

Computational chaos in complex networks

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Computational chaos reports the artificial generation or suppression of chaotic behaviour in digital computers. There is a significant interest of the scientific community in analysing and understanding computational chaos of discrete and continuous systems. Notwithstanding, computational chaos in complex networks has received much less attention. In this article, we report computational chaos in a network of coupled logistic maps. We consider two types of networks, namely the Erdős–Rényi random network and the Barabási–Albert scale-free network. We show that there is an emergence of computational chaos when two different natural interval extensions are used in the simulation. More surprisingly, we also show that this chaos can be suppressed by an average of natural interval extensions, which can thus be considered as a filter to reduce the uncertainty stemming from the inherent finite precision of computer simulations.

Keywords: computational chaos; chaos suppression; non-linear dynamics; complex networks; computer arithmetic; complex systems.

1. Introduction

Chaos theory is intimately linked to the computational simulation of dynamical systems [1–3]. Although many researchers indicate the work of Henry Poincaré as one of the first to study the theory of dynamical systems and indicate sensitivity to initial conditions, the work of Lorenz [4] is commonly regarded as a milestone development of this theory. While Lorenz [4] has received a huge attention from the scientific community, a later work [5] has not received the same concern. In this work, Lorenz defines the term *computational chaos*. More than 20 years after discovering the butterfly effect, Lorenz suggests that there are chaotic behaviours that are not due to the dynamic system, but in fact, to the process of discretization.

In fact, the existence of chaos due to discretization processes had previously been identified in works of [6–8], but it was with the work of Lorenz that this prospect gained breath and dozens of other studies have emerged in the literature discussing or approving the issue of computational chaos [9–34].

The perspective of computational chaos encompasses both situations in which the original system presents regularity/periodicity and its discretized version, on the other hand, presents chaos, as well as the inverse, when the system is considered chaotic and its computational simulation shows regularity. The latter phenomenon has been given the name of *chaos suppression* [35–48]. In fact, the suppression of expected chaos in complex networks has been widely investigated. It has been noticed that networks with biological background have been found to suppress chaos more successfully than other networks. The interested reader is referred to [49–52] for more details.

In principle, the questions raised by Lorenz were mostly discussed from the perspective of discretization, that is, a numerical method problem. Even in previous works, as in [6], the authors show that original equations give rise to chaos due to a discretization process. In the 1970s, these authors have shown that simple differential equations may have difference equations with complicated dynamic behaviours and associate this phenomenon with the recent so-called chaos [6], as understood by Li and York a few years before [53]. These solutions have been called *ghost solutions* in several subsequent works [6–10, 14, 18, 21, 47, 54–56] or *spurious solutions* [12, 57]. The question of discretization in particular has been the subject of several works, both in the perspective of robustness of results, as well as in the line of trying to reduce the error and avoid spurious results [54, 57–64].

Parallel to the research based on numerical methods, it is also perceived a significant effort to bring security to computational simulations. In the 1980s, Hammel *et al.* [65] are considered pioneers in proving a theorem, with computational aid, that the logistic map could be simulated by millions of iterations with the security that the error would be at most in the seventh decimal place. Recently, however, Nepomuceno *et al.* [66] have relevant limitations of the theorem proposed in [65] based on an in-depth study of arithmetic computation. Much of these works make use of the shading theorem, applicable to hyperbolic systems [65, 67–72]. More recently, scientific computation also becomes more rigorous with the use of interval arithmetic and affine arithmetic, which allow to incorporate errors associated with finite precision intrinsic to numerical computation [73–78]. This line of research has received great prominence, particular attention should be given to the formulation of a standard by IEEE to treat specifically under the arithmetic interval [73]. Examples of application of interval arithmetic for rigorous simulation of discrete maps were proposed in [79, 80]. Researchers such as Galias, have performed important studies using interval arithmetic to broaden the rigour in computational calculus, particularly of non-linear dynamical systems [81–88]. Importantly, however, that interval arithmetic can also be influenced by software/hardware implementation and inconsistent results were observed in [89].

