 0$ scales the amplitude of payoff variations. The payoff matrix is subjected to additive chaotic disturbances that are determined by a simple spatial extension of the classical Lorenz system [26] given by the equations

$$\mathrm{d}x/\mathrm{d}t = \sigma(y - x) + D\nabla^2 x, \tag{2}$$

$$\mathrm{d}y/\mathrm{d}t = rx - y - xz, \tag{3}$$

$$\mathrm{d}z/\mathrm{d}t = xy - bz. \tag{4}$$

For simplicity, but without loss of generality, only the variable x of the Lorenz system is subjected to diffusive coupling, which is implemented via a first-order numerical approximation with periodic boundary conditions on a unitary two-dimensional lattice, whereby the diffusion coefficient $D = 2.0$ is chosen small-enough as to forbid synchronization amongst distant units. Note that strongly spatially correlated disturbances evoked by large D decrease the effect of cooperation promotion since they essentially act as payoff scaling that is equal over extensive areas of the spatial plane, thus not affecting the outcome of the game at all. On the other hand, some spatial correlations of environmental disturbances, warranted by $D > 0$, have to be taken into account since nearby neighbours surely experience the same fate with respect to external factors (*e.g.*, weather, exposure to sunlight, food supplies). In general, however, smaller D decrease spatial correlations of the chaotic payoff variations and thus increase unpredictability of the environment. The effect of cooperation promotion is therefore better pronounced by smaller than by larger D.

Importantly, the variable x satisfies $\langle x_i \rangle_t = 0$ for $\forall i$, where $\langle \ldots \rangle_t$ indicates average over time. Thus, the average addition to the payoffs, due to the introduction of chaotic variations, is zero which warrants the conditions for a "soft constraint" but not for the "hard constraint" PD game, meaning that the payoff ranking of the PD game is allowed to be violated locally in time, but is preserved on average over a long time span. If a is set small enough to ensure also

a "hard-constraint" PD game ($|ax_i| < \kappa$ for $\forall i, t$) then the facilitative effect on the survival of cooperators is minute at most.

Starting from random initial conditions for the spatially extended Lorenz system and uniformly distributed cooperators and defectors on the square lattice, each player can change its strategy according to the performance of neighbouring players, whereby the probability that a player P_i will adopt the strategy of one of its randomly chosen nearest neighbours P_j is determined by the cumulative payoffs S_i and S_j of both players according to

$$W(P_i \leftarrow P_j) = \frac{1}{1 + \exp[(S_i - S_j)/K]}, \tag{5}$$

where $K = 0.1$ is the uncertainty related to the strategy adoption. $0 < K \ll 1$ implies that the better performing player is readily adopted, whilst it is not completely impossible to adopt the strategy of a worse performing player.

The described spatial prisoner's dilemma game can be iterated forward in time using either a synchronous or a random Monte Carlo update scheme [11], whereby it has been reported that by non-deterministic player adoption rules obtained results do not differ substantially [8,9]. Since the synchronous update scheme converges more quickly to the equilibrium than the random iteration, we thus apply the former, letting all individuals interact pairwise with their nearest neighbours and then simultaneously update their strategy according to eq. (5). After each synchronous game iteration the profile of chaotic variations imposed on the payoff matrix varies as dictated by the numerical integration procedure that is currently implemented with $dt = 0.02$. Larger dt may induce numerical instability, whilst smaller dt simply delay the transition to equilibrium frequencies of cooperators and defectors and thus unnecessary burden the expensive numerical procedure.

It is well known that cooperation promotion by spatial structure depends heavily on the payoff parameters of the game [6,7]. By the currently applied game settings and non-varying payoffs ($a = 0$) only 10% of cooperators are able to survive if the temptation to defect equals $\kappa = 0.006$. Remarkably, the addition of weak chaotic variations to the payoffs is able to boost the fraction of cooperators to 50% by the same κ. Figure 1 captures this phenomenon. Note that with or without the addition of chaotic variations cooperators survive by forming clusters so as to protect themselves against being exploited by defectors. Cooperators located in the interior of such clusters enjoy the benefits of mutual cooperation and are therefore able to survive despite the constant exploitation by defectors along the cluster boundaries.

