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Reputation preferences resolve social dilemmas in spatial multigames

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Abstract. Heterogeneity and difference in the dynamics of individual reputation may strongly affect learning behavior, and hence also the evolution of cooperation within a population. Motivated by this, we propose here an evolutionary spatial multigames model, wherein the reputation of an individual increases if they cooperate and decreases if they defect. After the payoffs are determined, individuals with a higher reputation will be more likely to act as strategy sources for other individuals. We perform systematic Monte Carlo simulations to determine the transitions between cooperation and defection, as well as the parameter regions of strategic coexistence. We show that preferential learning, based on dynamic reputation changes, strongly promotes cooperation regardless of the interaction network's structure. The mechanism responsible for more favorable evolutionary outcomes is enhanced network reciprocity, which

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leads to more compact cooperator clusters and thus to more robust spatiotemporal dynamics that are resilient to invading defectors. Our research may improve the understanding of selection patterns that favor the emergence and persistence of cooperative behavior.

Keywords: evolutionary game theory

Contents

1. Introduction

With the fast development of science and technology, human society cannot operate well without a high degree of cooperation [1]. However, there is a cost associated with cooperation, and this has to be paid for others to benefit. This is at odds with the Darwinian theory and thus creates one of the great puzzles of nature [2]. Why should we cooperate? Why should an organism perform a costly action that benefits somebody else? The answer is, of course, in the common good, which eventually also yields benefits to those that cooperate. And in knowing that the alternative is a pointless existence, action without worthwhile long-term prospects will dominated by defection and selfishness. Obviously, therefore, individual intelligence, or some form of collective intelligence is needed for cooperation, as observed in humans, bees, ants, and indeed in many other organisms. Nonetheless, the theoretical search for mechanisms and reasons that favor cooperation is still ongoing and is, indeed, a vibrant field of research across the social and natural sciences, not excluding physics.

J. Stat. Mech. (2021) 013403

Evolutionary game theory has supplied a universal framework to clarify how cooperation emerges, maintains and spreads among selfish individuals [3, 4]. Within this framework, various famous game models including, but not limited to, the prisoner's dilemma game (PDG) [5, 6], the snowdrift game (SDG) [7, 8] and the public goods game (PGG) [9–12], have been commonly used to address various social dilemmas and make fruitful achievements [13–16], especially in the field of artificial intelligence [17, 18]. In particular, the pioneering work by Nowak and May [19] and the fast development of complex networks [20, 21] have further stimulated the investigation of evolutionary games using square lattice [14, 22], small-world [23, 24], and scale-free [25, 26] networks, although a previous work [27] demonstrated that a square lattice might inhibit the evolution of cooperation in the SDG. In contrast to well-mixed interactions, cooperators can spontaneously organize into compact clusters to fight against the invasion of defectors. In doing so, cooperation survives, even in social-dilemma games, which is the hallmark of network reciprocity [19]. In addition, other available mechanisms have also been proposed, including alliance [28, 29], aspiration [30], memory effect [31], punishment [32], tolerant [33] and asymmetric [34], to name but a few. During the past few decades, the related progress in the evolution of cooperation has been summarized in several comprehensive reviews [35–37].

In addition, among most of the previous studies mentioned above, when the strategy is updated, it is generally assumed that the focal individual randomly singles out one of his nearest neighbors to imitate. Nevertheless, in reality, due to the heterogeneity of the neighbors' influence on a focal individual, the rational individual usually shows certain preferences when they perform strategy imitation. For instance, students with better academic performance often have their behavior learned by others, meanwhile colleagues with higher-performing achievements tend to be imitated, and so on. In previous studies, Wu *et al* [38] integrated an adjustable preferential learning rule into a spatial PDG for the first time, wherein the probability of a neighbor being selected was positively correlated with the number of times that his strategy was imitated by the focal individual. Wang and Perc [39] found that cooperation could be improved by increasing the probability that the fittest individual would be chosen to refer to, and further verified the robustness under various conditions. Based on the degree of difference of Barabási–Albert scale-free and Erdös–Rényi network structures, Huang *et al* [40] came up with another preferential selection rule for the PDG to study the impact of preferential choice on cooperation, and found that the level of evolutionary cooperation is closely associated with the degree of difference between the selected neighbor and the focal individual. Wu *et al* [41] further studied the impact of social-influence preferences on a spatial PGG. Inspired by dynamic social relationships, Sun *et al* [42] introduced a novel preferential-learning model into a spatial PDG, and discovered that the proposed model was able to enhance cooperation with different kinds of network structures. More recently, Wang *et al* [43] devised a preferential model based on payoff satisfaction and memory to research their influence on cooperation in several networks.