Although there is great effort from the scientific community to understand the theory of chaos, and in particular of computational chaos, it is perceived that there are still issues in open, raised since the first years of development of this theory. Ford [90] with the intriguing title: ‘Chaos: solving the unsolvable, predicting the unpredictable’ is one of the first authors to raise questions about the conclusions obtained from computational simulations. Years later, Lozi [56] questions whether in the simple case of the discrete dynamic system of the Hénon map there are long periodic orbits or strange attractors. Similarly, Galias [86] has suggested that the importance of developing methods for the proof of chaos, which he says, exists for the Lorenz attractor, citing the work of [91], but that for the circuit of Chua is still an open question. It is important to emphasize that the evidence of the existence of the Lorenz attractor makes use of computational simulation, and although Galias assert the proof for the attractor of Chua is still an open question, Chua *et al.* [92] indicate a proof for the existence of the double attractor, also using computational resources.

One of the first most accurate indications of the study of chaos theory in conjunction with computational simulation was done by Corless [55]: ‘In summary, there are four levels of abstraction used here: the physical reality of the problem under study, the mathematical continuous model of physical reality, the

numerical discretization of the mathematical model, and the simulation using floating-point discretization'. Similarly, Oberkampff *et al.* [93] suggest five levels in simulation: conceptual model, mathematical model, discretization, computational programming, numerical solution and numerical solution representation. In the passage cited in [55], the floating-point refers to the pattern of representation of the real numbers and realization of arithmetic operations in a digital system, indicated by the IEEE 754-2008 [94–97] standard. According to [98], arithmetic computation is a field of knowledge that encompasses the definition and standardization of arithmetic systems for computers. It also includes hardware and software implementation issues, worrying about the testing and reliability of these implementations. Nannarelli *et al.* [98] also affirm that it is an area with great concern in scientific computing, and finally, they affirm that it is an interdisciplinary area that involves mathematics, computer science and electrical Engineering. Palmore and Herring [99] were pioneers to relate the theory of chaos with arithmetic computation, which explores aspects of arithmetic computation from the point of view of dynamic systems. In that work, the effects of finite precision of computer arithmetic have been evaluated in uniformly hyperbolic chaotic systems. Even more interesting is the assertion that the authors analyse the computer as a dynamic system, a similar assertion found in [72]. The conclusion of the study is that the use of floating-point representation can strongly influence the results obtained from computational simulations. Followed work reiterated this position, such as [72, 100–103]. A demonstration of this is in [103], which states that a good analysis of the relationship between arithmetic (floating-point) and digital dynamic systems is given in [99], because such authors demonstrate that even trivial changes in arithmetic computation can significantly modify the structures of the pseudo-orbits. Nepomuceno and Martins [34] have observed that the change of natural interval extensions, that is, functions mathematically equivalent, but written differently from elementary mathematical operations, produced qualitatively different results in the Mackey-Glass system simulation. Important to note that long before, Fryska and Zohdy [72] have claimed that some chaotic systems can produce attractors with different topologies when integrated into different levels of precision to the floating point pattern. The problem of finite precision in the investigation of chaos theory has also been expressed in a clear way in [17]. In that work, the author states that for chaotic systems any error in the initial conditions or computational errors propagate and grow rapidly due to the properties of the dynamic system itself. Due to truncation operations, it is critical to observe safely the behaviour of chaotic or even periodic trajectories for a long period of time. Another interesting observation was made by Adler *et al.* [3] as they claim that the logistic map exhibits unexpected behaviour dependent on the precision used. In [3], it is demonstrated, contrary to the general intuition, that the use of greater precision (greater number of bits) does not necessarily lead to greater accuracy in the results, and on the contrary, the authors show that the use of a larger number of bits can lead to a less accurate result; concludes the article saying that the mathematical software cannot be treated as a black box for an adequate understanding of the computational simulation results. As to the precision similar conclusions were also found in [104]. Still regarding the connection between chaotic systems and finite precision, Mendes and Nepomuceno [105] propose a method for calculating the exponent Lyapunov, one of the most used indexes to verify whether the system is chaotic or not, from the simulation error.