To quantify the ability of chaotic variations to facilitate and maintain cooperation in the studied spatial PD game more precisely, we calculate the fraction of cooperators over a broad range of κ in the absence and by a fixed magnitude a of chaotic variations. Results presented in fig. 2 show that additive chaotic variations promote cooperation even for defection temptation values that are an order of magnitude larger than the threshold for cooperation extinction by $a = 0$. Importantly, while in the absence of explicit payoff variations cooperators never dominate, they do so over a broad range of κ if chaotic variations are added. Nevertheless, it also appears that there exists an upper bound of κ that can still be compensated by chaotic variations in order to prevent the extinction of cooperators.

The existence of the upper bound of κ can be studied accurately by calculating the fraction of cooperators separately for various κ in dependence on a. Results presented in fig. 3 show that although larger values of κ require stronger chaotic variations for cooperation promotion, the positive effect is still well pronounced even for $\kappa = 0.06$ if only a is large enough. Remarkably, cooperators dominate only for intermediate values of a, whilst for large a unpredictable chaotic variations completely overrule the game. Under such circumstances no strategy is
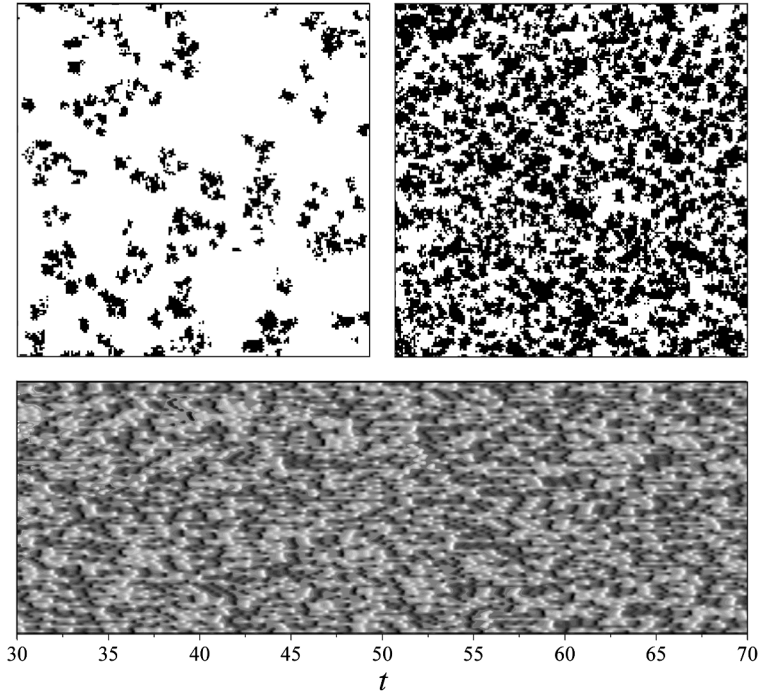
Fig. 1 – Characteristic equilibrium spatial distributions of cooperators (black) and defectors (white) obtained by $a = 0$ (left panel) and $a = 0.0034$ (right panel) for the defection temptation value $\kappa = 0.006$. Both panels are depicted on a 200×200 portion of a larger spatial grid. The lower panel features additive chaotic variations in dependence on time for a one-dimensional column of the spatial grid. The colour profile is linear, black marking -0.06 and white 0.06 values of additive payoff disturbances.
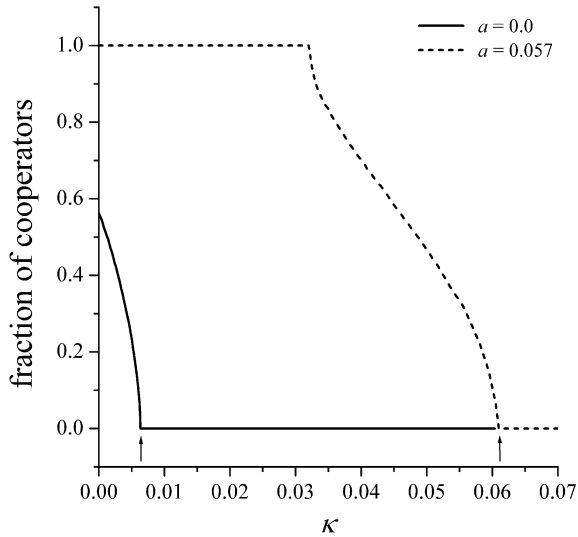

