Design of Resilient Reliable Dissipativity Control for Systems With Actuator Faults and Probabilistic Time-Delay Signals via Sampled-Data Approach

R. Manivannan⬡, R. Samidurai, Jinde Cao⬡, *Fellow, IEEE*, and Matjaž Perc⬡

Abstract—The issue of resilient reliable dissipativity performance index for systems including actuator faults and probabilistic time-delay signals via sampled-data control approach is investigated. Specifically, random variables governed by the Bernoulli distribution are examined in detail for the random time-delay signals. By using the Lyapunov–Krasovskii functionals together with the Wirtinger double integral inequality approach and reciprocally convex combination technique, which reflects complete information on the certain random sampling; as a result, a new set of sufficient criterion is launched to ensure that the proposed closed-loop system is strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$-dissipative. The proposed criterion for dissipativity-based resilient reliable controller is expressed in the form of linear matrix inequalities. The major contributions of this paper is $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$-dissipativity concept can be adopted to analyze more dynamical performances simultaneously, such as $\mathcal{H}_\infty$, passivity, mixed $\mathcal{H}_\infty$, and passivity performance for the proposed system model by choosing the weighting matrices $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$. Finally, an interesting simulation example is demonstrated to showing the applicability and effectiveness of the theoretical results together with proposed control law by taking the experimental values of the high-incidence research model and rotary servo system.

Index Terms—Actuator faults, dissipativity, high-incidence research model (HIRM), probabilistic time delays, reciprocally convex approach, resilient reliable control, Wirtinger integral inequality.

R. Manivannan is with the Department of Mathematics, Faculty of Natural Sciences, National Institute of Technology Calicut, Kozhikode 673601, Kerala, India, and also with the Advanced Research Center of Electronics and Information, Division of Electronic Engineering, Chonbuk National University, Jeonju-Si 54896, South Korea (e-mail: manimath7@gmail.com).

R. Samidurai is with the Department of Mathematics, Thiruvalluvar University, Vellore 632115, India (e-mail: samidurair@gmail.com).

J. Cao is with the School of Mathematics, Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210096, China, and also with the School of Mathematics and Statistics, Shandong Normal University, Ji'nan 250014, China, also with the School of Electrical Engineering, Nantong University, Nantong 226019, China (e-mail: jdcao@seu.edu.cn).

M. Perc is with the Faculty of Natural Sciences and Mathematics, University of Maribor, 2000 Maribor, Slovenia (e-mail: matjaz.perc@uni-mb.si).

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I. INTRODUCTION

RECENT years, many modern practical systems rely on advanced control techniques to hike their good safety and performance (see [1], [2], [12], [14], [15], [19]–[21], [33], [39]–[42]). In the circumstance of system quality failings, the usual control techniques may result in insufficient performances or even instability or some other poor performances, specifically for complex safety critical systems, e.g., spacecraft, aircraft, nuclear power plant, and so on. This has exploded immense research accomplishments in the recent inspection of novel control scheme approaches, for accepting the component repairs and keeping the agreeable system safety, such as stability and excellent performances, so that sudden changes and complete system failings can be avoided. Therefore, in order to secure the comfort and safety of system performance, different control approaches have been designed in the recent investigation, such as robust control [1], adaptive control [12], sampled-data control [14], impulsive control [15]–[17], nonfragile control [33], state feedback control [42]. Howbeit, the above-mentioned control techniques are achieved based on the supposition that the system is well set up together with actuators, which is a usual supposition. In many practical situations, the system structure cannot be uniform, as well as the proposed system model, turn into poor performance. In order to enhance system safety and reliability, it is necessary to construct a reliable controller; as a result the performance and stability of the proposed system model can operate well together with some actuator faults [3]. For this purpose, recently the issue of reliable control design has admitted tremendous research attention from the researchers and many interesting results have been available on this issues (see [4]–[6]).

Generally, in many practical systems, time delay usually occur in a probabilistic mode, that means, a few values of the delay signals are very abundant but the random delays catching such abundant values are very small and it gives a more conservative result (see [7]). Furthermore, its probabilistic distinguishing, such as the Poisson distribution and the Bernoulli distribution [8], which can be frequently achieved by some statistical procedures. Owe to these real facts, many researchers have been incorporated the idea of random or probabilistic delay effects in the dynamical system model (see [9], [10]). Besides, data-driven control techniques are generally concentrated on modeling a control parameters which is entirely formulated according to their characteristic evaluation

of a plant and it is independent of the natural behavior of the plant [11]. In the view of all these techniques do not demand the design of a plant in control parameter, the proposed techniques, the unmodeled system dynamics and the theoretical suppositions on the dynamic behaviors of the plant is disappeared. For this reason, different data-driven control techniques have been currently launched on this research issues but the researchers named them in various, like data-driven control, model-free adaptive control, data-based control, iterative learning control, and so on, for more details on this issues (see [12], [13]). However, owing to the digitalized modern technology in control theory, the sampled-data control strategy has been gathering rich interest from the researchers together with continuous time-delay signals by suggesting a sawtooth function structure for the time-delay (see [14]). In recent years, the sampled-data control scheme together with input time-delay signal has been well adapted to many glorious dynamical models (see [17], [22], [42]). Thus, we should also paid more interest to the importance of the sampled-data scheme and probability-distribution delay when the real-world control problem is designed.

Likewise, in many practical systems, uncertainties are mostly cannot be avoided in controller utilization. Due to this fact, the closed-loop system becomes instability or other poor performances. Therefore, it is necessary to consider a controller which should permit the uncertainties in its parameters. In addition, in many physical systems, the errors may occur in gains exist in an arbitrary form owing to the distribution of an network interaction. Thus, it is mentioned that in many practical applications, when the nonfragility cannot be assured, then the applications of the controllers leads to much costly due to the demand of introducing more parameters in the control designing (see [33]). Therefore, nowadays this leads an advanced and challenging research issue on this topic: how to design a control parameter for the proposed system model is unresponsive to some bulk of error subject to its gain (resilient controller gain). To overcome this difficulty, resilient (or nonfragile) controller has been proposed for diverse kinds of control systems, and some important results can be found in (see [29]–[33] and references therein). Among them, some sufficient criteria for the issue of nonfragile control of parametric-based uncertain system model was extensively investigated in [29]. Recently, Song and Niu [30] have focused on the issue of resilient finite-time boundedness for Takagi–Sugeno (T–S) fuzzy stochastic system model including randomly occurring gain fluctuations and uncertainties. More recently, the dissipativity-based resilient sampled-data control scheme for stochastic system model including actuator failures have been explored in [31]. Currently, Sakthivel *et al.* [32] established new results on finite-time passivity-based $\mathcal{H}_\infty$ control for T–S fuzzy systems subject to gain variations and randomly occurring uncertainties via resilient reliable state feedback sampled-data scheme. Furthermore, the issue of nonfragile $\mathcal{H}_\infty$ fuzzy-based control scheme for T–S fuzzy models including external disturbance, uncertainties, and unmeasurable state variables have been launched in [33].

Recently, the dissipativity theory has received more research interest in synthesis and analysis of system behaviors in both theory and practical implementation. Particularly, in the physical systems, dissipativity issue is deeply associated with the concept of energy dissipation. That means this theory has provides an elementary structure for the control-based issues on designing in the study of linear and nonlinear system models via an input–output representation through the energy-related characterization. Generally, the dissipative system is distinguished by the amount of energy which the system can possibly supply to its environment cannot exceed the amount of energy that has been supplied to it (see [23]). Since 1972, Willems introduced this concept (see [24]), it has been received outstanding research attention from the researchers due to its wide range of practical realizations (see [25], [27], [47]). For this purpose, in modern control engineering environment, the consideration of dissipativity control has validated to be indispensable and cordial informative tool for control engineering realizations, such as robotics, circuit theory, damping, combustion engines, and electromechanical system modelings. Moreover, the dissipativity theory suggests a new designing performance index through the flexible parameters $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$ compared with other performance indexes, such as passivity and $\mathcal{H}_\infty$ control. For this reason, numerous take an interest of the linear matrix inequality (LMI) framework analysis together with the dissipativity-based investigation of time-delay systems in recent years (see [34]–[41]). For instance, to contribute a more interesting information on realistic NN models, utilizing some new approaches and the time-delayed combination has been taken into consideration to displaying usefulness of a real-world application problem, which was extensively investigated in [34] and [35]. Furthermore, recently Manivannan *et al.* [36], [37], in order to enhance the practical significance, some new methodology was developed for the study of NN model to showing practical application on a biological network model. Recently, by using the lifting technique and LMI approach to investigate the stability and dissipativity of stochastic system model to describe the control and learning actions of the repetitive controller were proposed in [38].

Howbeit, until now, the design of resilient reliable dissipativity-based control design for the system subject to actuator faults and probabilistic time delayed signal via sampled-data control scheme has not yet been explored. Motivated by this observation, in this paper, a new set of sufficient criterion is explored for systems including actuator faults and probabilistic time-delay signal is obtained in terms of LMIs via dissipativity-based resilient reliable sampled-data controller. The highlights of this paper lie in the subsequent standpoints.

1) The issue of $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$-dissipativity-based resilient reliable sampled-data control scheme for systems including actuator faults and probabilistic time-delay signal has been launched in the system model.
2) This is the first effort to investigate the issue of dissipativity-based resilient reliable sampled-data feedback controller design together with actuator faults and probabilistic time-delay signals for the proposed system model.

TABLE I
NOTATIONS AND DESCRIPTIONS

Notation	Description
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{p \times q}$	set of $p \times q$ matrices
$\mathcal{I}$	identity matrix
$\mathcal{X}^{-1}$	inverse of matrix $\mathcal{X}$
$\mathcal{X}^T$	transpose of matrix $\mathcal{X}$
$\mathcal{X} > 0$	$\mathcal{X}$ is a symmetric and positive definite matrix
$\mathcal{X} \geq 0$	$\mathcal{X}$ is a symmetric and positive semi-definite matrix
$\| \cdot \|$	Euclidean norm in $\mathbb{R}^n$
$\maltese$	symmetric term in symmetric matrix
$Pr\{\alpha\}$	the occurrence probability of an event α
diag$(\dots)$	signifies a block diagonal matrix
$\mathcal{E}\{\cdot\}$	mathematical expectation

Fig. 1. Block diagram of controller design procedure.

3) A more generalized resilient reliable sampled-data controller is designed for the solvability of proposed system model including actuator faults and probabilistic time-delay signals.

4) By the implementation of Lyapunov stability theory together with the Wirtinger double integral inequality (WDII) technique, a novel set of sufficient criterion is derived in the form of LMIs to sure that the proposed system is strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$-dissipative.