It is worthy of note that indirect reciprocity [44, 45] is a significant mechanism for resolving the tragedy of commons. Under this mechanism, cooperative individuals will accumulate a good image score [46] through helping others in previous game rounds, so as to be very likely to get help from others in the near future in turn. Thus, the

J. Stat. Mech. (2021) 013403

level of cooperation can still be dramatically improved, even without the assumption that cooperative individuals are asked to repetitively interact with each other. Hence, one can conclude that individual reputation [47–51], which is closely related to one's social behavior, plays a pivotal role in facilitating cooperation. To be specific, during past interactions, the reputation of an individual will gradually increase if they always adopt a cooperative strategy, and decrease if they often betray others. In reality, individuals with high reputation will attract the attention of others, which is in line with the requirement for rational individuals to pursue long-term interests. Furthermore, each individual within structured populations may have his own understanding of social dilemmas, which gives rise to the concept of multigames [28, 52, 53]. However, up until now, there have been no works on reputation preference rules in the environment of spatial multigames. Thus, we attempt a deep investigation of the impact of reputation preferences on cooperation in spatial multigames. In detail, each individual will be assigned an additional reputation attribute in addition to their strategy and fitness attributes, that is, every individual has a reputation value (i.e. an image score), which can be dynamically adjusted according to his behavior during the process of evolution. Given that the pro-social (or anti-social) behavior is generally positive (or negative), an individual's reputation value will be increased (or decreased) by one unit Δ when they adopt a strategy of cooperation (or defection). Meanwhile, to explore the influence of antisocial behavior [54, 55] on evolutionary cooperation, we also talk about the opposite situation, that is, cooperation is denied while defection is affirmed. At the end of each game round, each individual can accurately acquire the information about his neighbors' payoff and reputation information to update his current strategy for the next game round. Instead of randomly choosing neighbors to compare payoffs, every individual is supposed to have a certain amount of rationality, so that the neighbor with the highest reputation can preferentially be selected as the target of strategy imitation with certain preferences. Here, it is noted that our research is different from [44], which emphasizes that individuals with high reputation are more likely to get others' help. Although our study is similar to [56], there is also a minor difference, that is, the latter stresses that prestige-seeking behavior is costly, but can be culturally transmitted; while our study focuses on the imitation of strategies influenced by reputation information.

The remaining sections of this paper are organized as follows. In section 2, the Monte Carlo simulation (MCS) process of the reputation–preference model of spatial multigames will be amply introduced. Following that, we analyze and discuss the related results in section 3. Finally, we further give a summary of the conclusions and point out the future outlook in section 4.

2. Mathematical model

In this section, we will describe the multigames model with reputation preference and the experimental design in detail, which principally covers the following main sub-steps within one basic MCS procedure: (i) each individual has a pure strategy (cooperation or defection) for taking part in the game where there is more than one social dilemma; (ii)

J. Stat. Mech. (2021) 013403

each individual gains an accumulated payoff as his fitness by interacting with hisdirect neighbors according to his own payoff matrix; (iii) each individual has the opportunity to update his current strategy through imitating the strategy of a neighbor who is selected based on reputation preferences.

2.1. Initial setup

Within this research, the model is mainly conducted on a $N = L \times L$ square lattice with periodic boundary conditions and von Neumann neighbors, that is, $\langle k \rangle = 4$, where each node can be only occupied by an individual and the links stand for interactive relationships. Meanwhile, the robustness of this model is also tested using other potential interaction networks including a square lattice with a Moore neighborhood, a small-world network generated by a square lattice with a rewiring probability of 0.03 [57, 58], and a random regular graph [58, 59] where $\langle k \rangle$ is 4 in the small-world and random regular graphs.