The literature on chaos in networks is plenty of works [106–116]. The interest has been even amplified after the discovery of the chimera states and many other works have been devoted to such study [117–127]. Nevertheless, computational chaos in complex networks has received much less attention. Indeed, a search for 'computational chaos' AND network in *Web of Science*, *Scopus* and *IEEEExplore* has found only five [106, 108, 128, 129], whereas computational chaos is only investigated in [106, 129]. Barhen *et al.* [108] have reported a methodology based on the stability of asynchronous computation for the prevention of computational chaos. Barhen *et al.* [106] have reported computational chaos in artificial

neural networks with asynchronous regime, which impedes the efficient retrieval of information usually stored in the system's attractors.

In this article, we have reported computational chaos in a complex network. We have investigated a network of coupled logistic map [113, 114]. In our study, we choose two complex network models: the Erdős–Rényi random networks and the Barabási–Albert scale-free networks. Li *et al.* [113] have reported that even when the parameters of the nodes are not in chaotic regions, a coupled large-scale network can exhibit chaotic behaviour. Here, we show that there is an emergence of computational chaos. Using different natural interval extensions [34, 66, 130], we show different qualitative behaviour in such network. More surprisingly, we have also shown that this chaos can be suppressed by an average of natural interval extensions, which can be seen as filter to reduce the noise derived from the finite precision of the simulation.

The remainder of the article is organized as follows: Section 2 presents the network of coupled logistic maps. The emergence of computations chaos is shown in Section 3 and suppression of chaos is shown in Section 4. Final remarks are given in Section 5.

2. Logistic Network

Let a node of a network given by the logistic map in the form of

$$x^{k+1} = \mu x^k (1 - x^k), \quad (1)$$

where $\mu \in [0, 4]$. Consider a connected dynamical network of N coupled identical nodes given by Eq. (1). The state equations of the logistic network are given by [114, 115]:

$$x_i^{k+1} = f(x_i^k) - c \sum_{j=1}^N a_{ij} f(x_j^k), \quad i = 1, 2, \dots, N, \quad (2)$$

where $x_i^k \in \mathbb{R}$ is the state variable of node i at time step k , and $c > 0$ is a positive real parameter that represents the coupling strength of the network. The coupling matrix, also called Laplacian Matrix [131], $A \in \mathbb{R}^{N \times N}$ is represented by

$$A = \begin{pmatrix} d_{11} & a_{12} & a_{13} & \dots & a_{1N} \\ a_{12} & d_{11} & a_{23} & \dots & a_{2N} \\ a_{13} & a_{23} & d_{33} & \dots & a_{3N} \\ \vdots & \vdots & \vdots & \ddots & \dots \\ a_{1N} & a_{2N} & a_{3N} & \dots & d_{NN} \end{pmatrix}. \quad (3)$$

If there is a connection between node i and node j , then $a_{ij} = 1$; otherwise, $a_{ij} = 0 (i \neq j)$ and let

$$d_{ii} = - \sum_{j=1, j \neq i}^N a_{ij} = - \sum_{j=1, j \neq i}^N a_{ji}, \quad i = 1, 2, \dots, N \quad (4)$$

which means that the network is fully connected with no isolated clusters. A matrix form of Eq. (2) is

$$X^{k+1} = (I - cA)f(X^k), \quad (5)$$

where $X^k = [x_1^k, x_2^k, \dots, x_N^k]$ and $I \in \mathbb{R}^{N \times N}$ is identity matrix.

Here, we have investigated two coupling configurations according to [113]. The networks models have been generated using the routines developed in [132]. First, a scale-free network generated by the Barabási–Albert model with parameters $N = 1000$, $m = 3$ and $n_0 = 3$. Second, an Erdős–Rényi random network with $N = 1000$ nodes and $K = 3000$ edges. We have set $\mu = 2.5$, which have been used to generate the coupling matrix A according to Eq. (3). All nodes have been initiated around the fixed point $x^* = (\mu - 1)/\mu = 0.6$.