The reminder of this paper is designed as follows. The system model is addressed and some necessary preliminaries are designed in Section II. The new criteria for dissipativity-based resilient reliable sampled-data controller is proposed in Section III. In Section IV, in order to show the applicability, we dedicate a numerical simulation example. Finally, some conclusions are made in Section V. For clarity, we provide notations and descriptions in Table I.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this section, we propose the following system model with control input and noise disturbance:

$$\dot{\delta}(t) = \mathcal{A}\delta(t) + \mathcal{B}_u \overline{u}^F(t) + \mathcal{C}_\omega \omega(t) \tag{1}$$

where $\delta(t)$ is the state vector; $\overline{u}^F(t)$ is the control input; $\omega(t)$ is the external noise input vector belonging to $\mathcal{L}_2[0, \infty)$; and $\mathcal{A}$, $\mathcal{B}_u$, and $\mathcal{C}_\omega$ are known constants matrices with compatible dimensions.

Remark 1: It is mentioned that, the control signals may experience failings in many practical situations, in such a situations the ordinary state feedback control design will not be applicable in modern society of science and engineering applications. Therefore, it is of great importance to designing a proper control that can permit such failings in practise. This is our main purpose and focusing of this paper. During the control signal (actuators) experience failings, we use $\overline{u}^F(t)$ to characterize the control signal in the system model. Moreover, a block diagram of the controller design procedure can be shown in Fig. 1.

Therefore, by using this fact, in this paper, we prefer the actuator fault model which takes the subsequent structure

$$\overline{u}^F(t) = \mathcal{G}\overline{u}(t) \tag{2}$$

where $\mathcal{G}$ is the actuator fault matrix with $\mathcal{G} = $ diag$\{g_1, g_2, \dots, g_p\}$, $0 \leq \underline{g}_m \leq g_m \leq \overline{g}_m$, $\overline{g}_m \leq 1$ ($m = $
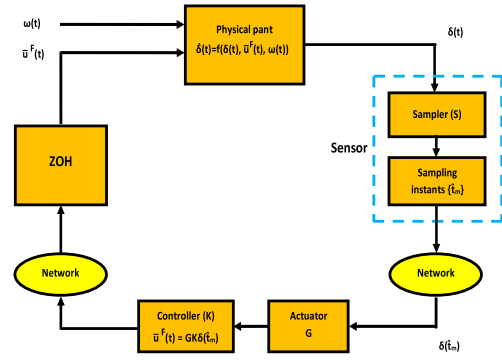
$1, 2, \dots, p$), $\underline{g}_m$ and $\overline{g}_m$ are some given scalars. It is mentioned that when $g_m = 0$, the mth actuator is completely fails, $g_m = 1$ denotes that mth actuator is usual. If $0 < g_m < 1$, the mth actuator has partial failure. Defining fault matrices $\mathcal{G}_0 = $ diag$\{g_{10}, g_{20}, \dots, g_{p0}\}$, $\mathcal{G}_1 = $ diag$\{g_{11}, g_{21}, \dots, g_{p1}\}$, where $g_{m0} = [(\overline{g}_m + \underline{g}_m)/2]$ and $g_{m1} = [(\overline{g}_m - \underline{g}_m)/2]$. Hence, the matrix $\mathcal{G}$ takes the form as $\mathcal{G} = \mathcal{G}_0 + \mathcal{G}_1 \Delta \mathcal{J}$, where $\Delta \mathcal{J} = $ diag$\{j_1, j_2, \dots, j_p\}$, $-1 \leq j_m \leq 1$.

Now, by adopting the actuator fault model (2) into the dynamic model (1), we have the following:

$$\dot{\delta}(t) = \mathcal{A}\delta(t) + \mathcal{B}_u \mathcal{G}\overline{u}(t) + \mathcal{C}_\omega \omega(t). \tag{3}$$

In this paper, the controller $\overline{u}(t)$ is sampling earlier arriving into the networks together with the sampled procedures and zero-order-hold circuit. Additionally, $\widehat{t}_m$ ($m = 0, 1, \dots$) are the sampled instants fulfilling $0 \leq \widehat{t}_0 < \widehat{t}_1 < \cdots < \widehat{t}_m < \dots$ and $\lim_{t \longrightarrow \infty} \widehat{t}_m = \infty$, for clarity, suppose that $\widehat{t}_{m+1} - \widehat{t}_m \leq \eta_1$, where $\eta_1 > 0$ is the sampled range, now the controller can be formed as

$$\overline{u}(t) = \overline{u}(\widehat{t}_m) = \mathcal{K}\delta(\widehat{t}_m), \quad \widehat{t}_m \leq t < \widehat{t}_{m+1} \tag{4}$$

where $\mathcal{K}$ is the state feedback gain matrix.

Remark 2: It is noteworthy that, the time delayed signals in control input usually experience in realistic control models, as a result it becomes oscillation, instability and some other poor performance of the dynamical model. Therefore, $\overline{u}^F(t)$ is damaged by the networked intercommunication, the intercommunication influenced the delay effects in the measurement and control signals are transferred through a wide-area network. Thus, both sampling of the communication signal and the probabilistic time-delay signals $h(t)$ are considered in this paper when designing the proposed system model.

Assume $h(t) = t - t_k$ and utilizing (4), the dynamical system (3) becomes

$$\dot{\delta}(t) = \mathcal{A}\delta(t) + \mathcal{B}_u \mathcal{G}\mathcal{K}\delta(t - h(t)) + \mathcal{C}_\omega \omega(t) \tag{5}$$

where $0 \leq h(t) \leq \eta_1$.

By incorporating the probabilistic time delays in the system model, we adopt the suppositions in [7]. Therefore, in order to discuss better, in this paper, the subsequent suppositions are made on the system model (5).

Assumption (A1): The probability distribution can be observed, i.e., the functions and two sets are described as

$\mathbb{C}_1 = \{t : h(t) \in [0, \eta_0]\}$, $\mathbb{C}_2 = \{t : h(t) \in [\eta_0, \eta_1]\}$ and

$$h_1(t) = \begin{cases} h(t), & \text{for } t \in \mathbb{C}_1, \\ 0, & \text{else} \end{cases}, \quad h_2(t) = \begin{cases} h(t), & \text{for } t \in \mathbb{C}_2 \\ \eta_0, & \text{else} \end{cases}$$

where $\eta_0 \in [0, \eta_1]$. It is not difficult to verify that $t \in \mathbb{C}_1$, which denotes that the event $h(t) \in [0, \eta_0]$ exist and $t \in \mathbb{C}_2$ denotes that the event $h(t) \in (\eta_0, \eta_1]$ exists. Thus, a stochastic term $\theta(t)$ can be described as

$$\theta(t) = \begin{cases} 1, & \text{for } t \in \mathbb{C}_1 \\ 0, & \text{for } t \in \mathbb{C}_2. \end{cases}$$

Assumption (A2): $\theta(t)$ is a Bernoulli distributed sequence with $\mathcal{P}_r\{\theta(t) = 1\} = \mathcal{P}_r\{h(t) \in [0, \eta_0]\} = \mathcal{E}\{\theta(t)\} = \theta_0$ and $\mathcal{P}_r\{\theta(t) = 0\} = \mathcal{P}_r\{h(t) \in (\eta_0, \eta_1]\} = 1 - \mathcal{E}\{\theta(t)\} = 1 - \theta_0$, where $0 \leq \theta_0 \leq 1$ is a scalar and $\mathcal{E}\{\theta(t)\}$ is the mathematical expectation of $\theta(t)$. Also, we have $\mathcal{E}\{\theta(t) - \theta_0\} = 0$, $\mathcal{E}\{(\theta(t) - \theta_0)^2\} = \theta_0(1 - \theta_0)$.

Then, by taking the probabilistic time-delay signals into the system model (5), it comes as

$$\dot{\delta}(t) = \mathcal{A}\delta(t) + \theta(t)\mathcal{B}_u\mathcal{GK}\delta(t - h_1(t)) \\ + (1 - \theta(t))\mathcal{B}_u\mathcal{GK}\delta(t - h_2(t)) + \mathcal{C}_\omega\omega(t) \quad (6)$$

or equivalently in other form

$$\dot{\delta}(t) = \mathcal{A}\delta(t) + \theta_0\mathcal{B}_u\mathcal{GK}\delta(t - h_1(t)) \\ + (1 - \theta_0)\mathcal{B}_u\mathcal{GK}\delta(t - h_2(t)) + \mathcal{C}_\omega\omega(t) \\ + (\theta(t) - \theta_0)\mathcal{B}_u\mathcal{GK}[\delta(t - h_1(t)) - \eta(t - h_2(t))]. \quad (7)$$

Remark 3: It is noted that in the dynamical model (7), there are four factors that complicate the design of resilient reliable controller, actuator failure, external disturbance, and probabilistic time-varying delay parameters. In particular, the random variable $\theta(t)$ is incorporated to realize the issue of the randomly occurring time-delay signal and it is more comfortable for giving back parameter fluctuation of a random nature, especially in the networked intercommunication environment. Moreover, for the network-based control environment, the usual values of the component parameters in control devices may experience randomly sudden variations because of probabilistic time delays in the networked environment with impact on the controller parameter utilization. To overcome these issues, we offer the random variable $\theta(t)$ in the system model (7), which reflects the resilient reliable sampled-data performance via dissipativity theory.

Before completing this section, we introduce the subsequent definition and lemmas which are needed in the proof of our main results.

Definition 1: System (7) is said to be strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipative, if for any $t > 0$ and some scalar $\alpha > 0$ together with zero initial state, the subsequent supposition is holds

$$\mathcal{E}[\langle\delta, \mathbb{Q}_\delta\rangle_t + 2\langle\delta, \mathbb{S}_\omega\rangle_t + \langle\omega, \mathbb{R}_\omega\rangle_t] \geq \alpha\mathcal{E}[\langle\omega, \omega\rangle_t] \quad (8)$$

where $\mathbb{Q} \leq 0$, $\mathbb{S}$ and $\mathbb{R}$ are real matrices of appropriate dimensions with $\mathbb{Q}$ and $\mathbb{R}$ symmetric.

