Adaptive Reputation Promotes Trust in Social Networks
Zhengyang Hu, Xiaopeng Li, Juan Wang∗, Chengyi Xia∗, Member, IEEE, Zhen Wang∗, Member, IEEE, and Matjaž Perc∗

Abstract—Trust has played a pivotal role in the evolution of modern human societies, and it continues to be an essential underpinning of our social interactions. It is therefore important that we develop rigorous mathematical foundations that will enable us to better understand what promotes and what erodes trust and how to best preserve trustworthiness. To that effect we here propose a trust game, wherein investors, trustworthy trustees, and untrustworthy trustees compete for assets subject to a third-party evaluation system that oversees and modifies each individual reputation. We use Monte Carlo simulations on social networks to determine critical values of the degree of rationality and the reputation threshold that warrant high levels of trust and social wealth. We show that if investors have access to the reputation scores of trustees, the fraction of untrustworthy trustees drops if only the degree of rationality is sufficiently large, and this irrespective of the reputation threshold that determines the cutoff for untrustworthiness. But even though investors are allowed irrational investments, trust can still proliferate if the reputation threshold is sufficiently high. Our results thus formalize essential mechanisms of trust in social networks, which also outline policies to diminish untrustworthiness that can be employed in real life.

Index Terms—Evolutionary game theory, networked population, trust game, adaptive reputation.

Manuscript received April 21, 2021; revised June 26, 2021; accepted August 2, 2021. Date of publication August 10, 2021; date of current version December 9, 2021. This work was supported by the National Natural Science Foundation of China (NSFC) under Grants 62025602, 62173247, 61773286 and U1803263. Hu and Li would like to thank for the support of Tianjin graduate research and innovation project under Grants. 2020YJS08073 and 2019YJSB005. The work of Matjaž Perc was supported by the Slovenian Research Agency under Grants P1-0403 and J1-2457. Recommended for acceptance by Prof. Herbert H. C. Iu. (Corresponding authors: Juan Wang and Chengyi Xia.)

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Digital Object Identifier 10.1109/TNSE.2021.3103771

I. INTRODUCTION

T RUST, as an essential concept, is the favorable lubricant of society and economy [1], which is found almost in all aspects of realistic lives [2], including institution managing [3], [4], formation of intimate relationships [5], [6], interpersonal communication and cooperation [7]–[9], economic behavior [10], [11] and so on. Thus, trust is so important that it has caught a lot of attention of scholars in cognitive neuroscience [12]–[14], behavioral experiment economics [15], [16], social psychology [17]–[19] and economic management [20], [21]. However, compared with a great number of studies on classical game models, such as the prisoner’s dilemma game (PDG) [22]–[24] and public goods game (PGG) [25]–[27] on social networks, but networked trust game has not drawn a considerable concern of researchers in the area of evolutionary game theory [28], [29].