We have focused our attention on the node with the highest degree, which for the scale-free and random network are 81 and 15, respectively. Figure 1 presents the bifurcation diagrams for these nodes with the highest degrees. Similar result has been reported in [113]. In both network topology, the system is found to exhibit a period doubling cascade route to chaos. Nevertheless, in the Erdős–Rényi network it is visible the presence of some outlier points from normal pattern. The Erdős–Rényi network is going to be investigated in more detail in the next section.

3. Computational chaos in Erdős–Rényi network

To observe the computational chaos in Erdős–Rényi network, we have examined the computer simulation using two different natural extensions. Such approach has been used in other of our works [66, 130, 133–135]. In this work, we have analysed the following natural interval extensions [136]:

$$x^{k+1} = \mu x^k (1 - x^k) \quad (6)$$

$$x^{k+1} = \mu (x^k (1 - x^k)) \quad (7)$$

$$x^{k+1} = \mu x^k - \mu x^k x^k \quad (8)$$

$$x^{k+1} = (\mu - \mu x^k) x^k \quad (9)$$

$$x^{k+1} = \mu (x^k - x^k x^k) \quad (10)$$

$$x^{k+1} = \mu x^k - \mu (x^k x^k). \quad (11)$$

Equations (7) to (11) are mathematically equivalent. However, it has already been reported qualitatively different simulation outcome when interval extensions have been used [34]. In this section, we have used Eqs (7) and (8) to observe different qualitative behaviour in the network of coupled logistic map. In the next section, all the interval extensions have been used to present a chaos suppression technique.

Figure 2 shows different qualitative behaviour in simulation of a network of coupled logistic maps due to the use of different natural interval extensions. In this simulation, we have used the natural interval extensions described in Eqs (7) and (11). The coupling matrix has been generated using Erdős–Rényi random network with $N = 1000$ nodes and $K = 3000$ edges. All simulations have used the same initial condition for all nodes, which are slightly different from the fixed point $x^* = 0.6$. Each node is described by a logistic map with $\mu = 2.5$. In Fig. 2, the first column presents the results of the simulation using the first interval extension, that is, $\mu x^k (1 - x^k)$, as shown in (a), (c) and (e). The second interval extension,