Lemma 1 [48]: For any matrix $\mathbb{M} \in \mathbb{R}^{n \times n}$ and $\mathbb{M} = \mathbb{M}^T > 0$ and given scalars $\gamma > 0$, the vector function is $\chi:[0, \gamma] \rightarrow$

$\mathbb{R}^n$ such that the subsequent relation satisfied

$$-\gamma \int_0^\gamma \chi^T(s)\mathbb{M}\chi(s)ds \leq -\left[\int_0^\gamma \chi(s)ds\right]^T \mathbb{M}\left[\int_0^\gamma \chi(s)ds\right].$$

Lemma 2 [49]: Let $a_1, a_2, \ldots, a_N:\mathbb{R}^m \longmapsto \mathbb{R}$, which has positive values and belongs to an open subset $\mathbb{D}$ of $\mathbb{R}^m$. Then, the reciprocally convex combination (RCC) of a_i over $\mathbb{D}$ satisfies

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} a_i(t) = \sum_i a_i(t) + \max_{b_{i,j}(t)} \sum_{i \neq j} b_{i,j}(t)$$

subject to

$$\left\{b_{i,j} : \mathbb{R}^m \longmapsto \mathbb{R}, b_{j,i}(t) \triangleq b_{i,j}(t), \begin{bmatrix} a_i(t) & b_{i,j}(t) \\ b_{j,i}(t) & a_j(t) \end{bmatrix} \geq 0\right\}.$$

Lemma 3 [50]: Let $\mathbb{M} > 0$ be any scalar matrix, and for any constants m and n with $m < n$, the subsequent inequality is holds for any differentiable function ω in $[m, n] \rightarrow \mathbb{R}^n$

$$\frac{(n - m)^2}{2} \int_m^n \int_\theta^n \dot{\omega}^T(s)\mathbb{M}\dot{\omega}(s)dsd\theta$$

$$\geq \left(\int_m^n \int_\theta^n \dot{\omega}(s)dsd\theta\right)^T \mathbb{M}\left(\int_m^n \int_\theta^n \dot{\omega}(s)dsd\theta\right)$$

$$+ 2\Theta_\vartheta^T \mathbb{M}\Theta_\vartheta$$

where

$$\Theta_\vartheta = -\int_m^n \int_\theta^n \dot{\omega}(s)dsd\theta + \frac{3}{n - m} \int_m^n \int_\theta^n \int_v^n \dot{\omega}(v)dvdsd\theta.$$

III. MAIN RESULTS

This segment is devoted to showing a novel criterion for system (7) is dissipative via resilient reliable sampled-data control based on the suppositions developed by recently available integral inequality.

Theorem 1: Under the Assumptions (A1) and (A2), for given positive scalars $0 < \eta_0 < \eta_1$, $\theta_0 > 0$ and matrices $\mathbb{Q}, \mathbb{S}, \mathbb{R}$ with $(\mathbb{Q} \leq 0)$ and $\mathbb{R}$ is real symmetric and the gain matrix $\mathcal{K}$, then the system (7) is stabilized to the equilibrium point and strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipative, if there exist positive definite symmetric matrices $\mathcal{P} > 0$, $\mathcal{Q}_i > 0$ $(i = 1, 2)$, $\mathcal{R}_i > 0$ $(i = 1, 2)$, $\mathcal{S}_i > 0$ $(i = 1, 2)$ and real matrices $\mathcal{M}_i$ $(i = 1, 2)$ of compatible dimensions such that the subsequent LMIs hold

$$\Psi = \begin{bmatrix} [\Psi_{ij}]_{10 \times 10} & \Pi_1 & \Pi_2 \\ \maltese & -\Lambda & 0 \\ \maltese & \maltese & -\theta_0(1 - \theta_0)\Lambda \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} \mathcal{R}_1 & \mathcal{M}_1 \\ \maltese & \mathcal{R}_1 \end{bmatrix} > 0, \quad \begin{bmatrix} \mathcal{R}_2 & \mathcal{M}_2 \\ \maltese & \mathcal{R}_2 \end{bmatrix} > 0 \quad (10)$$

where

$$\Psi_{11} = 2\mathcal{P}\mathcal{A} + \mathcal{Q}_1 - \mathcal{R}_1 - \frac{3}{2}\eta_0^2\mathcal{S}_1 - \frac{3}{2}\eta_1^2\mathcal{S}_2 - \mathbb{Q}$$

$$\Psi_{12} = \theta_0\mathcal{P}\mathcal{B}_u\mathcal{GK} + \mathcal{R}_1 - \mathcal{M}_1, \quad \Psi_{13} = (1 - \theta_0)\mathcal{P}\mathcal{B}_u\mathcal{GK}$$

$$\Psi_{14} = \mathcal{M}_1, \quad \Psi_{18} = 3\mathcal{S}_1, \quad \Psi_{19} = 3\mathcal{S}_2$$

$$\Psi_{1_{10}} = \mathcal{P}\mathcal{C}_\omega - \mathbb{S}, \quad \Psi_{22} = -2\mathcal{R}_1 + \mathcal{M}_1 + \mathcal{M}_1^T$$
$$\Psi_{24} = -\mathcal{M}_1 + \mathcal{R}_1, \quad \Psi_{33} = -2\mathcal{R}_2 + \mathcal{M}_2 + \mathcal{M}_2^T$$
$$\Psi_{34} = \mathcal{R}_2^T - \mathcal{M}_2^T, \quad \Psi_{35} = -\mathcal{M}_2 + \mathcal{R}_2$$
$$\Psi_{44} = -\mathcal{Q}_1 + \mathcal{Q}_2 - \mathcal{R}_1 - \mathcal{R}_2, \quad \Psi_{45} = -\mathcal{M}_2$$
$$\Psi_{55} = -\mathcal{Q}_2 - \mathcal{R}_2, \quad \Psi_{66} = -3\mathcal{S}_1, \quad \Psi_{68} = \frac{6}{\delta_0}\mathcal{S}_1$$
$$\Psi_{77} = -3\mathcal{S}_1, \quad \Psi_{79} = \frac{6}{\delta_1}\mathcal{S}_1, \quad \Psi_{88} = -\frac{18}{\eta_0^2}\mathcal{S}_1$$
$$\Psi_{99} = -\frac{18}{\eta_1^2}\mathcal{S}_2, \quad \Psi_{10_{10}} = -(\mathbb{R} - \alpha\mathcal{I})$$
$$\Pi_1 = \begin{bmatrix} \mathcal{A}\Lambda & \theta_0\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K} & (1-\theta_0)\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K}\Lambda & \underbrace{0 \dots 0}_{6 \text{ times}} & \mathcal{C}_\omega\Lambda \end{bmatrix}^T$$
$$\Pi_2 = \begin{bmatrix} 0 & \theta_0(1-\theta_0)\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K}\Lambda \\ & -\theta_0(1-\theta_0)\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K}\Lambda & \underbrace{0 \dots 0}_{7 \text{ times}} \end{bmatrix}^T.$$

Proof: We construct the subsequent Lyapunov–Krasovskii functional (LKF) for the system (7), as follows:

$$\mathbb{V}(\delta(t)) = \sum_{p=1}^{4} \mathbb{V}_p(\delta(t)) \tag{11}$$

where

$$\mathbb{V}_1(\delta(t)) = \delta^T(t)\mathcal{P}\delta(t)$$
$$\mathbb{V}_2(\delta(t)) = \int_{t-\eta_0}^{t} \mathcal{G}(s, \mathcal{Q}_1)ds + \int_{t-\eta_1}^{t-\eta_0} \mathcal{G}(s, \mathcal{Q}_2)ds$$
$$\mathbb{V}_3(\delta(t)) = \eta_0 \int_{-\eta_0}^{0} \int_{t+\beta}^{t} \mathcal{H}(s, \mathcal{R}_1)dsd\beta$$
$$+ (\eta_1 - \eta_0) \int_{-\eta_1}^{-\eta_0} \int_{t+\beta}^{t} \mathcal{H}(s, \mathcal{R}_2)dsd\beta$$
$$\mathbb{V}_4(\delta(t)) = \frac{\eta_0^2}{2} \int_{-\eta_0}^{0} \int_{\theta}^{0} \int_{t+\beta}^{t} \mathcal{H}(s, \mathcal{S}_1)dsd\beta d\theta$$
$$+ \frac{\eta_1^2}{2} \int_{-\eta_1}^{0} \int_{\theta}^{0} \int_{t+\beta}^{t} \mathcal{H}(s, \mathcal{S}_2)dsd\beta d\theta$$

with $\mathcal{G}(s, \mathcal{Q}) = \delta^T(s)\mathcal{Q}\delta(s)$, $\mathcal{H}(s, \mathcal{R}) = \dot{\delta}^T(s)\mathcal{R}\dot{\delta}(s)$, $\mathcal{H}(s, \mathcal{S}) = \dot{\delta}^T(s)\mathcal{S}\dot{\delta}(s)$.

Define the infinitesimal operator $\mathcal{L}$ of $\mathbb{V}(\delta(t))$ as follows:

$$\mathcal{L}\mathbb{V}(\delta(t)) = \lim_{h \longrightarrow 0^+} \frac{1}{h} \left\{ \mathbb{E}\{\mathbb{V}(\delta(t+h) \mid_{\delta(t)}\} - \mathbb{V}(\delta(t)) \right\}. \tag{12}$$

Calculating the derivatives of $\mathbb{V}_p(\delta(t))$ $(p = 1, 2, 3, 4)$ for the system (9), we obtain

$$\mathcal{L}\mathbb{V}(\delta(t)) \le \sum_{p=1}^{4} \mathbb{V}_p(\delta(t)) \tag{13}$$

where

$$\mathcal{L}\mathbb{V}_1(\delta(t)) = 2\delta^T(t)\mathcal{P}\big[\mathcal{A}\delta(t) + \theta_0\mathcal{B}_u\mathcal{G}\mathcal{K}\delta(t - h_1(t))$$
$$+ (1-\theta_0)\mathcal{B}_u\mathcal{G}\mathcal{K}\delta(t - h_2(t)) + \mathcal{C}_\omega\omega(t)$$
$$+ (\theta(t) - \theta_0)\mathcal{B}_u\mathcal{G}\mathcal{K}$$
$$\times [\delta(t - h_1(t)) - \delta(t - h_2(t))]\big] \tag{14}$$
$$\mathcal{L}\mathbb{V}_2(\delta(t)) = \delta^T(t)\mathcal{Q}_1\delta(t) + \delta^T(t - \eta_0)\big[-\mathcal{Q}_1 + \mathcal{Q}_2\big]$$
$$\times \delta(t - \eta_0) - \delta^T(t - \eta_1)\mathcal{Q}_2\delta(t - \eta_1) \tag{15}$$
$$\mathcal{L}\mathbb{V}_3(\delta(t)) = \dot{\delta}^T(t)\big[\eta_0^2\mathcal{R}_1 + (\eta_1 - \eta_0)^2\mathcal{R}_2\big]\dot{\delta}(t)$$
$$+ \mathfrak{R}_1(t) + \mathfrak{R}_2(t) \tag{16}$$

where

$$\mathfrak{R}_1(t) = -\eta_0 \int_{t-\eta_0}^{t} \dot{\delta}^T(s)\mathcal{R}_1\dot{\delta}(s)ds$$
$$\mathfrak{R}_2(t) = -(\eta_1 - \eta_0) \int_{t-\eta_1}^{t-\eta_0} \dot{\delta}^T(s)\mathcal{R}_2\dot{\delta}(s)ds.$$