Actually, trust game is the most foundational sequential game, which involves two kinds of players (trustee and trustee) with two strategies. The former (trustee) can firstly decide whether to trust the latter or not. If the trustee decides not to trust the trustee, then the game will stop and both players get 0. Yet, the trustee must transfer a certain amount of money c (usually set c = 1) to the trustee and take risks of being betrayed if he decides to trust. Then, the follower (trustee) must choose to return some funds (be trustworthy) or keep all of them himself (be untrustworthy). If the trustee selects to be trustworthy, both of them receive the payoff r (0 < r < 1) because of the mutual benefit and reciprocity. If the trustee decides to defect for his selfishness, then the trustee will lose the principal, getting the net payoff −1, while the trustee harvests the total payoff 1. Obviously, the Nash equilibrium of the trust game is that both trustee and trustee choose to defect [30], and not to be trustworthy, respectively. However, the rational decision of two players is inconsistent with the pervasive phenomenon of trust in our social life. Therefore, there must exist some realistic factors that promote and maintain the trust among individuals. Over the past few decades, the topic of trust and trust game has attracted more and more attention [31], [32]. As an example, from the perspective of evolutionary equilibrium, Guth et al. [33] theoretically illustrated the sub-game perfect equilibrium of “game of trust” being non-trust or non-trustworthy. Meanwhile, they found that the trust rarely occur under the context of randomly and fairly allocating positions through extensive behavior experiments. In addition, Gokhale et al. [34] proposed a method
generalizing two-player games with two strategies to multi-player games with more than two strategies. In their work, the precise mathematical proof of evolutionary game’s equilibrium has been exhibited, which shows that there could be at most \((N - 1)^{k-1}\) isolated internal equilibria for \(N\) players with \(s\) strategies. Afterwards, Abbass et al. [35] modeled an evolutionary \(N\)-player trust game. At the same time, they studied the dynamic evolution of trust in a well-mixed or unstructured demographic context, finding that the untrustworthy individuals finally dominated the whole population even if there were a small number of untrustworthy agents presented within the initial population. Undoubtedly, the whole population only composed of trusters and trustworthy individuals should be the most ideal and optimal case. It is well known that, being a general tool or framework, complex network is widely used to investigate many social [36], [37], engineering [38] and biological systems [39]. Therefore, it is great meaningful to extend the trust game to the social network. Chica et al. [40] further investigated the emergence of trust and dynamic evolutionary mechanism in the \(N\)-player trust game—being composed of three kinds of individuals: 1) investor (or truster); 2) trustworthy trustee; and 3) untrustworthy trustee—which is played within a social network with the specific spatial topology, demonstrating that \(N\)-player trust game on the structured social network is able to boost the trust to a higher level even if there are numerous untrustworthy individuals inside the initial population, which was different from the results displayed in [35]. However, not all trust games defined on structured networks [41]–[43] are capable of improving the level of trust. As an example, Kumar et al. [44] discussed a similar networked \(N\)-player trust game with two types of strategies and were surprised to find that the structured network had almost little impact on the dynamic evolution of both trust and trustworthiness except for scale-free networks with un-normalized replicator dynamics. Looking forward to reaping high returns, the truster invariably exposes himself to a risky circumstance. For instance, the seller is likely to market a counterfeit good or simply not even transport the commodity to the buyer in an online transaction [45], and this self-interested phenomenon is frequently found in some well-known games such as prisoners dilemma game (PDG) and public goods game (PGG). At present, several effective mechanisms, such as the reward, punishment and reputation and so on, have been devised to make agents voluntarily discard a part of individual benefit and then take the pro-social behavior. Taking two examples here, Xia et al. [46] investigated the evolution and risk analysis of cooperation under public goods game, showing that the risk of the player can be exploited effectively decline under the action of individual reputation, and the evolution of cooperation can also be greatly enhanced when taking reputation into consideration. Li et al. [47] proposed a reputation-based changing intensity of interaction mechanism to study the evolution of cooperation by considering the evaluation level of individual behavior. Both rigorous mathematical proof and extensive numerical simulations indicated that the reputation evaluation can effectively promote the cooperation level. In the trust game of buyers and sellers in online marketplaces, it is conceivable that reputation mechanism can enhance the behavior of buyer’s trust and the seller’s cooperation, which can alleviate the social dilemma to a certain extent. Since a couple of individual moral-related mechanisms [48]–[53] have been demonstrated to be capable of promoting the cooperation in spatial networks with the help of indirect reciprocity. Thus, the reputation mechanism, as a common rule related with the indirect reciprocity, may be responsible for the promotion of trust at play in the trust game.

In addition, in the traditional trust game, the truster will transfer the same amount of money to each trustee if he decides to trust, which is unwise for the truster. Henceforth, in this paper, we consider a networked \(N\)-player trust game with a dynamic and adaptive individual reputation mechanism based on the third-party evaluation system [54]–[56] to probe into the dynamic evolution of the trust and trustworthiness under a more realistic context. To be remarkable, in the stock investment, a rational individual usually anticipates the payoff of stocks in advance, and then reasonably allocates the capital to buy the corresponding stocks. Thus, we introduce the degree of rationality \(\alpha\) to describe whether or how rational the truster is in the trust game. Besides, we also introduce the reputation threshold \(R_c\) to distinguish the trustee’s reputation level or judge whether it is trustworthy, because the phenomenon that individuals are unwilling to interact with others with bad reputation is ubiquitous. For example, people prefer to consume much more in the online shops with good reputation and positive comment than those with poor reputation and negative comment. Specifically, within a group of trust game players, the truster rationally chooses whether to trust the trustee or not according to the reputation status of the trustee with the probability of \(\alpha\) (only the truster is allowed to give the opportunity and priority to evaluate the trustee’s reputation), or to just trust the trustee irrationally and unconditionally with the complementary probability. In addition, the reputation threshold \(R_c\) also acts as a classifier, which enables the trusters to distinguish between trustees with high credibility and to avoid the contact with untrustworthy trustees. The main contributions in this work can be outlined as the following aspects:

- A new networking trust game model, which is based on the reputation evaluation from the third party, is put forward to promote the evolution of trust behavior within the population.
- An adaptive reputation mechanism is integrated into the networked trust game, where the reputation of trustees will be evaluated according to their returning strategy for the truster’s investment.
- The individual irrationality is introduced into the investment decision of trustee (i.e., investor), who may irrationally donate to the trustees without considering their reputation.

The remainder of this paper is organized as follows. Firstly, we explain the networked \(N\)-player trust game with a dynamic and adaptive individual reputation mechanism, and depict the payoff calculation as well as the evolutionary update methods of both individual strategies and reputation in Section II. Secondly, plentiful simulation experiment results and analyses
are illustrated in Section III. In the end, the conclusions and future works are presented in Section IV.

II. MODEL

A. Networked Trust Game

In this section, we will define the networked N-player trust game with a dynamic and adaptive reputation mechanism based on the third-party evaluation system in detail. The model consists of $N$ (a limited natural number) agents occupying social network nodes, and the links represent the contact and interaction between them. The topology of social network accommodating game agents does not change over time, which means that the network in our model is static. Following the model proposed by Abbass et al. [35], during the evolutionary process, each agent can choose from three possible strategies $S = \{I, T, U\}$, where the strategy $I$ denotes the investor who commits to provide assets in the trust game, the strategy $T$ stands for a trustworthy trustee who uses an asset handed by an investor in a credible, economically appropriate manner and the strategy $U$ represents an untrustworthy trustee who reaps an asset handed by an investor in an incredible and inappropriate manner.