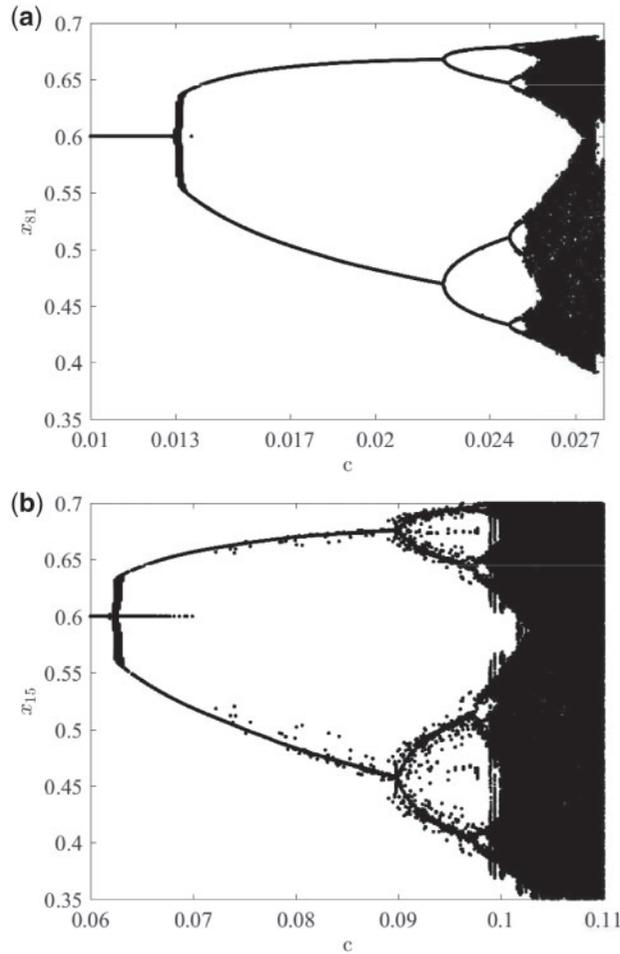


FIG. 1. Bifurcation diagram for the nodes with highest degree in two networks of coupled logistic maps according to Eq. (5). (a) The coupling matrix A is generated by a scale-free network following the Barabási–Albert model with $N = 1000$, $m = 3$ and $n_0 = 3$. (b) The coupling matrix A is generated by Erdős–Rényi random network with $N = 1000$ nodes and $K = 3000$ edges. In both cases, $\mu = 2.5$ and all nodes have been initiated close to the fixed point $x^* = 0.6$. The parameter bifurcation is the coupling strength c .

$\mu(x^k(1 - x^k))$, has been used to generate the results exhibited in Fig. 2 (b), (d) and (f). The bifurcation parameters are $c = 0.06994$, $c = 0.0990$ and $c = 0.100002$ for first, second and third line, respectively.

The results show different qualitative behaviour as outlined in Table 1. Although, it is expected the same outcome, different sequence of arithmetic operations presented in the natural interval extension is responsible for this qualitative different behaviour. The chaotic behaviour has evidenced by Lyapunov exponent in Fig. 3. It is an evident case of computational chaos. In fact, there is no reason to choose one of Eqs (7) and (8) as the correct outcome. At this point, we may analyse these results as case of computational chaos or chaos suppression.