Initially, every individual is randomly designated to adopt either a strategy of cooperation (C) or defection (D). In addition, all individuals are also randomly divided into two categories at the initial stage, i.e. type A (with a proportion of τ) and type B (with a proportion of $1 - \tau$), respectively, in which the former takes part in the PDG, while the latter participates in the SDG. During the game process, all individuals must simultaneously make decisions for payoffs without knowing the information of the opponent. In detail, when two individuals encounter, they will obtain a reward R (punishment P) for mutual cooperation (defection). However, if they choose different strategies, the defector reaps the reward of the temptation to defect, T while the cooperator endures the sucker's payoff, S. In the PDG, the sequence of payoff satisfies $T > R > P > S$ and $2R > T + S$, which signifies that defection is always the best choice. Thus, the social dilemmas are quite strong in the PDG. However, in the case of the SDG, the order could be readjusted to $T > R > S > P$, which indicates that the optimal action for an individual strongly relies on the opponent's action, that is, the optimal strategy is defection if the rival adopts cooperation, and vice versa. Hence, the social dilemmas are relatively palliative in SDG. The difference of the payoff order makes it clear that the individuals playing the PDG or the SDG have a corresponding understanding of social dilemmas. To be specific, if the sucker's payoff is positive, the individual can participate in SDG; while if the sucker has a negative return, the individual will take part in PDG. Thus, the environment of multigames can be controlled by the positive or negative values of the sucker's payoffs, in which individuals participating in PDG or SDG can play the game with the following matrixes, as in [28, 52, 53],

$$M_{\mathrm{PD}} = \begin{pmatrix} 1 & -\delta \\ b & 0 \end{pmatrix} \quad \text{or} \quad M_{\mathrm{SD}} = \begin{pmatrix} 1 & +\delta \\ b & 0 \end{pmatrix} \tag{1}$$

where the parameter b $(1 < b < 2)$ represents the intensity of the social dilemmas. Without any doubt, the parameter $\delta \in (0, 1)$ controls the type of game. When $\delta = 0$, only the traditional weak PDG is drawn into the system. For $\delta \neq 0$, the system enters the environment of multigames no matter what value of τ is. Here, we must emphasize that the situation is that when different types of individuals encounter, the type of game that they play is determined by the type of the focal individual. It also should be noted that the type of an individual will not change after having been set at the initial process of the evolution. Furthermore, each individual x will be assigned an integer R_x to indicate the information of his reputation, which can also be called the image score. Moreover, all individuals' reputations R_x uniformly and randomly fall into the interval $[R_{\min}, R_{\max}]$ at the initial stage, where $R_{\min}$ (or $R_{\max}$) denotes the potentially lowest (or highest) reputation value.

2.2. Fitness and reputation

According to an individual's own game type, and regardless of the opponent's one, a random individual x being selected as the focal individual calculates his payoffs through playing the games with one of his direct neighbors (say y), which can be denoted as π_{xy}. During each round of the game, after interacting with all of his neighbors, the randomly selected individual x takes the accumulated payoff as his fitness F_x, which can be summed up according to,

$$F_x = \sum_{y \in \Gamma_x} \pi_{xy} \tag{2}$$

where Γ_x is the group of the nearest neighbors of x.

Next, when individual x has completed a round of games, his reputation R_x will be dynamically adjusted according to his current strategy. In detail, if an individual x adopts a strategy of cooperation (defection), his reputation R_x will be increased (decreased) by one unit Δ at the end of this round. It should be noted that individual $x's$ reputation can be evaluated by a third-party institution and then communicated to all the others. Thus, the reputation of an individual has the same value for any neighbors around x. Moreover, considering that assessment is usually within a certain range, we assume that the individual $x's$ reputation has a maximum $R_{\max}$ and a minimum $R_{\min}$, according to previous works [60–62], that is, the value of the reputation R_x can only be adjusted within the given interval. To be specific, during the process of evolution, if individual $x's$ reputation reaches the maximum $R_{\max}$ (or the minimum $R_{\min}$), then the value of the reputation will not change, even if x continues to adopt a cooperative (or defective) strategy, that is, the maximum reputation cannot exceed $R_{\max}$, meanwhile the minimum must not be lower than $R_{\min}$, which can be described by the following equation,

$$R_x(t+1) = \begin{cases} R_x(t) + \Delta, & s_x = C \\ R_x(t) - \Delta, & s_x = D \end{cases} \tag{3}$$

where Δ is a non-negative integer if not clearly specified and indicates the amount of reputation adjustment. It is worth pointing out that we also talk about the impact of anti-intellectual social interaction on cooperation, that is, the situation where $\Delta < 0$.