At the beginning, three classes of strategies are randomly assigned within the population and the normalized condition is also fulfilled. Without lacking the generality, we can set $ProI = 0.3$, $ProT = 0.3$ and $ProU = 0.4$, where $ProI$, $ProT$ and $ProU$ are proportions of agents holding the strategy $I$, $T$ and $U$ in the population, respectively. In particular, each agent $i$ will be endowed with a random number to denote one’s reputation value $R_i$, which is set within the interval between $(0, 4)$ and basically follows the Gaussian-distribution $R_i \sim G(\mu, \sigma^2)$ where $\mu = 2$ and $\sigma^2 = 0.6^2 = 0.36$. The reputation value will be dynamically and adaptively adjusted depending on the agent’s behavior. Reputation owned by an agent is assessed through a third-party evaluation system (similar to the role of credit agency), thus the information of an agent’s reputation is open to public group members, and all focal agents have access to the reputation of other neighbors. As shown in Fig. 1, 10 agents occupy nodes of a social network, and the group of focal agent 2, 5 and 8 are specially labeled with the shadowed area. Every focal agent can interact with any one of its direct neighbors within a single trust game.

After initializing the model, each focal agent in the network starts to interact with the direct neighbors in the group. During the process of interaction with their group neighbors, since the investor has the priority in the trust game, it will decide whether to trust the trustee or not according to the reputation of the trustee and the probability of $\alpha$, or to trust the trustee unconditionally with the complementary probability $1 - \alpha$, as shown in Fig. 2. We call the former rational trust and the latter irrational trust, respectively. The rational trust is a mechanism to protect investors and avoid bad trustees, which means that investors will decide whether to trust the trustee or not and the trust extent to the trustee in terms of their reputation. But it is worth mentioning two points when deciding on rational trust: first of all, only the trustee with a high reputation (i.e., $R_i > R_c$) has access to earn the trust of investors, on the contrary, the investor will not trust a trustee with low reputation (i.e., $R_i < R_c$); another one is that the higher the reputation of the trustee is, the more trust investors will devote to, namely, the trustee will obtain more money. Besides, if the investor is not rational enough or makes a wrong decision, that is, opting for irrational trust, then the investor chooses to trust every trustee in the group unconditionally and will distribute the funds equally. In addition, here we do not consider the heterogeneity of individual degree of rationality, and it is assumed that the degree of rationality $\alpha$ of all investors is identical in an independent simulation. Fig. 3 shows the specific interaction process between the

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**Fig. 1.** A social network consists of 10 agents with three types of strategies $I$, $T$ and $U$. The blue, green and red cartoon denote an investor, a trustworthy trustee and an untrustworthy trustee, respectively. The areas shadowed in blue, green and red denote the game interaction groups with agent 2, 5 and 8 as the focal agent. Each agent owns an individual reputation $R_i$.

**Fig. 2.** Detailed interaction process between the investor and trustees with agent 2 in Fig. 1 as the focal agent. The upper branch indicates that the investor decides to trust trustees rationally, and the lower branch denotes the case of the investor to trust trustees irrationally. The investor pays $tv$, then it will be allocated to trustees in the group according to whether the investor is rational or irrational. After that, the trustworthy trustees return half of its payoffs, while the untrustworthy trustees retain all its payoffs.
Investor and trustees in the trust game, the trustworthy trustee will return half of payoffs, which are equal to the funds received from the investor multiplied by the trustworthy trustee’s gaining factor $2 \cdot R_T$, while the untrustworthy trustee will keep all of the funds received from the investor multiplied by the untrustworthy trustee’s gaining factor $R_U$ and will return nothing, where $1 < R_T < R_U < 2R_T$. Investors have no information about whether the trustee is trustworthy before making the decision, and thus they have to run some risks for getting higher returns.