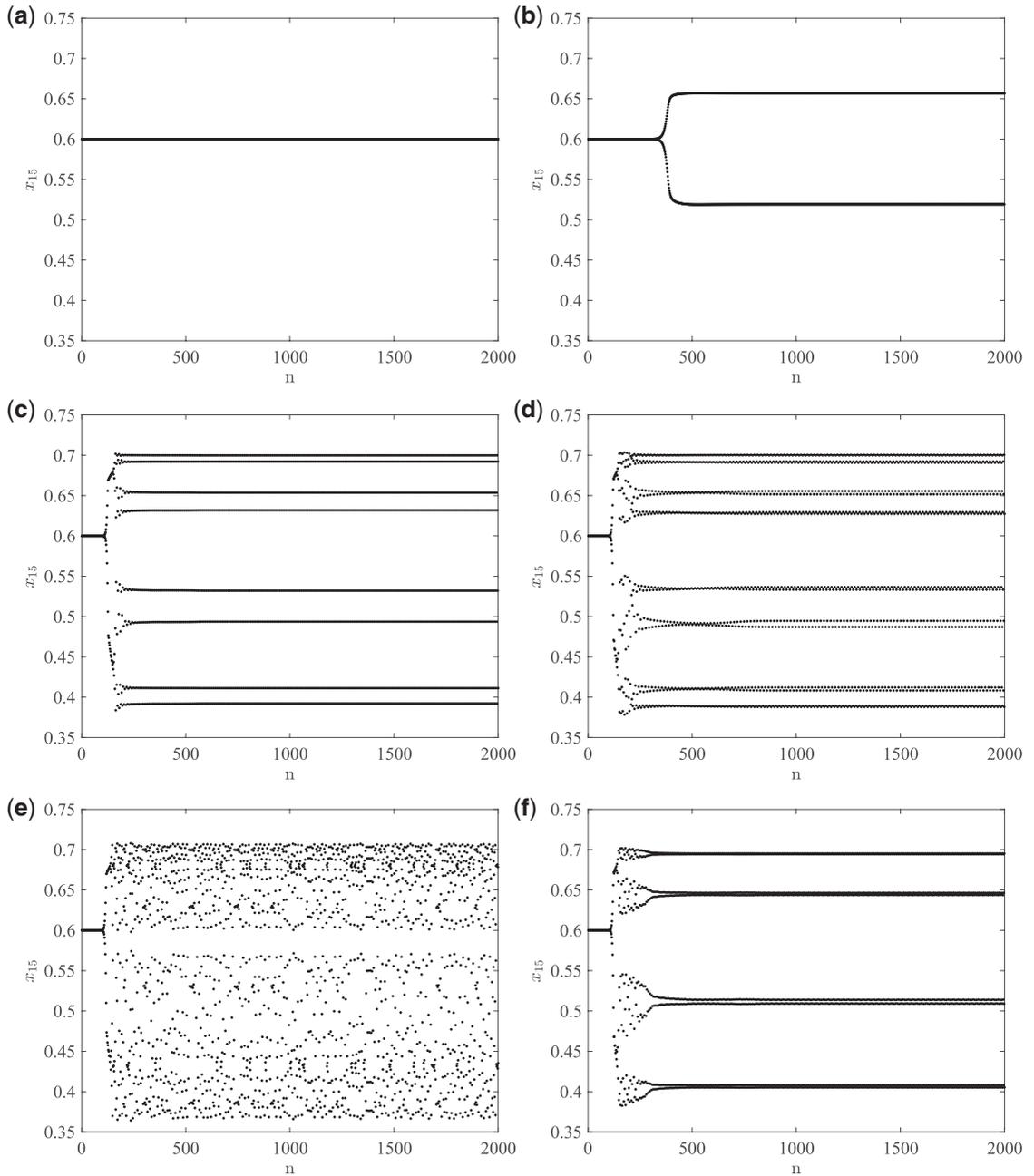


FIG. 2. Different qualitative behaviour in simulation of a network of coupled logistic maps. The coupling matrix has been generated using Erdős–Rényi random network with $N = 1000$ nodes and $K = 3000$ edges. All simulations have used the same initial condition for all nodes, which are slightly different from the fixed point $x^* = 0.6$. Each node is described by a logistic map with $\mu = 2.5$. The first column (a, c, e) and second column (b, d, f) have been simulated using the interval extensions $\mu x^k (1 - x^k)$ and $\mu(x^k(1 - x^k))$, respectively. The bifurcation parameter is $c = 0.06994$, $c = 0.099$ and $c = 0.100002$ for first, second and third line, respectively. Different number of fixed periods and even computational chaos can be noticed.

TABLE 1 *Different qualitative behaviour in coupled logistic map network. c is the bifurcation parameter. Two natural interval extensions have been analysed. Although, it is expected the same outcome, different sequence of arithmetic operations is the reason for these outcomes. We have adopted a tolerance of 10^{-5} to calculate the period. The chaotic behaviour has evidenced by Lyapunov exponent in Fig. 3. It is an evident case of computational chaos*

c	Eq. (7)	Eq. (8)
0.069940	Fixed point	Period 2
0.099000	Period 8	Period 16
0.100002	Chaos	Period 16

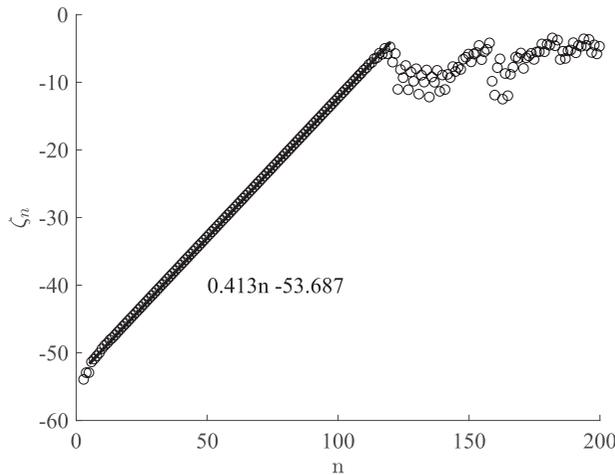


FIG. 3. Computation of the largest positive Lyapunov exponent to the node with the highest degree in the network of coupled logistic map. We adopted the procedure developed in [105]. In this case, the Lyapunov exponent is $0.413 \text{ bit}/n$, where n is iteration. It is an evidence that this node presents a chaotic behaviour using the interval extension as in Eq. (7).