To facilitate new upper bounds for the integral terms $\mathfrak{R}_1(t)$ and $\mathfrak{R}_2(t)$, we apply the Lemmas 1 and 2, we have

$$\mathfrak{R}_1(t) \le -\begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix}^T \begin{bmatrix} \mathcal{R}_1 & \mathcal{M}_1 \\ \maltese & \mathcal{R}_1 \end{bmatrix} \begin{bmatrix} \Upsilon_1 \\ \Upsilon_2 \end{bmatrix} \tag{17}$$
$$\mathfrak{R}_2(t) \le -\begin{bmatrix} \Upsilon_3 \\ \Upsilon_4 \end{bmatrix}^T \begin{bmatrix} \mathcal{R}_2 & \mathcal{M}_2 \\ \maltese & \mathcal{R}_2 \end{bmatrix} \begin{bmatrix} \Upsilon_3 \\ \Upsilon_4 \end{bmatrix} \tag{18}$$

where

$$\Upsilon_1 = \delta(t) - \delta(t - h_1(t))$$
$$\Upsilon_2 = \delta(t - h_1(t)) - \delta(t - \eta_0)$$
$$\Upsilon_3 = \delta(t - \eta_0) - \delta(t - h_2(t))$$
$$\Upsilon_4 = \delta(t - h_2(t)) - \delta(t - \eta_1)$$
$$\mathcal{L}\mathbb{V}_4(\delta(t)) = \dot{\delta}^T(t) \left\{ \left(\frac{\eta_0^2}{2}\right)^2 S_1 + \left(\frac{\eta_1^2}{2}\right)^2 S_2 \right\} \dot{\delta}(t)$$
$$+ \wp_1(t) + \wp_2(t) \tag{19}$$

where

$$\wp_1(t) = -\frac{\eta_0^2}{2} \int_{-\eta_0}^{0} \int_{t+\beta}^{t} \dot{\delta}^T(s)\mathcal{S}_1\dot{\delta}(s)dsd\beta$$
$$\wp_2(t) = -\frac{\eta_1^2}{2} \int_{-\eta_1}^{0} \int_{t+\beta}^{t} \dot{\delta}^T(s)\mathcal{S}_2\dot{\delta}(s)dsd\beta.$$

To facilitate new upper bounds for the integral terms $\wp_1(t)$ and $\wp_2(t)$, we apply Lemma 3, we have

$$\wp_1(t) \le -\begin{bmatrix} \Upsilon_5 \\ \Upsilon_6 \end{bmatrix}^T \begin{bmatrix} \mathcal{S}_1 & 0 \\ \maltese & 2\mathcal{S}_1 \end{bmatrix} \begin{bmatrix} \Upsilon_5 \\ \Upsilon_6 \end{bmatrix} \tag{20}$$
$$\wp_2(t) \le -\begin{bmatrix} \Upsilon_7 \\ \Upsilon_8 \end{bmatrix}^T \begin{bmatrix} \mathcal{S}_2 & 0 \\ \maltese & 2\mathcal{S}_2 \end{bmatrix} \begin{bmatrix} \Upsilon_7 \\ \Upsilon_8 \end{bmatrix}. \tag{21}$$

where

$$\Upsilon_5 = \eta_0\delta(t) - \int_{t-\eta_0}^{t} \delta(s)ds$$
$$\Upsilon_6 = -\frac{\eta_0}{2}\delta(t) - \int_{t-\eta_0}^{t} \delta(s)ds + \frac{3}{\eta_0} \int_{-\eta_0}^{0} \int_{t+\beta}^{t} \delta(s)dsd\beta$$
$$\Upsilon_7 = \eta_1\delta(t) - \int_{t-\eta_1}^{t} \delta(s)ds$$
$$\Upsilon_8 = -\frac{\eta_1}{2}\delta(t) - \int_{t-\eta_1}^{t} \delta(s)ds + \frac{3}{\eta_1} \int_{-\eta_1}^{0} \int_{t+\beta}^{t} \delta(s)dsd\beta.$$

Let

$$\Lambda = \eta_0^2\mathcal{R}_1 + (\eta_1 - \eta_0)^2\mathcal{R}_2 + \left(\frac{\eta_0^2}{2}\right)^2 \mathcal{S}_1 + \left(\frac{\eta_1^2}{2}\right)^2 \mathcal{S}_2.$$

Then, we have

$$\mathcal{E}\{\dot{\delta}^T(t)\Lambda\dot{\delta}(t)\} = \zeta^T(t)\Xi_1^T\Lambda\Xi_1\zeta(t)$$
$$+ \theta_0(1 - \theta_0)\zeta^T(t)\Xi_2^T\Lambda\Xi_2\zeta(t) \tag{22}$$

where

$$\zeta(t) = \left[\delta^T(t) \quad \delta^T(t - h_1(t)) \quad \delta^T(t - h_2(t)) \quad \delta^T(t - \eta_0)\right.$$

$$\delta^T(t - \eta_1) \quad \int_{t-\eta_0}^{t} \delta^T(s)ds \quad \int_{t-\eta_1}^{t} \delta^T(s)ds$$

$$\left. \int_{-\eta_0}^{0}\int_{t+\beta}^{t} \delta^T(s)dsd\beta \quad \int_{-\eta_1}^{0}\int_{t+\beta}^{t} \delta^T(s)dsd\beta \quad \omega^T(t)\right]^T$$

$$\Xi_1 = \left[\mathcal{A} \quad \theta_0 \mathcal{B}_u \mathcal{G} \mathcal{K} \quad (1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{K} \quad \underbrace{0\ldots0}_{6 \text{ times}} \quad \mathcal{C}_\omega\right]$$

$$\Xi_2 = \left[0 \quad \mathcal{B}_u \mathcal{G} \mathcal{K} \quad -\mathcal{B}_u \mathcal{G} \mathcal{K} \quad \underbrace{0\ldots\ldots\ldots\ldots0}_{7 \text{ times}}\right].$$

Finally, combining from (14)–(22), it is obvious that

$$\mathcal{E}\{\mathcal{L}\mathbb{V}(\delta(t)) - \delta^T(t)\mathbb{Q}\delta(t) - 2\delta^T(t)\mathbb{S}\omega(t)$$
$$- \omega^T(t)(\mathbb{R} - \alpha\mathcal{I})\omega(t)\} < \mathcal{E}\{\zeta^T(t)\Psi\zeta(t)\} < 0. \quad (23)$$

Obviously, when $\omega(t) = 0$, the relation (23) gives that $\mathbb{E}\{\mathcal{L}\mathbb{V}(\delta(t))\} < 0$; thus, system (7) is asymptotically stable. Moreover, our main purpose is to display the dissipativity performance of system (7). To achieve this, we integrating the above relation (23) from 0 to t with initial condition $\mathbb{V}(\delta(t)) = 0$ yields

$$\mathbb{E}\left\{\mathbb{V}(\delta(t)) + \alpha\int_0^t \omega^T(s)\omega(s)ds\right\} < \mathbb{E}\left\{\int_0^t \left[\delta^T(s)\mathbb{Q}\delta(s)\right.\right.$$
$$\left.\left. + 2\delta^T(s)\mathbb{S}\omega(s) + \omega^T(s)\mathbb{R}\omega(s)\right]ds\right\}. \quad (24)$$

Considering $\mathbb{V}(\delta(t)) \geq 0$, we guarantee that the relation in (8) is holds. Thus, the system (7) is strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipative in the sense of Definition 1, and the proof is completed. ∎

Remark 4: It should be mentioned that, the sufficient LMI supposition developed in Theorem 1 is to analysis the stability performance of the system (7) only when the gain matrix $\mathcal{K}$ is known. When the gain matrix $\mathcal{K}$ is a matrix variable to be evaluated, the above supposition is not an LMI therefore we have to dealt with the control designing problem of the dynamical system.

Therefore, based on the Theorem 1 we present the subsequent theorem to design the resilient reliable sampled-data controller gain for the system model (7).

Theorem 2: Under the Assumptions (A1) and (A2), for given positive scalars $0 < \eta_0 < \eta_1$, $\theta_0 > 0$ and matrices $\mathbb{Q}, \mathbb{S}, \mathbb{R}$ with $(\mathbb{Q} \leq 0)$ and $\mathbb{R}$ is real symmetric, then the system (7) is stabilized to the equilibrium point and strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipative via the proposed resilient reliable controller (2), if there exist positive definite symmetric matrices $\mathcal{P} > 0$, $\mathcal{Q}_i > 0$ $(i = 1, 2)$, $\mathcal{R}_i > 0$ $(i = 1, 2)$, $\mathcal{S}_i$, $(i = 1, 2)$ and real matrices $\mathcal{J}, \mathcal{M}_i$ $(i = 1, 2)$ of compatible dimensions such that the subsequent LMI holds

$$\Psi = [\widehat{\Psi}_{i,j}]_{18\times18} < 0 \quad (25)$$

where

$$\widehat{\Psi}_{11} = 2\mathcal{P}\mathcal{A} + \mathcal{Q}_1 - \mathcal{R}_1 - \frac{3}{2}\eta_0^2\mathcal{S}_1 - \frac{3}{2}\eta_1^2\mathcal{S}_2 - \mathbb{Q}$$

$$\widehat{\Psi}_{12} = \theta_0 \mathcal{B}_u \mathcal{G} \mathcal{J} + \mathcal{R}_1 - \mathcal{M}_1, \quad \widehat{\Psi}_{13} = (1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{14} = \mathcal{M}_1, \quad \widehat{\Psi}_{18} = 3\mathcal{S}_1, \quad \widehat{\Psi}_{19} = 3\mathcal{S}_2, \quad \widehat{\Psi}_{110} = \mathcal{P}\mathcal{C}_\omega - \mathbb{S}$$

$$\widehat{\Psi}_{111} = \mathcal{A}^T\mathcal{P}, \quad \widehat{\Psi}_{112} = \mathcal{A}^T\mathcal{P}, \quad \widehat{\Psi}_{113} = \mathcal{A}^T\mathcal{P}$$

$$\widehat{\Psi}_{114} = \mathcal{A}^T\mathcal{P}, \quad \widehat{\Psi}_{22} = -2\mathcal{R}_1 + \mathcal{M}_1 + \mathcal{M}_1^T$$

$$\widehat{\Psi}_{24} = -\mathcal{M}_1 + \mathcal{R}_1, \quad \widehat{\Psi}_{211} = \theta_0 \mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{212} = \theta_0 \mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{213} = \theta_0 \mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{214} = \theta_0 \mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{215} = \theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{216} = \theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{217} = \theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{218} = \theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{33} = -2\mathcal{R}_2 + \mathcal{M}_2 + \mathcal{M}_2^T, \quad \widehat{\Psi}_{34} = \mathcal{R}_2^T - \mathcal{M}_2^T$$

$$\widehat{\Psi}_{35} = -\mathcal{M}_2 + \mathcal{R}_2, \quad \widehat{\Psi}_{311} = (1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{312} = (1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{313} = (1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{314} = (1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{315} = -\theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{316} = -\theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{317} = -\theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}$$

$$\widehat{\Psi}_{318} = -\theta_0(1 - \theta_0)\mathcal{B}_u \mathcal{G} \mathcal{J}, \quad \widehat{\Psi}_{44} = -\mathcal{Q}_1 + \mathcal{Q}_2 - \mathcal{R}_1 - \mathcal{R}_2$$