Henceforth, the question then arises since the investor’s two completely different decisions may lead to the distinction in calculating the net wealth (or return) of game players. Taking Fig. 2 for example, for the rational trust, the net wealth of agent 5 (strategy $T$) is $w_5 = R_T \cdot \frac{k_T}{k_T - 1} \cdot t_v$, where $k_T$ represents the number of investors (strategy $I$) in the group, and $S_r$ is the total reputation of all trustees with high reputation in the group, $t_v$ denotes the investor’s trust value being paid to trustees. As the game dynamics are independent of the value of $t_v$, we can set it to be 1 [35]. In particular, the investor’s payoff strongly depends on the return of trustworthy trustees, hence the net wealth of agent 2 (strategy $I$) can be calculated as $w_2 = (R_T \cdot \frac{S_r}{N} - 1) \cdot t_v$, where $S_r$ is the total reputation of all trustworthy trustees with high reputation in the group. The net wealth of agent 1 (strategy $U$) is 0 because of its low reputation, which makes it untrustworthy in the eyes of the investor. However, for the irrational trust, calculations of agents’ net wealth are considerably simple in contrast to that of rational trust. Investors will trust every trustee in the group unconditionally and assign funds within all agents of the group equally, thus every trustee in the group receives $\frac{1}{N} \cdot t_v$, where $k_U$ represents the total number of trustworthy trustees and untrustworthy trustees in the group. Therefore, the net wealth of agents 1, 2 and 5 are $w_1 = R_T \cdot \frac{k_U}{k_U} \cdot t_v$, $w_2 = (R_T \cdot \frac{k_U}{k_U} - 1) \cdot t_v$ and $w_5 = R_T \cdot \frac{k_U}{k_U} \cdot t_v$, respectively, where $k_T$ is the number of trustworthy trustee (strategy $T$) in the group. Table I shows the specific calculations of the net wealth of agents in Fig. 2 under the rational trust and the irrational trust, respectively.

From the above example, under given model parameters, we can know that the net wealth of one focal individual is determined according to the decision of investors, strategies and reputation values of itself and its direct neighbors.

\[
\text{Net wealth } w_i = \begin{cases} 
R_T \cdot \frac{S_r}{N} - 1 \cdot t_v & \text{if } s_i = I \\
R_T \cdot \frac{k_T}{k_T - 1} \cdot t_v & \text{if } s_i = T \& R_i > R_c \\
0 & \text{if } s_i = T \& R_i < R_c \\
R_U \cdot \frac{k_U}{k_U} \cdot t_v & \text{if } s_i = U \& R_i > R_c \\
0 & \text{if } s_i = U \& R_i < R_c \\
(R_T - 1) \cdot t_v & \text{if } s_i = I \\
R_T \cdot \frac{k_T}{k_T} \cdot t_v & \text{if } s_i = T \\
R_U \cdot \frac{k_U}{k_U} \cdot t_v & \text{if } s_i = U 
\end{cases}
\]  

Therefore, we define the net wealth $w_i$ of focal individual $i$ as Equation (1), shown at the bottom of the page, where $s_i$ is the current strategy of agent $i$. Particularly, the net wealth of focal agent $i$ is simply 0 when there is no trustee with high reputation within the neighborhood under the case of rational trust and is no trustee in the group under the case of irrational trust. Besides, $S_r$ and $S_r$ can be defined as follows:

\[
S_r = R_i + \sum_{j=1}^{n} R_j 
\]

\[
S_r = R_i + \sum_{j=1}^{n} R_j 
\]

\[
(s_i, s_j \in \{T, U\} \& R_i > R_c, R_j > R_c)
\]

\[
(s_i, s_j \in \{T\} \& R_i > R_c, R_j > R_c)
\]

where $R_i$ is the reputation of focal agent $i$, $R_j$ is the reputation of neighbor $j$ that is directly connected to the focal agent $i$, $n$ is the number of neighbors in the group, $s_j$ is the current strategy of neighbor $j$. Besides, we also pay attention to the global wealth $GW$ of the population [40], which is calculated in the following way:

\[
GW = \sum_{i=1}^{N} w_i
\]

where $N$ is the number of all nodes occupying the social network in the model.

B. Evolutionary Update of Strategy and Reputation

At each time step $t$ during the whole simulation, agents have chances to update their strategy. Each focal agent will

<table>
<thead>
<tr>
<th>Net wealth</th>
<th>Rational trust</th>
<th>Irrational trust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent 1 (U)</td>
<td>0</td>
<td>10 $\times \frac{1}{3}$</td>
</tr>
<tr>
<td>Agent 2 (I)</td>
<td>$6 \times \frac{3.4}{3.4+3.2} - 1$</td>
<td>$6 \times \frac{1}{3} - 1$</td>
</tr>
<tr>
<td>Agent 5 (T)</td>
<td>$6 \times \frac{3.4}{3.4+3.2}$</td>
<td>$6 \times \frac{1}{3}$</td>
</tr>
</tbody>
</table>
synchronously decide whether it will imitate its direct neighbor’s strategy, which ensures that the decision of every agent at a fixed step is not affected by others. We use a common update rule—the proportional imitation rule—in the model, for the reason that it is convenient for us to compare our results with those in [40]. A focal agent may choose one of three possible strategies (i.e., I, T or U) according to one randomly chosen neighbor at each time step $t$ while its strategy needs to be updated. Firstly, the focal agent $i$ randomly selects a neighbor $j$ from its nearest neighbors, and meanwhile agent $i$ and $j$ obtain their net wealth calculated by Equation (1). Secondly, the rule compares the net wealth $w_j$ of neighbor agent $j$ at this time step $t$ with that of the focal agent $i$, $w_i$. If $w_j$ is higher, then focal agent $i$ will adopt the strategy of agent $j$ (i.e., $s_j$) at next time step $t + 1$ with the following probability:

$$\text{prob}(s_{i+1} \leftarrow s_j) = \frac{\max(0, w_j - w_i)}{\varphi}$$

where $\varphi$ can be calculated by $w_{\text{max}} - w_{\text{min}}$. The minimum possible wealth $w_{\text{min}}$ is 1 in our model, which occurs when an investor is completely surrounded by untrustworthy trustees. On the contrary, when the focal agent is trustworthy trustees (strategy $U$) and its direct neighbors are all investors (strategy $I$), the maximum possible net wealth $w_{\text{max}}$ can be obtained and be equal to $R_T * k_{\text{max}}$, in which $k_{\text{max}}$ denotes the maximum degree of the network. Besides, the probability $\text{prob}(s_{i+1} \leftarrow s_j) \in [0, 1]$.