Fig. 2 – Promotion of cooperation by a chaotic environment. In the absence of chaotic payoff variations cooperators die out at $\kappa = 0.0063$ (left arrow) whereas by $a = 0.057$ they persist up to $\kappa = 0.061$ (right arrow).
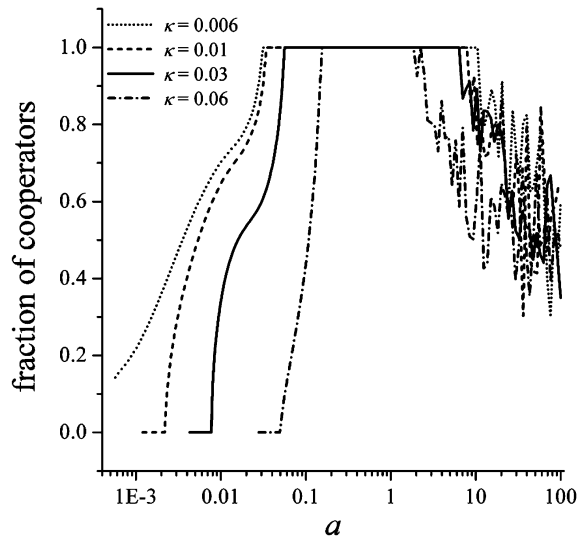
Fig. 3 – Promotion of cooperation by a chaotic environment for different values of κ. Note that the x-axis has a logarithmic scale.

privileged which is indicated by the irregular trend towards tantamount of cooperators and defectors. The fact that larger κ requires stronger chaotic variations for cooperation promotion, combined with the dominance of chance by large a, implies that there indeed exists an upper bound of κ for which environmental influences are still able to sustain total dominance of cooperators. After extensive calculations in the two-dimensional parameter space, spanned over κ and a, we found that $\kappa = 0.075$. Importantly though, even for defection temptation values exceeding $\kappa = 0.075$ chaotic variations are still able to sustain cooperation by very large a. It is highly questionable, however, whether the required large-amplitude chaotic variations are plausible.

Noteworthy, the transition from homogenous cooperation towards the mixed state occurring by large a is somewhat violent and unpredictable, thus strengthening the dominance of chance rather than determinism in that parameter range as well as fuelling speculations about plausibility of such large-amplitude variations. Evidently, average fractions of strategies for $a > 10$ are fluctuating heavily. We were unable to obtain smooth transitions in that parameter range with computationally feasible system sizes and number of game iterations.

In order to explain the reported phenomenon, we argue that cooperators are able to constructively exploit environmental influences due to their cooperative behaviour. In particular, if two neighbouring cooperators receive an unexpected positive and negative income respectively, mutual cooperation always decreases the relative difference between their cumulative payoffs, thus keeping the unlucky cooperator competitive despite of its temporary bad luck. On the other hand, the defecting strategy increases stratification among neighbours (or keeps it the same if all are defectors). Although in this sense defection threatens cooperators and defectors equally in that their strategy is rendered unsuccessful and thus unlikely to be adopted, clustered cooperators have an edge since they help each other out. Intuitively, the positive effect of chaotic payoff variations on the evolution of cooperation can be attributed to the introduced unpredictability. If two defectors meet they wind up empty handed, and at this point an unexpected or unforeseen negative income would be fatal since the defecting strategy would not be adopted in the future. Thus, in an unpredictable environment it is safer to cooperate

since then each player can always rely on a fix income. Although this income is often smaller as would be obtained by the defecting strategy, it is, on the other hand, reliable and this turns out to be the key enabling survival in an unpredictable environment. The conclusion is that in a chaotic world it appears better to receive smaller but reliable incomes than larger but infrequent ones. Noteworthy, that is true only for moderate uncertainties affecting the evolutionary process, whilst in a completely chaos-dominated environment prosperity is governed by chance.