$$\widehat{\Psi}_{55} = -\mathcal{Q}_2 - \mathcal{R}_2, \quad \widehat{\Psi}_{66} = -3\mathcal{S}_1, \quad \widehat{\Psi}_{68} = \frac{6}{\eta_0}\mathcal{S}_1$$

$$\widehat{\Psi}_{77} = -3\mathcal{S}_2, \quad \widehat{\Psi}_{79} = \frac{6}{\eta_1}\mathcal{S}_2, \quad \widehat{\Psi}_{88} = -\frac{18}{\eta_0^2}\mathcal{S}_1$$

$$\widehat{\Psi}_{99} = -\frac{18}{\eta_1^2}\mathcal{S}_2, \quad \widehat{\Psi}_{1010} = -\mathbb{R}_\alpha, \quad \widehat{\Psi}_{1011} = \mathcal{C}_\omega^T\mathcal{P}$$

$$\widehat{\Psi}_{1012} = \mathcal{C}_\omega^T\mathcal{P}, \quad \widehat{\Psi}_{1013} = \mathcal{C}_\omega^T\mathcal{P}, \quad \widehat{\Psi}_{1014} = \mathcal{C}_\omega^T\mathcal{P}$$

$$\widehat{\Psi}_{1015} = \frac{1}{\eta_0^2}[\mathcal{R}_1 - 2\mathcal{P}], \quad \widehat{\Psi}_{1016} = \frac{1}{(\eta_1 - \eta_0)^2}[\mathcal{R}_2 - 2\mathcal{P}]$$

$$\widehat{\Psi}_{1017} = \frac{4}{\eta_0^2}[\mathcal{S}_1 - 2\mathcal{P}], \quad \widehat{\Psi}_{1018} = \frac{4}{(\eta_1^2 - \eta_0^2)^2}[\mathcal{S}_2 - 2\mathcal{P}]$$

$$\widehat{\Psi}_{1515} = \frac{\theta_0(1 - \theta_0)}{\eta_0^2}[\mathcal{R}_1 - 2\mathcal{P}]$$

$$\widehat{\Psi}_{1616} = \frac{\theta_0(1 - \theta_0)}{(\eta_1^2 - \eta_0^2)^2}[\mathcal{R}_2 - 2\mathcal{P}]$$

$$\widehat{\Psi}_{1717} = \frac{4\theta_0(1 - \theta_0)}{\eta_0^2}[\mathcal{S}_1 - 2\mathcal{P}]$$

$$\widehat{\Psi}_{1818} = \frac{4\theta_0(1 - \theta_0)}{(\eta_1^2 - \eta_0^2)^2}[\mathcal{S}_2 - 2\mathcal{P}]$$

and the remaining terms in Ψ are zero. Let $\mathcal{J} = \mathcal{P}\mathcal{K}$, then we can achieve the reliable control gain matrix $\mathcal{K} = \mathcal{P}^{-1}\mathcal{J}$.

Proof: Let $\mathbb{R}_\alpha = (\mathbb{R} - \alpha\mathcal{I})$. In Theorem 1, considering the first ten rows and the first ten columns of Ψ and express it as $\widehat{\Psi}$, thus the relation (9) can be deduced as

$$\begin{bmatrix} [\widehat{\Psi}_{i,j}]_{10\times10} & \Gamma_1 & \Gamma_2 \\ \maltese & -\Lambda & 0 \\ \maltese & \maltese & -\theta_0(1 - \theta_0)\Lambda \end{bmatrix} < 0 \quad (26)$$

where

$$\Gamma_1 = \left[\Lambda\mathcal{A} \quad \theta_0\Lambda\mathcal{B}_u\mathcal{G}\mathcal{K} \quad (1 - \theta_0)\Lambda\mathcal{B}_u\mathcal{G}\mathcal{K} \quad \underbrace{0\ldots0}_{6 \text{ times}} \quad \Lambda\mathcal{C}_\omega\right]^T$$

and

$$\Gamma_2 = \left[0 \quad \theta_0(1 - \theta_0)\Lambda\mathcal{B}_u\mathcal{G}\mathcal{K} \quad -\theta_0(1 - \theta_0)\Lambda\mathcal{B}_u\mathcal{G}\mathcal{K} \quad \underbrace{0\ldots0}_{7 \text{ times}}\right]^T.$$

Now, we utilizing Schur compliment [51], the above relation (26) is given by

$$
\begin{bmatrix}
[\widehat{\Psi}_{i,j}]_{10\times10} & \widehat{\Gamma}_1 & \widehat{\Gamma}_1 & \widehat{\Gamma}_1 & \widehat{\Gamma}_1 & \widehat{\Gamma}_2 & \widehat{\Gamma}_2 & \widehat{\Gamma}_2 & \widehat{\Gamma}_2 \\
\maltese & \Theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \Theta_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \Theta_3 & 0 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \Theta_4 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \Theta_5 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \Theta_6 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \Theta_7 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \Theta_8
\end{bmatrix} < 0 \quad (27)
$$

where

$$
\Theta_1 = -\frac{\mathcal{R}_1^{-1}}{\eta_0^2}, \quad \Theta_2 = -\frac{\mathcal{R}_2^{-1}}{(\eta_1-\eta_0)^2}, \quad \Theta_3 = -\frac{4\mathcal{S}_1^{-1}}{\eta_0^2}
$$

$$
\Theta_4 = -\frac{4\mathcal{S}_2^{-1}}{(\eta_1^2-\eta_0^2)^2}, \quad \Theta_5 = -\frac{\theta_0(1-\theta_0)\mathcal{R}_1^{-1}}{\eta_0^2}
$$

$$
\Theta_6 = -\frac{\theta_0(1-\theta_0)\mathcal{R}_2^{-1}}{(\eta_1-\eta_0)^2}, \quad \Theta_7 = -\frac{4\theta_0(1-\theta_0)\mathcal{S}_1^{-1}}{\eta_0^2}
$$

$$
\Theta_8 = -\frac{4\theta_0(1-\theta_0)\mathcal{S}_2^{-1}}{(\eta_1^2-\eta_0^2)^2}
$$

$$
\widehat{\Gamma}_1 = \begin{bmatrix} \mathcal{A} & \theta_0\mathcal{B}_u\mathcal{G}\mathcal{K} & (1-\theta_0)\mathcal{B}_u\mathcal{G}\mathcal{K} & \underbrace{0\ldots\ldots0}_{6\text{ times}} & \mathcal{C}_\omega \end{bmatrix}^T \text{ and}
$$

$$
\widehat{\Gamma}_2 = \begin{bmatrix} 0 & \theta_0(1-\theta_0)\mathcal{B}_u\mathcal{G}\mathcal{K} & -\theta_0(1-\theta_0)\mathcal{B}_u\mathcal{G}\mathcal{K} & \underbrace{0\ldots\ldots0}_{7\text{ times}} \end{bmatrix}^T.
$$

Pre- and post-multiply both sides of (27) with $\text{diag}\{\underbrace{\mathcal{I}\ldots\mathcal{I}}_{10\text{ times}}, \underbrace{\mathcal{P}\ldots\mathcal{P}}_{8\text{ times}}\}$ and its transpose, respectively, we get

$$
\begin{bmatrix}
[\widehat{\Psi}_{i,j}]_{10\times10} & \widetilde{\Gamma}_1 & \widetilde{\Gamma}_1 & \widetilde{\Gamma}_1 & \widetilde{\Gamma}_1 & \widetilde{\Gamma}_2 & \widetilde{\Gamma}_2 & \widetilde{\Gamma}_2 & \widetilde{\Gamma}_2 \\
\maltese & \widehat{\Theta}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \widehat{\Theta}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \widehat{\Theta}_3 & 0 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \widehat{\Theta}_4 & 0 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \widehat{\Theta}_5 & 0 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \widehat{\Theta}_6 & 0 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \widehat{\Theta}_7 & 0 \\
\maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \maltese & \widehat{\Theta}_8
\end{bmatrix} < 0
$$

$$(28)$$

where

$$
\widehat{\Theta}_1 = -\frac{1}{\eta_0^2}\mathcal{P}\mathcal{R}_1^{-1}\mathcal{P}, \quad \widehat{\Theta}_2 = -\frac{1}{(\eta_1-\eta_0)^2}\mathcal{P}\mathcal{R}_2^{-1}\mathcal{P}
$$

$$
\widehat{\Theta}_3 = -\frac{4}{\eta_0^2}\mathcal{P}\mathcal{S}_1^{-1}\mathcal{P}, \quad \widehat{\Theta}_4 = -\frac{4}{(\eta_1^2-\eta_0^2)^2}\mathcal{P}\mathcal{S}_2^{-1}\mathcal{P}
$$

$$
\widehat{\Theta}_5 = -\frac{\theta_0(1-\theta_0)}{\eta_0^2}\mathcal{P}\mathcal{R}_1^{-1}\mathcal{P}, \quad \widehat{\Theta}_6 = -\frac{\theta_0(1-\theta_0)}{(\eta_1-\eta_0)^2}\mathcal{P}\mathcal{R}_2^{-1}\mathcal{P}
$$

$$
\widehat{\Theta}_7 = -\frac{4\theta_0(1-\theta_0)}{\eta_0^2}\mathcal{P}\mathcal{S}_1^{-1}\mathcal{P}, \quad \widehat{\Theta}_8 = -\frac{4\theta_0(1-\theta_0)}{(\eta_1^2-\eta_0^2)^2}\mathcal{P}\mathcal{S}_2^{-1}\mathcal{P}
$$

$$
\widetilde{\Gamma}_1 = \begin{bmatrix} \mathcal{P}\mathcal{A} & \theta_0\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K} & (1-\theta_0)\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K} & \underbrace{0\ldots0}_{6\text{ times}} & \mathcal{P}\mathcal{C}_\omega \end{bmatrix}^T \text{ and}
$$

$$
\widetilde{\Gamma}_2 = \begin{bmatrix} 0 & \theta_0(1-\theta_0)\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K} & -\theta_0(1-\theta_0)\mathcal{P}\mathcal{B}_u\mathcal{G}\mathcal{K} & \underbrace{0\ldots0}_{7\text{ times}} \end{bmatrix}^T.
$$

Define $\mathcal{J} = \mathcal{P}\mathcal{K}$ and using $-\mathcal{P}\mathcal{R}_m^{-1}\mathcal{P} < \mathcal{R}_m - 2\mathcal{P}$ and $-\mathcal{P}\mathcal{S}_m^{-1}\mathcal{P} < \mathcal{S}_m - 2\mathcal{P}$ for $m = 1, 2$ in relation (28), we can easily obtain LMI (25). Therefore, the proof is completed. ∎

Remark 5: It is mentioned that, in this paper, a new delay-dependent criterion for $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$-dissipativity of proposed error system (7) including noise distraction is obtained in terms of LMI in Theorem 2. This criterion is obtained by choosing a proper LKF candidate with triple integral terms as defined in (11), and estimating their derivative by the well-known Jensen's inequality, RCC approach and WDII approach. Particularly we mentioned that, the double integral terms in LKF was estimated by the WDII technique, which gives the effective less conservatism. Furthermore, according to the derived criterion, the resilient reliable sampled-data controller is performed.