What’s more, the reputation of an agent is also constantly updated during the process of the game. The value of the agent’s reputation at time step $t + 1$ relies on its strategy at the time step $t$. If an agent chooses to be the investor (strategy $I$), the value of reputation remains unchanged. Otherwise, the agent’s reputation will accordingly increase or decrease $\Delta$ if the agent adopts the strategy $T$ (i.e., be a trustworthy trustee) or strategy $U$ (i.e., be an untrustworthy trustee), where $\Delta = 0.04$ denotes the unit of reputation variation. The update rule of reputation can be defined in the following way,

$$R_{i+1} = \begin{cases} R_i & \text{if } s_i = I \\ R_i + \Delta & \text{if } s_i = T \\ R_i - \Delta & \text{if } s_i = U \end{cases}$$

In particular, without loss of generality, we try to keep the reputation within the reasonable scope. On the one hand, if $R_i + \Delta > 4$, then $R_{i+1} = 4$; on the other hand, $R_{i+1} = \frac{3}{2}$ if $R_i - \Delta \leq 0$.

Simulations of the trust game model are conducted by means of Monte Carlo simulation (MCS) on multiple types of network population of 1024 agents (i.e., $N = 1024$). In order to effectively reduce the impact of randomness, all simulation experiments are repeated for 50 independent Monte Carlo runs, which change both the agents’ initial strategies and initial reputation setup under the same social network topology and other parameters.

In the following section, we investigate the effects of different parameter settings, mainly including the degree of rationality $\alpha$, [Fig. 3. The average number of final agents ($K_T$, $K_I$ and $K_U$) and global wealth ($GW$) as a function of the degree of rationality $\alpha$ for different values of reputation threshold $R_c$ on an SF network. As $\alpha$ increases when $R_c$ is fixed, $K_T$ in panel (a), $K_I$ in panel (b) and $GW$ in panel (d) are all greatly enhanced and then reach the stable maximum, while $K_U$ in panel (c) is largely reduced. For each of $R_c$, we provide the results with respect to the degree of rationality $\alpha \in [0, 0.5]$, because there will be no significant impact on the results when $\alpha$ is greater than 0.5. Other parameters are set to be $R_I = 6$, $r_{IT} = 0.7$, the initial proportion of three types of agents $Prob = 0.3$, $ProI = 0.3$ and $ProU = 0.4$, respectively.]

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reputation threshold $Rc$ as well as the robustness verification through extensive and systematic MCS experiments.

### III. Results

Here, we have determined the stationary state when the average of various strategy densities on the network becomes time-independent, and it often takes within 10 000 MCS steps to arrive at this state. However, in order to ensure the higher accuracy, all results are calculated by averaging over the last quartile of total time steps (i.e., the last 5000-time steps) for each parameter setup converged to an equilibrium state within 20 000 MCS steps. Furthermore, the results displayed here have been averaged over 50 independent realizations to further reduce the perturbations. The trustworthy trustee’s gain factor $R_t$ is set to be 6, and the ratio of temptation to defect $r_{UT}$ is set to be 0.7 by default, which is much bigger than that in [40].

Thus, according to the function $r_{UT} = R_{UT} / R_{T}$, the trustworthy trustee’s gain factor $R_t$ is set to 10.2. Furthermore, the average degree of SF network is usually fixed to be $<k> \approx 4$.

First of all, the average number of three types of agents (i.e., $K_I$, $K_T$, and $K_U$) and global wealth ($GW$) are plotted as a function of the degree of rationality $\alpha$ for different values of reputation threshold $Rc$, which are shown in Fig. 3. It can be found that, with the increase of $\alpha$, the number of investors ($K_I$), trustworthy agents ($K_T$) and global wealth ($GW$) continue to increase while the number of untrustworthy agents ($K_U$) continues to decline, which indicates that the level of trust has been effectively promoted with the help of the proposed dynamic and adaptive individual reputation mechanism. When $\alpha$ is less than 0.1, there is no dramatic change in all subgraphs in Fig. 3 with different $\alpha$. Unexpectedly, when $\alpha$ further increases, this phenomenon converts greatly. The $K_U$ decreases monotonously and then gradually becomes almost extinct. In addition, no matter what the value of reputation threshold $Rc$ is, if the degree of rationality $\alpha$ is large enough, untrustworthy agents (Strategy $U$) can be wiped out of the system. Thus, we can confirm that the degree of rationality $\alpha$ can strongly affect the evolution process of the trust game, which helps investors and trustworthy trustees to form compact clusters more easily so as to resist the invasion of untrustworthy trustee. Therefore, we conclude that it is $\alpha$ that be the key factor to promote the evolution of trust.