Importantly, the above interpretation of results holds also for Monte Carlo update schemes despite the fact that they introduce an inherent infrequency of incomes due to the random pickings of players during each game iteration. Since for large numbers of game iterations all players interact with their neighbours equally often on average, the introduced infrequent incomes annihilate and the final result is qualitatively identical as obtained by the presently applied synchronous update scheme.

We suggest that payoffs, defining the interaction phase between two individuals, are a likely part of the evolutionary process where environmental uncertainties can take effect. Thereby, disturbances can arise either from the players themselves, for example by not adhering to the rules of the game in trying to make an illegal profit or due to different personalities, or from the environment which can either favour or hinder the success of each player. Either way, cooperators appear to be better suited for the trial of life.

REFERENCES

[1] AXELROD R. and HAMILTON W. D., *Science*, **211** (1981) 1390.
[2] WILKINSON G. S., *Nature*, **308** (1984) 181.
[3] SEYFARTH R. M. and CHENEY D. L., *Nature*, **308** (1984) 541.
[4] MILINSKI M., *Nature*, **325** (1987) 433.
[5] CLUTTON-BROCK T. H., O'RIAIN M. J., BROTHERTON P. N., GAYNOR D., KANSKY R., GRIFFIN A. S. and MANSER M., *Science*, **284** (1999) 1640.
[6] NOWAK M. A. and MAY R. M., *Nature*, **359** (1992) 826.
[7] NOWAK M. A. and MAY R. M., *Int. J. Bifurcat. Chaos*, **3** (1993) 35.
[8] HUBERMAN B. A. and GLANCE N. S., *Proc. Natl. Acad. Sci. U.S.A.*, **90** (1994) 7716.
[9] NOWAK M. A., BONHOEFFER S. and MAY R. M., *Proc. Natl. Acad. Sci. U.S.A.*, **91** (1994) 4877.
[10] KILLINGBACK T., DOEBELI M. and KNOWLTON N., *Proc. R. Soc. London, Ser. B*, **266** (1999) 1723.
[11] HAUERT C., *Int. J. Bifurcat. Chaos*, **12** (2002) 1531.
[12] IFTI M., KILLINGBACK T. and DOEBELI M., *J. Theor. Biol.*, **231** (2004) 97.
[13] TRIVERS R. L., *Q. Rev. Biol.*, **46** (1971) 35.
[14] NOWAK M. A. and SIGMUND K., *Nature*, **393** (1998) 573.
[15] WEDEKIND C. and MILINSKI M., *Science*, **288** (2000) 850.
[16] HAUERT C., DE MONTE S., HOFBAUER J. and SIGMUND K., *Science*, **296** (2002) 1129.
[17] SEMMANN D., KRAMBECK H. J. and MILINSKI M., *Nature*, **425** (2003) 390.
[18] HAUERT C. and SZABÓ G., *Am. J. Phys.*, **73** (2005) 405.
[19] MCNAMARA J. M., BARTA Z. and HOUSTON A. I., *Nature*, **428** (2004) 745.
[20] DALL S. R. X., HOUSTON A. I. and MCNAMARA J. M., *Ecol. Lett.*, **7** (2004) 734.
[21] SZABÓ G. and TŐKE C., *Phys. Rev. E*, **58** (1998) 69.
[22] SZABÓ G. and HAUERT C., *Phys. Rev. Lett.*, **89** (2002) 118101.
[23] SZABÓ G., VUKOV J. and SZOLNOKI A., *Phys. Rev. E*, **72** (2005) 047107.
[24] TRAULSEN A., RÖHL T. and SCHUSTER H. G., *Phys. Rev. Lett.*, **93** (2005) 028701.
[25] PERC M., *New J. Phys.*, **8** (2006) 22.
[26] LORENZ E. N., *J. Atmos. Sci.*, **20** (1963) 130. Parameters: $r = 25$, $b = 8/3$, $\sigma = 10$.
[27] PFEIFFER T., SCHUSTER S. and BONHOEFFER S., *Science*, **292** (2001) 504.
[28] PFEIFFER T. and SCHUSTER S., *Trends Biochem. Sci.*, **30** (2005) 20.