Remark 6: It is noteworthy that, if we taking $\mathbb{Q} = -\mathcal{I}$, $\mathbb{S} = 0$ and $\mathbb{R}_\alpha = \gamma^2\mathcal{I}$, strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipativity performance turns to the $\mathcal{H}_\infty$ performance, and if letting $\mathbb{Q} = 0$, $\mathbb{S} = \mathcal{I}$, and $\mathbb{R}_\alpha = \gamma\mathcal{I}$, strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipativity performance reduces to the strictly passivity performance. Furthermore, if choosing $\mathbb{Q} = -\vartheta\mathcal{I}$, $\mathbb{S} = (1-\vartheta)\mathcal{I}$, and $\mathbb{R}_\alpha = \gamma^2\vartheta\mathcal{I}$ where $\vartheta \in (0, 1)$, strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipativity performance leads to the mixed $\mathcal{H}_\infty$ and passivity performance.

IV. NUMERICAL EXAMPLES

Example 1: In this section, we offer a numerical simulation study by inspecting an HIRM [44] to expose the usefulness and applicability of the suggested methodology. First we develop the dynamical equations of motion for a general aircraft model, and then the HIRM problem will be established. Consider a general aircraft model expressed by the subsequent dynamical equations of motion

$$
\dot{\mathcal{V}} = \frac{\mathcal{F}_{\omega x}}{m}g\sin\gamma
$$

$$
\dot{\alpha} = q_b - \frac{q_\omega}{\cos\beta} - p_b\cos\alpha\tan\beta - r_b\sin\alpha\tan\beta
$$

$$
\dot{\beta} = r_\omega + p_b\sin\alpha - r_b\cos\alpha
$$

$$
\dot{\gamma} = q_\omega\cos\varphi - \gamma_\omega\sin\varphi
$$

$$
\dot{\varphi} = p_\omega + (q_\omega\sin\varphi + r_\omega\cos\varphi)\tan\gamma
$$

$$
\dot{\psi} = \frac{q_\omega\sin\varphi + r_\omega\cos\varphi}{\cos\gamma}
$$

$$
\dot{q}_b = \frac{1}{\mathcal{I}_y}\left[\mathcal{M}_b + \mathcal{I}_{xy}(r_b^2 - p_b^2) + (\mathcal{I}_z - \mathcal{I}_x)r_bp_b\right]
$$

$$
\begin{bmatrix} \dot{p}_b \\ \dot{r}_b \end{bmatrix} = \begin{bmatrix} \mathcal{I}_x & -\mathcal{I}_{xz} \\ -\mathcal{I}_{xz} & \mathcal{I}_z \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{L}_b + \mathcal{I}_{xz}p_bq_b + (\mathcal{I}_y - \mathcal{I}_z)q_br_b \\ \mathcal{N}_b - \mathcal{I}_{xz}q_br_b + (\mathcal{I}_x - \mathcal{I}_y)p_bq_b \end{bmatrix}
$$

where the above parameters and their descriptions are given in Table II. It is supposed that the transformations among the different axial system models utilized in the above mentioned dynamic equations are general. As discussed in [45], suppose that the aircraft is in a wings-level steady-state flight situation, in this situation, the roll angle becomes 0. If the sideslip angle is trivial and the roll and yaw rates are low, the above-discussed dynamic equations of motion can be modified to an original longitudinal motion. Assume that the flight path velocity is invariable, by taking $\delta = [\alpha \ q_b \ \theta]^T$ as the state

TABLE II
PARAMETERS AND DESCRIPTIONS

Parameter	Description
$\mathcal{V}$	signifies a flight path velocity
$\alpha,\ \beta$	signifies angle of attack and angle of sideslip
$\gamma,\ \varphi,\ \psi$	signifies wind-axis Euler angles
$p_b,\ q_b,\ r_w$	signifies body-axis angular rates
$p_w,\ q_w,\ r_w$	signifies wind-axis angular rates
$g,\ m$	signifies gravity acceleration and mass
$\mathcal{F}_{wx}$	signifies wind-axis total force about x body axis
$\mathcal{I}_x,\ \mathcal{I}_y,\ \mathcal{I}_z$	signifies moments of inertia about z body axes
$\mathcal{L}_b,\ \mathcal{M}_b$	signifies body axis total rolling and pitching
$\mathcal{N}_b$	signifies yawing moments
$\mathcal{I}_{xz}$	signifies cross product of inertia with respect to x and z body axes

Fig. 2. State trajectories and control input of the HIRM based on dissipativity analysis.

signal of the longitudinal motion and by linearizing the above system with $z = 0$, then the subsequent state-space dynamic system model can be proposed

$$\dot{\delta}(t) = \begin{bmatrix} \mathcal{Z}_a & 1 & -g\sin(\frac{\mu_*}{\mathcal{V}_*}) \\ \mathcal{M}_\alpha & \mathcal{M}_q & 0 \\ \mathcal{M}_\alpha & \mathcal{M}_q & 0 \end{bmatrix}\delta(t) + \begin{bmatrix} \mathcal{Z}_{\delta_z} \\ \mathcal{M}_{\delta_z} \\ 0 \end{bmatrix}\bar{u}^F(t) + \mathcal{C}_\omega\omega(t)$$

where θ is the pitch angle; the parameters $\mathcal{Z}_\alpha$, $\mathcal{M}_\alpha$, $\mathcal{M}_q$, $\mathcal{Z}_{\delta_z}$, and $\mathcal{M}_{\delta_z}$ are the moment dimensional derivatives and force; the subscript δ_z signifies the equivalent elevator deflection; $\mathcal{V}_*$ and μ_* denote the velocity and flight-path angle on the equilibrium point, respectively. Moreover, $u(t)$ is the elevator deflection; $\omega(t)$ is the external noise. Therefore, one can refer for more details in [45]. By considering the above the facts and aircraft model in [44] and [46], we examine the six-degree-of-freedom nonlinear HIRM together with sensor models and nonlinear actuator. The dynamic equations of the physical model use aerodynamic data achieved from flight tests of an unpowered, scaled drop model and wind tunnel. The comprehensive investigation about the HIRM system can be launch in [46]. The linearization dynamics of the HIRM together with an altitude of 1600 m and mach number 0.3 and is governed by the subsequent dynamical model [45], [47]

$$\dot{\delta}(t) = \begin{bmatrix} -0.5427 & 1 & 0 \\ -1.069 & -0.4134 & 0 \\ 0 & 1 & 0 \end{bmatrix}\delta(t)$$
$$+ \begin{bmatrix} -0.113 \\ -3.259 \\ 0 \end{bmatrix}\bar{u}^F(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\omega(t).$$

By considering facts that the probabilistic time-varying delay, actuator faults, and resilient reliable sampled-data control law into the above state-space model, which is equivalent to (7), then we have the following dynamic model:

$$\dot{\delta}(t) = \mathcal{A}\delta(t) + \theta(t)\mathcal{B}_u\mathcal{G}\mathcal{K}\delta(t - \eta_1(t))$$
$$+ (1 - \theta(t))\mathcal{B}_u\mathcal{G}\mathcal{K}\delta(t - \eta_2(t)) + \mathcal{C}_\omega\omega(t)$$

where

$$\mathcal{A} = \begin{bmatrix} -0.5427 & 1 & 0 \\ -1.069 & -0.4134 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \mathcal{B}_u = \begin{bmatrix} -0.113 \\ -3.259 \\ 0 \end{bmatrix}, \mathcal{C}_\omega = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Moreover, the additional parameters are chosen as $\mathcal{G} = 0.5$, $\eta_0 = 0.25$, $\eta_1 = 0.658$, and $\gamma = 4.2057$. With these
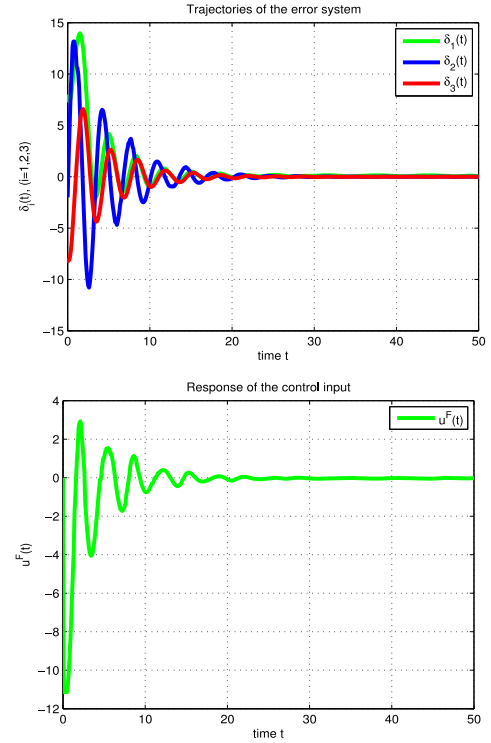
parameters, our major purpose is to establish our conclusion on dissipativity realization of the HIRM is exploited, which unifies the all dynamic analysis of HIRM, such as $\mathcal{H}_\infty$, passivity and mixed $\mathcal{H}_\infty$ and passivity performances as follows. However, until now, this has not been exploited in the previous works on this issues (see [44]–[47]).

For dissipative performance, we setting $\mathbb{Q} = -0.3$, $\mathbb{S} = -1.2$, and $\mathbb{R}_\alpha = 3$. Including these values, utilizing the MATLAB LMI control toolbox, some sufficient criterion developed in Theorem 2 is found to be feasible for given $\theta_0 = 0.95$ and its correspondent resilient reliable control gain matrix is determined as