In order to further scrutinize the role of degree of rationality $\alpha$ in the networked trust game model, the time series of both $K_I$ and $GW$ at each MCS step for different values of $\alpha$ on $L = 32$ regular lattice network. Each value on the curve is the average over 50 Monte Carlo simulations and the maximum time step $t$ is 20 000. This is a double Y-axis graph to which the left axis denotes the average number of agents while the right axis represents the global wealth. It is worth noting that the first panel (a) corresponds to the traditional case (as in [40]) in which $\alpha$ is set to be 0. In the middle panel (b) and the third panel (c), $\alpha$ is set to be 0.2 and 0.4, respectively. Besides, the blue, green and red full line, denote $K_I$, $K_T$ and $K_U$, respectively, and the grey dotted line indicates $GW$, which records the detailed evolution of four parameters at each MCS step. Other parameters are fixed to be $Rc = 3.0$, $R_T = 6$ and $r_{UT} = 0.7$, the initial proportion of three types of agents $ProI = 0.3$, $ProT = 0.3$ and $ProU = 0.4$, respectively.

To deeply analyze why the rational trust based on dynamic and adaptive individual reputation mechanism can promote the evolution of trust more effectively than the traditional case, here we inspect the characteristic snapshots from a micro perspective, which is shown in Fig. 5. It should be firstly noted that a regular square lattice with $N = L \times L$ has been used as
a potential network of interactions. From Fig. 5, one can intuitively observe that the evolutionary process of snapshots presents the great difference under different rational conditions. When $\alpha = 0$ (the first row in Fig. 5), investors are totally irrational, which degenerate to the traditional networking case [40]. Thus, investors are undiscerned in the process of the trust game, which renders that they have to put up with the deception from untrustworthy trustees. Due to the high ratio of temptation to defect, untrustworthy trustees with low reputation continues to increase and investors gradually decrease. Finally, investors are thoroughly swept away, and there are a large number of untrustworthy trustees and a small number trustworthy trustees in the system. Moreover, once investors completely disappear, the net wealth of every agent is simply 0 and they do not change its strategy any more, leading to the cases that snapshots at MCS = 10 000 and MCS = 20 000 are almost the same. However, if investors are not totally irrational (i.e., $\alpha \neq 0$), that is, they can invest according to the reputation of the trustee, then the situation will be changed. As for $\alpha = 0.2$, we can observe the second row in Fig. 5, compared to the panel (b) in Fig. 4, we can infer that the system in a state of dynamic balance and investors will be kept in the system at last. Furthermore, when the degree of rationality $\alpha$ increase to 0.4, untrustworthy trustees will be be purged from the system, and investors with high reputation as well as trustworthy trustees with high reputation will rule the system. In fact, the proportion of agents with high reputation ($R > R_c$) becomes lower and lower at the first 40 MCS steps, however, trustees with high reputation are able to gain payoffs while trustees with low reputation can not do like this. Furthermore, due to the impact of the dynamic and adaptive individual reputation mechanism, the individual reputation will be plus or minus one unit of $\Delta$ if the agent chooses to be trustworthy or untrustworthy, respectively. Thus, within several limited steps, with the value of reputation being declined, the untrustworthy trustees (strategy $U$) gain fewer payoffs, or even gain 0 when $R < R_c$. Under these cases, both the trustworthy trustees with high reputation and investors can gain considerable payoffs, and thus they can own enough space or chance to persuade the neighbors with low reputation or untrustworthy trustees to adopt the prosocial behaviour, which will create the favorable environment much more for trustworthy clusters. With the increase of $\alpha$, the proposed mechanism restricts trustees with low reputation to gain high payoffs and ensures the beneficial environment for trustworthy trustees with high reputation, which can further increase the return to investors. What is more, they form compact clusters to further resist the invasion of untrustworthy trustees under the enhanced network reciprocity.