4. Chaos suppression

In this section, we work with an idea of variance reduction of the simulation. This idea has been appeared in [137]. According to [138], the variance of a stochastic process can have its variance reduced by

$$s(k) = \frac{s}{\sqrt{k}}, \quad (12)$$

where s is the variance of one realization and k is the number of realizations. In our case, we are going to consider each interval extension, shown in Eqs (7) to (11), as an realization. We are not going to calculate the variance, but rather, we are interested in the effects of the simulation by using this rationale. Following this idea, we are going to simulate the network of coupled logistic map with the same parameters as in

Fig. 1(e). Let the following auxiliary variables:

$$\begin{aligned} L_1 &= \mu x^k (1 - x^k) \\ L_2 &= \mu (x^k (1 - x^k)) \\ L_3 &= \mu x^k - \mu x^k x^k \\ L_4 &= (\mu - \mu x^k) x^k \\ L_5 &= \mu (x^k - x^k x^k) \\ L_6 &= \mu x^k - \mu (x^k x^k) \end{aligned}$$

and Eq. (1) has been replaced by

$$x^{k+1} = \frac{1}{m} \sum_{i=1}^m L_i, \quad m = 1, 2, \dots, 6, \quad (13)$$

which is nothing but an average of the natural interval extensions. It is important to stress that from the point of view of dynamical systems theory based on mathematical rules, Eq. (13) is equivalent to (1), independent for the value of m . Nevertheless, Fig. 4 shows the result of the network of coupled logistic map using Eq. (13), in which the chaotic behaviour has been suppressed for the case of $m = 6$. In other cases, the system has been changed as well. Figure 4(d) seems to have a lower period, however a close look reveals a non-periodic behaviour. It is, however, in Fig. 4(f) that the chaos suppression occurs, in which a zoom shows the period 8.

5. Conclusion

The simulation of networks of dynamic systems are widespread. In many situations, it has been reported the emergence of chaos. Here, we have reported the emergence of computational chaos in complex networks. We have examined two types of networks: Barabási–Albert scale free and Erdős–Rényi random network. Our heuristic search has not found computational chaos in Barabási–Albert scale free. It does not mean that there is no computational chaos, but at least, it suggests that it less evident that the computational chaos in Erdős–Rényi random network. Our attention has been given to the node with highest degree. We have applied natural interval extensions to observe such phenomena and two simulations have been used. Although, these extensions represent the same equation, in one has been observed chaos, whereas in the other, period 16. Different periods have been also noticed.

The presence of computational chaos has been also evidenced as we have been able to suppress the chaos in the node with the highest degree. To achieve such goal, we have applied six different natural interval extensions for the logistic map. In each iteration for each node, the value of the next iteration has been calculated as the average of these six extensions. This procedure has been shown sufficient to suppress the chaotic behaviour. We believe that this procedure works as a filter to reduce the variance of noise caused but finite-precision error. Although, the computational chaos in many situations should be mitigated, it has been considered useful in applications such as encryption schemes [3, 33, 139].

Despite the variety of attributes, we have considered, our scope has been restricted. A more extensive investigation on the topological effects should be carried out. The use of continuous systems, such as Lorenz, or oscillators as Kuramoto in each node also seems a natural add-on of this work. We also have not considered soft computing techniques, such as interval arithmetic, to measure the error propagation.