$$\mathcal{K} = [-0.6119 \quad 0.3192 \quad 2.0923]. \tag{29}$$

Owe to the construction of page limit, we have omitted the feasible matrices. In order to explore simulation results, the noise parameter taken as $w(t) = -0.05(1 + \cos(t))$ along three initial states $\delta(0) = [8, -2, -8]^T$, then the controlled closed-loop system performance and control input of dynamic system (7) are displayed in Fig. 2. Therefore, the simulation result, we confirm that the error state signals and control input converges. Moreover, Fig. 3 shows the phase trajectories of the HIRM. Hence, by Theorem 2, there exists a proposed controller for the closed-loop system (7) with noise $w(t)$ is strictly $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-dissipative. In addition, Table III shows the maximum allowable upper bound (MAUB) of η_1 for different values of η_0. It is clear from Table III that the MAUBs of η_1 increase as η_0 increases. Moreover, in spite of deliberate the control performance of different cases, we realize the subsequent three cases from the acquired dissipative result according to Remark 6.
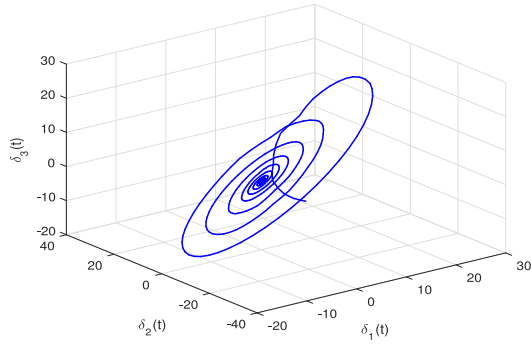
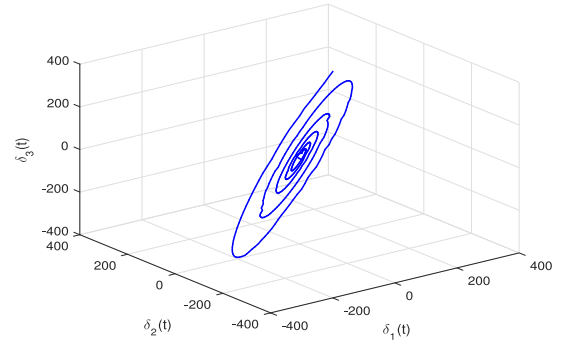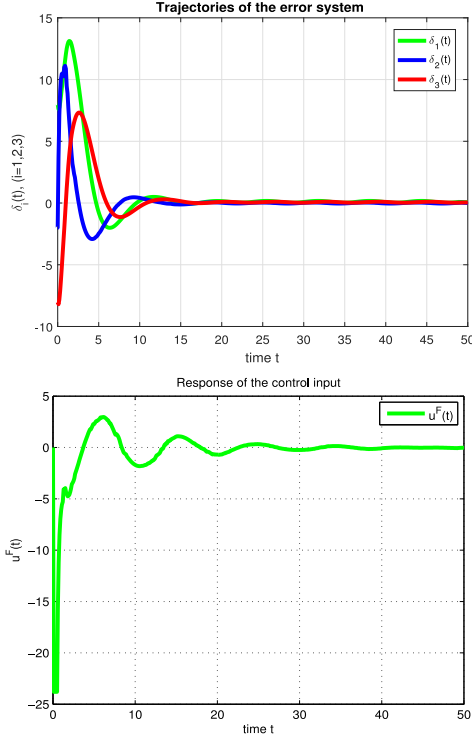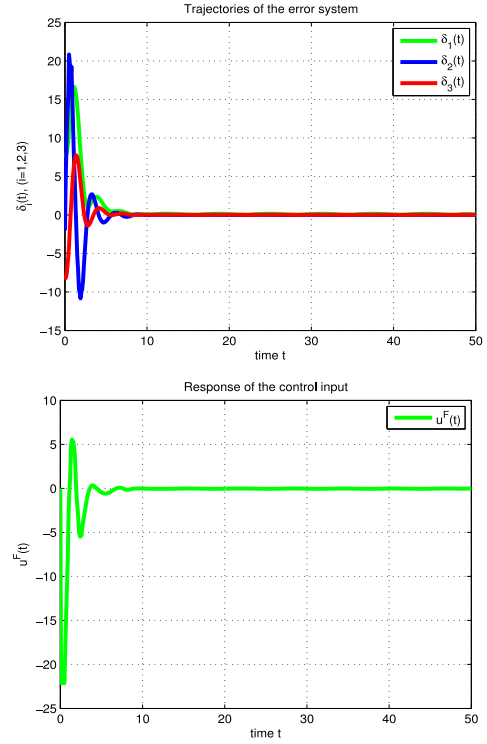Fig. 3. Phase trajectories based on dissipativity analysis.

Fig. 5. Phase trajectories based on $\mathcal{H}_\infty$ analysis.

Fig. 4. State trajectories and control input of the HIRM based on $\mathcal{H}_\infty$ analysis.

Fig. 6. State trajectories and control input of the HIRM based on passivity analysis.

TABLE III
MAUB OF η_1 FOR DISSIPATIVE PERFORMANCE
WITH VARIOUS VALUES OF η_0

η_0	0.02	0.04	0.06	0.08	0.1
η_1	1.2150	1.3425	1.4824	1.7280	1.8564

A. $\mathcal{H}_\infty$ Performance of HIRM

For this case, we let $\mathbb{Q} = -\mathcal{I}$, $\mathbb{S} = 0$, and $\mathbb{R}_\alpha = \gamma^2 \mathcal{I}$. Then, we applying the LMI (25) in Theorem 2, we obtain the subsequent gain matrix

$$\mathcal{K} = [-3.1503 \quad 2.1764 \quad 2.2543]. \tag{30}$$

With the help of above gain matrix $\mathcal{K}$, the simulation studies of the closed-loop system is displayed in Figs. 4 and 5. From Fig. 4, the state responses and control input of systems are really well stabilized via $\mathcal{H}_\infty$ control. This indicates that the closed-loop system (7) with noise $w(t)$ is asymptotically stable

under the same initial states as mentioned above. Furthermore, for this case the minimum attenuation level $\gamma_{\min}$ is 1.1037.

B. Passivity Performance of HIRM

We let $\mathbb{Q} = 0$, $\mathbb{S} = \mathcal{I}$, $\mathbb{R}_\alpha = \gamma \mathcal{I}$ and applying Theorem 2, the corresponding gain matrix $\mathcal{K}$ can be obtained as follows:

$$\mathcal{K} = [-1.3728 \quad 1.1802 \quad 3.8610]. \tag{31}$$

By employing the convex optimization algorithm, its minimum disturbance attenuation level is estimated as $\gamma_{\min}$= 0.4562. Fig. 6 interpret the state responses and control input of system (7) based on the passivity control performance. In addition, Fig. 7 display the phase trajectories of the HIRM. Therefore, from Fig. 6 that the error system (7) is strictly passive.
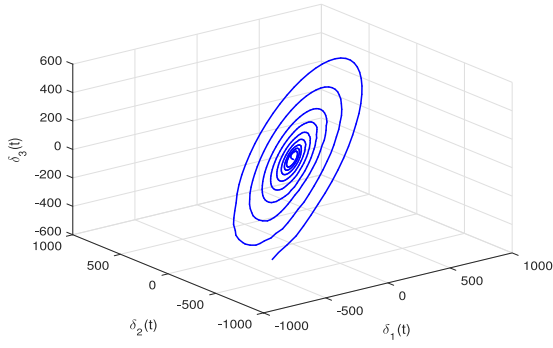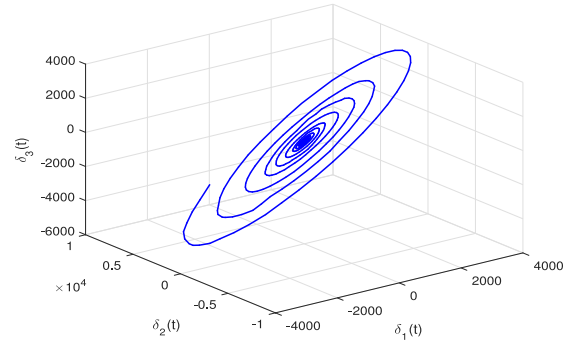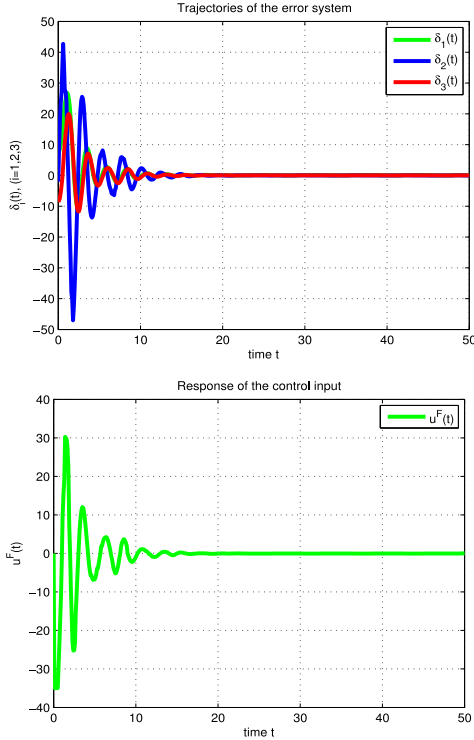
Fig. 7.   Phase trajectories based on passivity analysis.

Fig. 9.   Phase trajectories based on mixed $\mathcal{H}_\infty$ and passivity analysis.

Fig. 8.   State trajectories and control input of the HIRM based on mixed $\mathcal{H}_\infty$ and passivity analysis.

C. Mixed $\mathcal{H}_\infty$ and Passivity Performance of HIRM

Based on the Theorem 2 with free matrices $\mathbb{Q} = -0.5I$, $\mathbb{S} = 0.6$, and $\mathbb{R}_\alpha = 0.6\gamma^2 \mathcal{I}$, and its equivalent resilient reliable gain matrix can be determined as to the feasibility of LMI (25) is obtained by

$$\mathcal{K} = [-2.4051 \quad 1.0087 \quad 6.0872] \tag{32}$$

and the minimum attenuation level $\gamma_{\min}$ is 0.9886. Moreover, the mixed $\mathcal{H}_\infty$ and passivity analysis of the closed-loop system (7) utilizing the proposed control gain can be established from Figs. 8 and 9. Utilizing LMI control toolbox and Theorem 2, Table IV indicates the corresponding minimum attenuation of γ for various η_0. From the table, it is confirm that, $\gamma_{\min}$ decreases when increasing the values η_0 in mixed $\mathcal{H}_\infty$ and passivity and $\mathcal{H}_\infty$ performances. Therefore, in a conclusion, $\gamma_{\min}$ increases when increase the η_0 values in the passive performances.

TABLE IV
$\gamma_{\min}$ FOR DIFFERENT VALUES OF η_0

η_0	0.75	0.80	0.85	0.90	0.95
$\mathcal{H}_\infty$	1.4325	1.3520	1.2011	1.1807	1.1037
Passivity	0.1157	0.1256	0.2768	0.3276	0.4862
Mixed $\mathcal{H}_\infty$ and Passivity	1.4256	1.3211	1.2056	1.1011	0.9886

Remark 7: It is very interesting that, the simulation results of the state responses of the closed-loop system (7) for different control indices and the responses of the control input for different performances is depicted in Figs. 2–9. Therefore, from the figures, it is not difficult to justify that the passivity control law admits the excellent realization of the error system (7) with $w(t)$ among the four performances. Likewise, in the passivity case only we obtained least possible control effort compared with other performances. In addition, the attenuation level γ earns its least possible value meanwhile in the same case which is listed in Table IV. Hence, in conclusion, it can possibly save cost and time in the passivity performance. Therefore, from Figs. 2–9, our conclusion is that the proposed control law can stabilize the HIRM system more effectively.