Fig. 5. Characteristic snapshots for different values of the degree of rationality $\alpha$ on L = 32 regular lattice network. From top to bottom, $\alpha$ is set to be 0.0, 0.2 and 0.4, respectively. From left to right, the MCS steps are set to be $0, 1 \times 10^2, 1 \times 10^3, 1 \times 10^4$ and $2 \times 10^4$, respectively. In all panels, the dark blue dots represent investors with high reputation (HR-I), the light blue dots represent investors with low reputation (LR-I), the dark green dots represent trustworthy trustees with high reputation (HR-T), the light green dots represent trustworthy trustees with low reputation (LR-T), the dark red dots represent untrustworthy trustees with high reputation (HR-U), the light red dots represent untrustworthy trustees with low reputation (LR-U), respectively. The other parameters are the same as those in Fig. 4.
the degree of rationality $\alpha$ are considered. When $\alpha = 0$ (the first row in Fig. 6), one can find that nonzero $GW$ can be obtained only in the absence of a untrustworthy trustee at the initial population, which indicates that the trust is hard to spread in this case. Similarly, only when there is no untrustworthy trustee in the system, the other type of agents can survive in the system, otherwise, investors will be wiped out from the system [see panel (a1) in Fig. 6]. All these results have been explained in the previous discussion, which are coincident with those obtained in [40]. When investors choose the rational trust with a certain small probability $\alpha = 0.2$ (the second row in Fig. 6), the situation has been dramatically changed. Under this scenario, even if there are few investors ($ProI = 0.1$) and trustworthy agents ($ProT = 0.1$) in the initial population, $K_I$ [see panel (b1)] can be enhanced and maintained at about 91, $K_U$ [see panel (b3)] declines to about 621, and $GW$ [see panel (b4)] can be achieved to approximately 863. As a further step, when we set $\alpha = 0.4$ (the third row in Fig. 6), it means that investors are more rational and intelligent, and at this moment the system will evolve into the higher level of trust. Investors (strategy $I$) and trustworthy trustees (strategy $T$) are eliminated only when the initial population consists of untrustworthy trustees (strategy $U$) and the other one of the two (investor or trustworthy trustee). In this case, with the initial population being set to be $ProI = 0.1$ and $ProT = 0.1$, $K_I$ [see panel (c1)], $K_T$ [see panel (c2)] and $GW$ [see panel (c4)] increase up to approximately 450, 574 and 3897, respectively. For almost any initial population setting, the trust can be significantly promoted and there are no untrustworthy trustees (strategy $U$) at the stationary state. It is worth mentioning that the dynamic and adaptive individual reputation mechanism can greatly enhance the trust level even if the social dilemma is quite hard ($r_{UT} = 0.7$), which seems to be almost impossible to achieve a rough task in the traditional case [40].

To further compare with the results of Chica et al. in [40], the average number of final agents (i.e., $K_I$, $K_T$ and $K_U$) and global wealth ($GW$) as a function of the initial proportion of population by setting $ProI$, $ProT$ and $ProU$ for different values of the degree of rationality $\alpha$ on an SF network. From top to bottom, $\alpha$ is set to be 0, 0.2 and 0.4, respectively. Four columns of graphs represent $K_I$, $K_T$, $K_U$ and $GW$, respectively. Other parameters are set to be $Re = 3.0$, $R_T = 6$ and $r_{UT} = 0.7$, respectively.

![Fig. 6. The average number of final agents($K_I$, $K_T$ and $K_U$) and global wealth($GW$) as a function of the initial proportion of population by setting $ProI$, $ProT$ and $ProU$ for different values of the degree of rationality $\alpha$ on an SF network. From top to bottom, $\alpha$ is set to be 0, 0.2 and 0.4, respectively. Four columns of graphs represent $K_I$, $K_T$, $K_U$ and $GW$, respectively. Other parameters are set to be $Re = 3.0$, $R_T = 6$ and $r_{UT} = 0.7$, respectively.](image-url)
increase of investors’ degree of rationality, the mutual benefit relationship between investors and trustworthy trustees can be more stable and persistent under the effect of dynamic and adaptive individual reputation mechanism so that they can maintain even in a harder social dilemma. Meanwhile, the dynamic and adaptive individual reputation update will punish untrustworthy trustees by reducing their reputation, which makes them gain fewer and fewer payoffs. Hence, the untrustworthy trustees tend to change their strategies, and then the trust and trustworthiness inside the system are promoted to a higher level under this context.

The results shown above aim at discussing the effect of the rationality of degree $\alpha$ on the dynamic evolution of the networked trust game. Furthermore, in order to understand the impact of the threshold $Rc$ on the evolution of trust and trustworthiness, the results of the average number of final agents ($K_I$, $K_T$ and $K_U$) and global wealth ($GW$) as a function of reputation threshold $Rc$ for different values of the degree of rationality $\alpha$ on an SF network is illustrated in Fig. 7. For making the comparison with the traditional model in [40], the black curve ($\alpha = 0$) is also plotted here. When the reputation threshold $Rc$ is 0, it means that investors will no longer distinguish whether the trustee has a high reputation or not, and will unconditionally trust every trustee within the group. However, it is worth noting that investors will still allocate their funds according to the trustee’s reputation, that is, the greater reputation the trustee has, the more funds it will get, which is quite different from the traditional model. As one can see in Fig. 8, with the increase of $Rc$, $K_I$ declines mildly when the value of $\alpha$ is small ($\alpha = 0.1$ or $\alpha = 0.2$), and declines dramatically when the value of $\alpha$ is large enough ($\alpha = 0.3$ or $\alpha = 0.4$). Yet, when $\alpha = 0.5$, with the increase of $Rc$, there is no obvious change and all curves can reach a higher stable state since the higher degree of rationality $\alpha$ encourages the trustees to hold the good image (i.e., the higher reputation), which is helpful for them to own the higher investment proportion from the trustees.