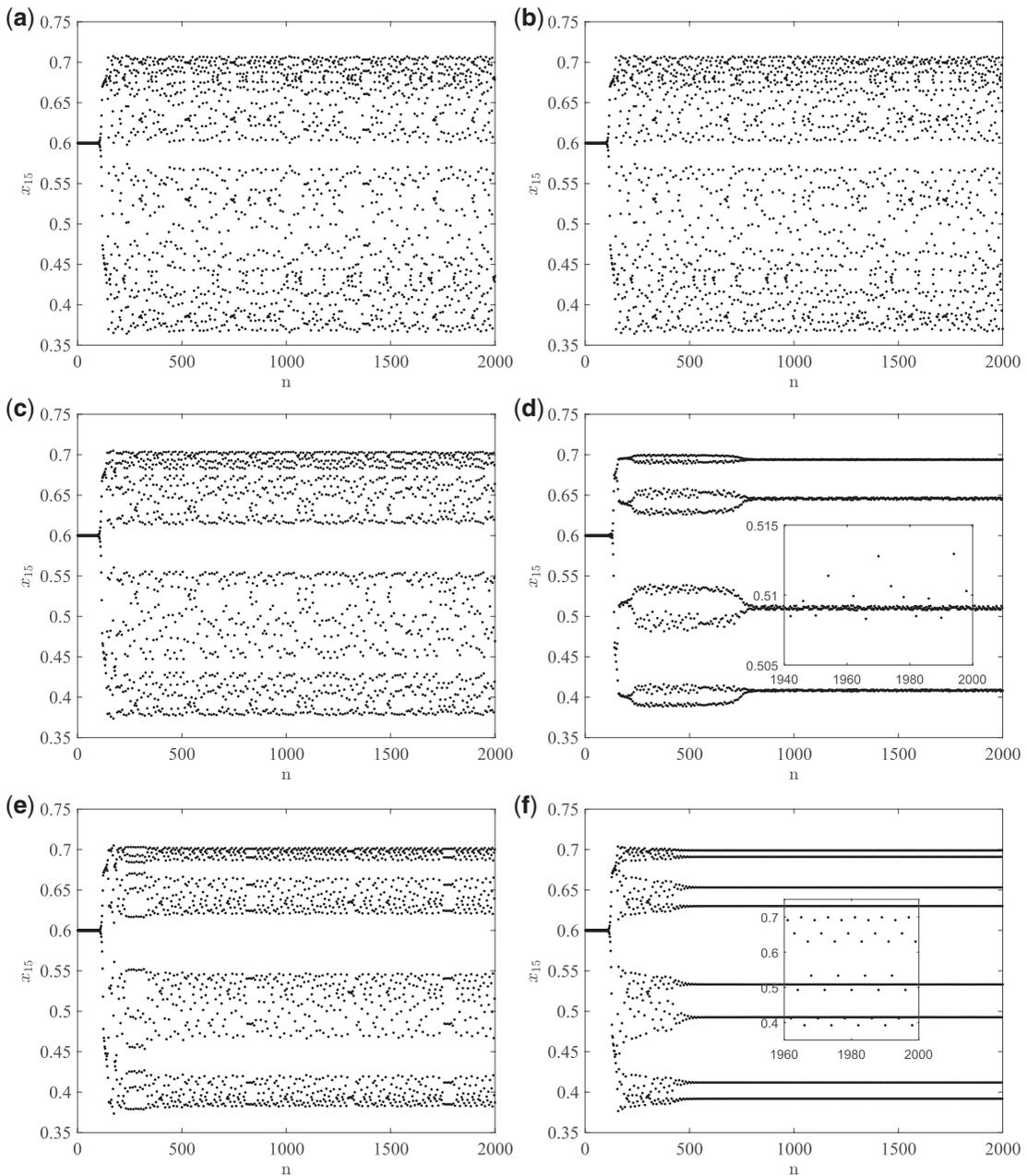


FIG. 4. Chaos suppression of the node with highest degree in the network of coupled logistic map. Here, we present the simulation using different number of interval extensions, from 1 to 6. In other words, $m = 1, 2, \dots, 6$. Equations (7) to (11) have been adopted. Although, Eq. (13) is equivalent to (1), the average of interval extensions works as a reduction of the variance in the simulation. (Panel d) looks period 4, but a close analysis reveals a no periodic behaviour. (Panel g), on other hand, shows a period 8, which is a clear case of chaos suppression.

To the best of authors' knowledge, this is the first time that computational chaos and chaos suppression have been observed in complex network. Lorenz [5] has been realistic in saying that 'in working with a simple system one generally affords to avoid it by making τ [step size] very small. When a system has many variables, however, economy may demand a larger τ .' This is even more so true in working with complex networks [132] or network of networks [140].

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