Remark 8: It is quite interesting that, the above example is demonstrated to showing the proposed control methodology can be applied to the practical applications of HIRM under dissipativity performance. In recent years, the HIRM has gained much interest from the research communities, for the reason that it can be easy to use and realize the practical applications. Therefore, until now, no research investigation has been explored in the literature with respect to the dissipativity performance of HIRM including actuator faults and probabilistic time-delay signals via the resilient reliable sampled-data controller (see [44]–[47]). That is to say, all the works mentioned above have dealt with the robust control, $\mathcal{H}_\infty$ control and passivity control. Therefore, the results reported in [44]–[47] have some demands. Thus, in this paper, we have proposed unified framework analysis in terms of LMIs and the numerical investigations are explored in the sense of more other dynamical performance of HIRM, such as $\mathcal{H}_\infty$ control, passivity control, and $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$ dissipative control performances. Thus, in the viewpoint of practical applications, the developed results in this paper are essentially significant. Hence, the analysis technique and control law developed in this paper is more general.
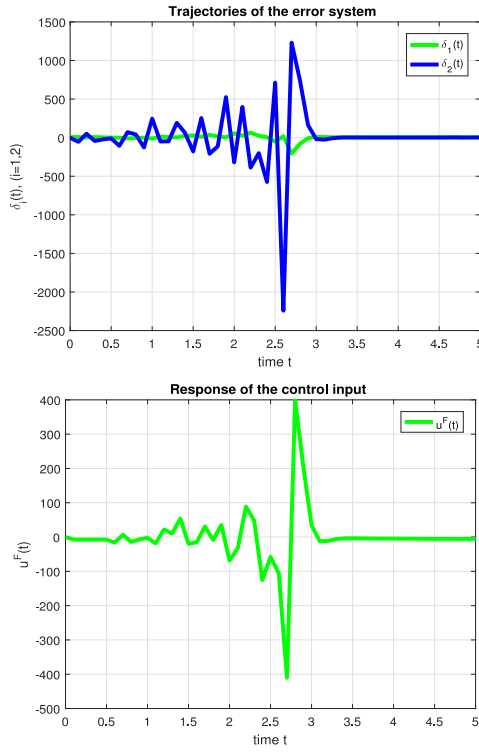
Fig. 10. State trajectories and control input of the RSS based on dissipativity analysis.
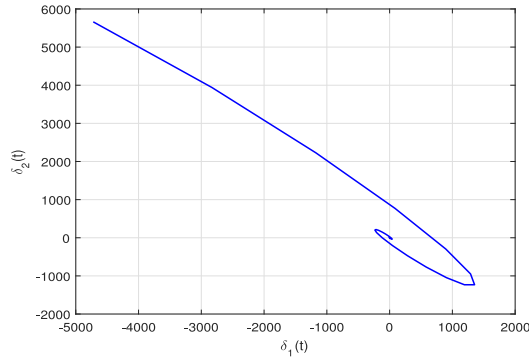

Fig. 11. Phase trajectories of the RSS based on dissipativity analysis.

Example 2: In order to show the effectiveness of the proposed methodology, in this paper, we consider the rotary servo system (RSS) as one more example (see [52]). The mathematical model of RSS is described as

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = (\mathcal{A} + \Delta\mathcal{A})\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \mathcal{B}u(t) + d(t) \tag{33}$$

where θ and $\dot{\theta}$ are the system states that represents the angular position and velocity, respectively; $u(t)$ is the control signal; $d(t)$ is the external disturbance. Now, we compare the above system with (1), it is clear that $\delta(t) = [\theta \quad \dot{\theta}]^T$ with $\Delta\mathcal{A} = 0$ and $\mathcal{C}_\omega = \mathcal{I}$, then the system (33) is equal to (1) together with (7). For the simulation purpose, the system parameters are borrowed form [52], we have $\mathcal{A} = \begin{bmatrix} 0 & 0 \\ 0 & -30 \end{bmatrix}$, $\mathcal{B}_u = [0 \quad 80]^T$, and $\omega(t) = [0 \quad 80d_2(t)]^T$ with $d_2(t) = 100(t-4)$. Due to the page limit, in this example, we interested in only

to discuss the more generalized case the dissipativity of the RSS is exploited. Furthermore, the additional parameters are chosen as $\mathbb{Q} = -0.2$, $\mathbb{S} = -1.1$, $\mathbb{R}_\alpha = 2$, $\mathcal{G} = 0.8$, $\eta_0 = 0.01$, $\eta_2 = 0.5$, and $\gamma = 2.8927$. Then, we applying the LMI (25) in Theorem 2, we obtain the subsequent gain matrix

$$\mathcal{K} = [-1.1119 \quad 0.2192]. \tag{34}$$

Under this gain matrix $\mathcal{K}$ and initial condition $\delta(0) = [8, -2]^T$, the relative simulations are depicted in Figs. 10 and 11. Fig. 10, illustrates the state responses and control input of the RSS are well stabilized via dissipative control. Also, Fig. 10 display the responses of the phase trajectories of the RSS. Therefore, from Figs. 10 and 11, we conclude that the RSS is stable via dissipative control.

V. CONCLUSION

In this paper, we have investigated the resilient reliable dissipativity-based control issue for systems including actuator faults and probabilistic time-delay signals via sampled-data control scheme. By utilizing the input time-delay procedure, the resilient reliable sampled-data control system is converted into usual system model including probabilistic time-delay signals. Accordingly, an LMI-based new version of delayed dependent stabilization criterion has been acquired by choosing a proper LKF and estimating their upper bounds some improved techniques. Consequently, the criterion is enlarged to design the resilient reliable sampled-data control scheme which establishes the better system performance. Moreover, $\mathcal{H}_\infty$ resilient reliable control, mixed $\mathcal{H}_\infty$ and passivity resilient reliable control, and passivity resilient reliable control performances have been discussed as the special cases of developed $(\mathbb{Q}, \mathbb{S}, \mathbb{R})$-$\gamma$-dissipativity-based control problem. In addition, a real-world example is illustrated to showing applicability and usefulness of the proposed control scheme on an HIRM and RSS. Moreover, the proposed system model and control approach has played an important role in many engineering disciplines, such as an offshore steel jacket platforms [5] and vehicles dynamics [42]. In addition, the proposed method in this paper can be extendable to many famous dynamical systems, such as fuzzy systems [3], networked control systems [7], stochastic systems [31], switched system [39], Markovian jump systems [41], and tracking problem [45]. This will occur in the near future.

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R. Manivannan received the B.Sc. degree in mathematics from Sacred Heart College (Autonomous), Tiruppattur, India (affliated to Thiruvalluvar University, Vellore, India), in 2009, the postgraduation degree in mathematics from Islamiah Men's College (Autonomous), Vaniyambadi, India (affliated to Thiruvalluvar University, Vellore, India), in 2012, and the M.Phil. degree in mathematics with the specialization of numerical studies in fluid dynamics from Bharathiar University, Coimbatore, India, in 2014, and the Ph.D. degree in mathematics in dynamical behaviors of generalized delayed neural networks from Thiruvalluvar University, Vellore, India, in 2017.

He was a Junior Research Fellow and a Senior Research Fellow with the Department of Science and Technology-Science and Engineering Research Board, Government of India, New Delhi, India, from 2014 to 2016. He was an Assistant Professor with the Department of Mathematics, School of Natural Sciences, National Institute of Technology Calicut (NITC), Kozhikode, Kerala, India, from 2017 to 2018. He is currently a Post-Doctoral Research Fellow with the Advanced Research Center of Electronics and Information, Division of Electronic Engineering, Chonbuk National University, Jeonju-Si, South Korea. He has authored or co-authored over 15 research articles in various SCI journals holding impact factors. His current research interests include stochastic hybrid systems, networked control systems, state estimation, Markovian jump systems, extended dissipativity analysis, nonfragile control, sampled-data control, impulsive and switched neural networks, numerical studies in fluid dynamics and their applications.

Dr. Manivannan serves as an active reviewer for various SCI journals including IEEE journals.

Jinde Cao (F'16) received the Ph.D. degree in applied mathematics from Sichuan University, Chengdu, China, in 1998.

He is an Endowed Chair Professor, the Dean of School of Mathematics, and the Director of the Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing, China. From 1989 to 2000, he was with the Yunnan University, Kunming, China. In 2000, he joined the School of Mathematics, Southeast University, Nanjing, China. From 2001 to 2002, he was a Post-Doctoral Research Fellow with the Chinese University of Hong Kong, Hong Kong.

Prof. Cao was a recipient of the Highly Cited Researcher Award in Engineering, Computer Science, and Mathematics by Thomson Reuters/Clarivate Analytics and the National Innovation Award of China in 2017. He was an Associate Editor of the IEEE TRANSACTIONS ON NEURAL NETWORKS, and *Neurocomputing*. He is an Associate Editor of the IEEE TRANSACTIONS ON CYBERNETICS, IEEE TRANSACTIONS ON COGNITIVE AND DEVELOPMENTAL SYSTEMS, *Journal of the Franklin Institute*, *Mathematics and Computers in Simulation*, *Cognitive Neurodynamics*, and *Neural Networks*. He is a member of the Academy of Europe, a member of European Academy of Sciences and Arts, and a Fellow of Pakistan Academy of Sciences.

R. Samidurai received the B.Sc. degree in mathematics from Thiruvalluvar Government Arts College, Rasipuram, India, (affiliated to Periyar University, Salem, India), in 2003, the postgraduation degree in mathematics from Bharathiar University, Coimbatore, India, in 2005, and the Master of Philosophy degree in abstract differential equations and the Doctor of Philosophy degree in mathematics (area of control theory) from Periyar University, in 2007 and 2010, respectively.

Since 2011, he has been an Assistant Professor with the Department of Mathematics, Thiruvalluvar University, Vellore, India. He has authored or co-authored over 40 research articles in various SCI journals holding impact factors. His current research interests include stochastic differential equations, fractional differential equations, nonlinear systems, neural networks, time delay systems, complex-valued neural networks, networked control systems, extended dissipativity analysis, nonfragile control, sampled-data control, and control theory in partial differential equations and their applications.

Prof. Samidurai serves as a reviewer of various SCI journals including IEEE journals.

Matjaž Perc completed his doctoral thesis on noise-induced pattern formation in spatially extended systems with applications to the nervous system, game-theoretical models, and social complexity. He is a Full Professor of physics with the University of Maribor, Maribor, Slovenia, and the Director of the Complex Systems Center, Maribor.

Mr. Perc was a recipient of the Young Scientist Award for Socio- and Econophysics in 2015. He was among top 1% most cited physicists according to Thomson Reuters Highly Cited Researchers. He is also the 2017 USERN Laureate, which honors outstanding research serving the humanity in all fields of research. He is an Outstanding Referee of the *Physical Review* and *Physical Review Letters* journals and a Distinguished Referee of EPL. He is on the editorial board of *Physical Review E*, *New Journal of Physics*, *EPL*, *Scientific Reports*, *Royal Society Open Science*, *Applied Mathematics and Computation*, *Advances in Complex Systems*, *Chaos, Solitons and Fractals*, *Journal of Complex Networks*, and *Palgrave Communications*. In 2018, he also became a Specialty Chief Editor of *Frontiers in Physics* and *Social Physics*. He is a member of Academia Europaea and the European Academy of Sciences and Arts.