In order to verify the robustness of these results on different networks, we next implement the model by using ER random networks with the average degree $<k> \approx 4$ for the trust game. We find that the positive effect of the degree of rationality $\alpha$ and the reputation threshold $Rc$ on the evolution of networked N-player trust game persists, which is also compatible with those on SF network. Fig. 9 depicts the colored phase diagram for the average number of final agents ($K_I$, $K_T$ and $K_U$) and global wealth ($GW$) when the reputation threshold $Rc$ and the degree of rationality $\alpha$ are varied on an ER network. Obviously, with the increase of $\alpha$, the number of investor (strategy I) and trustworthy trustee (strategy T) as well as global wealth ($GW$) increase monotonously, which means that trust and trustworthiness in the model are promoted, see panel (a), (b) and (d) in Fig. 9, respectively. When $\alpha$ is small ($\alpha < 0.13$) or large ($\alpha > 0.5$) enough, it can be seen that there is no dramatic but mild change in all subgraphs in Fig. 9 with the increase of $Rc$. Hence, we infer that $\alpha$ is the key factor to promote the evolution of trust, which is consistent with our previous discussion. However, when $0.13 < \alpha < 0.5$, this phenomenon is greatly varied, in which, with the increase of $Rc$, there is a considerable promotion in $K_I$ and $K_T$, and less $K_U$ exists for a larger $Rc$. Compared with the SF network, the promotion of trust in ER network is more difficult. For this reason, with the increase of $\alpha$, the speed of untrustworthy agents (strategy U) disappearing in the population is lower than that in the SF network under the same condition for other parameters, which can be observed in panel (c) of Fig. 9 and in Fig. 3.

Fig. 7. The average number of final agents ($K_I$, $K_T$ and $K_U$) and global wealth ($GW$) as a function of the ratio of temptation to defect $r_{IT}$ for different values of the degree of rationality $\alpha$ on an SF network. For each of $\alpha$, we change the value of $r_{IT}$ over the range of 0.1 to 0.9 by using a step size 0.02. Other parameters are set to be $Rc = 3.0$, $R_T = 6$, the initial proportion of three types of agents $Prod = 0.3$, $ProT = 0.3$ and $ProU = 0.4$, respectively.

Fig. 8. The average number of final agents ($K_I$, $K_T$ and $K_U$) and global wealth ($GW$) as a function of reputation threshold $Rc$ for different values of the degree of rationality $\alpha$ on a SF network. For each of $\alpha$, the threshold $Rc$ is changed over the range of 0 to 4.0 by using a step size 0.1. As $Rc$ increases when $\alpha$ is smaller than 0.5, $K_I$, $K_T$ and $GW$ increase gradually. What’s more, when $\alpha$ is equal to 0.3 and 0.4, the dynamic of curves change more dramatically. Other parameters are set to be $R_{IT} = 6$, $r_{IT} = 0.7$, the initial proportion of three types of agents $Prod = 0.3$, $ProT = 0.3$ and $ProU = 0.4$, respectively.
K\frac{1}{4} can be easily observed only when the value of ProU and the degree ProI of the truster is large enough. The simulation results, being exciting and significant, show that making the truster have the opportunity to consider the reputation of the trustee, and trust the trustee ratio-

IV. CONCLUSION

In this paper, we propose a networked N-player trust game model with a dynamic and adaptive individual reputation update based on the third-party evaluation system to investigate the dynamic evolution of trust and trustworthiness under a more realistic context. We introduce the degree of rationality \( \alpha \) to describe whether or how rational the truster is. Specifically, for a group of agents in the trust game, the rational investor chooses whether to trust the trustee or not according to their reputation with the probability of \( \alpha \) (only the trustee is allowed to have the priority to evaluate the trustee’s reputation), otherwise trust the trustee irrationally and unconditionally with the complementary probability \( 1 - \alpha \). The simulation results, being exciting and significant, show that making the truster have the opportunity to consider the reputation of the trustee, and trust the trustee rationally can effectively reduce the existence of untrustworthy agents, thus promote the trust level in the structured population.

If we only focus on the influence of degree of rationality \( \alpha \) on the model, we can see that the behavior of trust and trustworthiness as well as global wealth in the social network are significantly improved with the increase of \( \alpha \). Even though the social dilemma is extremely hard (e.g., the ratio of temptation to defect \( r_{UT} > 0.66 \)), the agent in the social network can still reach the state of full trust or trustworthiness only if the degree of rationality \( \alpha \) of the truster is large enough.

We also explore the role of reputation threshold \( R_e \) in the networked trust game. When the degree of rationality \( \alpha = 0 \) (i.e., the traditional case in [40]), \( R_e \) does not affect the model evolution because the dynamic and adaptive individual reputation mechanism does not work under this case. Inspiringly, when \( \alpha \) is approximately between 0 and 0.5 (0 < \( \alpha \) < 0.5), at the same time other parameters are set to be \( R_T = 6 \), \( r_{UT} = 0.7 \), the initial proportion of three types of agents ProI = 0.3, ProT = 0.3 and ProU = 0.4, respectively.

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