2.3. Strategy updating

The strategies adopted by individuals influence their reputation, and in return, the alteration of their reputation can ultimately have an impact on their behavioral characteristics with a modification of imitation preferences on account of reputation. Generally, an individual's high reputation is accumulated by his pro-social behavior, however, one's reputation is bound to be low if one always adopted a strategy of defection during past rounds. In the real world, for any rational individual, selecting a target to imitate often reflects certain preferences. Thus, the focal individual x will refer to an individual y from his direct nearest neighbors to imitate with the following probability,

$$\psi_{xy} = \frac{R_y(t)}{\sum_{z\in\Gamma_x} R_z(t)} \tag{4}$$

where the sum $\sum_{z\in\Gamma_x} R_z(t)$ runs over all the nearest neighbors of individual x.

After neighbor y is selected, his fitness F_y can be accumulated by the focal individual x. The system enters the strategy-updating phase. If and only if their strategies are different, x will have the opportunity to learn the strategy of y according to the Fermi rule [14],

$$W = \frac{1}{1 + \exp[(F_x - F_y)/K]} \tag{5}$$

where K characterizes irrational choices. Here, it should be emphasized that whether the focal individual x imitates the selected neighbor y's strategy is strongly related to their payoffs' difference, not on the type of game. It is worth mentioning that a random number q is generated in the uniform distribution interval $(0, 1)$; if $q < W$, then x will accept $y's$ strategy; otherwise, x will discard $y's$ strategy.

2.4. The Monte Carlo method

So far, a full MCS is finished when all the aforementioned sub-steps are implemented. The consequences are mainly acquired on a square lattice a network size of $L = 200$ and run for at least 5×10^4 full MCS steps. The key quality characterizing the abundance of cooperators (ρ_c), which is averaged over the last 5×10^3 MCS steps after the initial 4.5×10^4 MCS steps are discarded, is acquired at the stationary stage. In addition, to avoid the finite-size effect, other lattice sizes, e.g. smaller $(L = 100)$ or larger $(L = 400)$, are also measured and qualitatively identical results are confirmed. Therefore, within this research, the results are only shown for lattices with a size $L = 200$. Moreover, all simulations adopt an asynchronous strategy-update scheme. Furthermore, to demonstrate the robustness of our model, we also take advantage of a square lattice with a Moore neighborhood, small-world [57, 58] and random regular graphs [58, 59] as potential network interactions to verify the robustness of the proposed model, where $\langle k\rangle$ is 4 in the small-world and random regular graphs. It needs to be pointed out that all of the final results are averaged over at least 20 autocephalous initial states to ensure the precision. Additionally, the relevant results of the study are obtained at $K = 0.1$, $\tau = 0.5$, $\mathrm{MCS} = 5 \times 10^4$, $R_{\min} = 1$, $R_{\max} = 100$, $\Delta = 1$, and $\delta = 0.3$ if not explicitly pointed out.

J. Stat. Mech. (2021) 013403

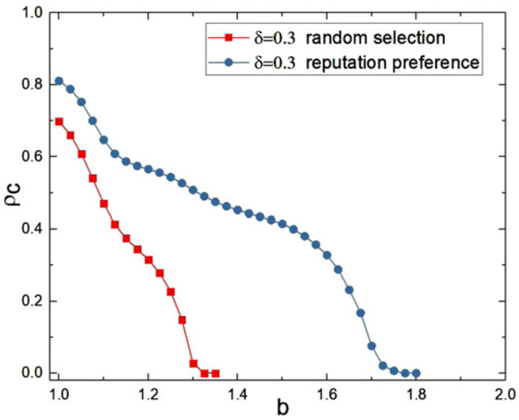

Figure 1. The dependence of the fraction of cooperators ρ_c on the temptation to defect b, as obtained with reputation preference (blue circles) and without reputation preference (red squares), both at $\delta = 0.3$. It can be observed that the inclusion of reputation preference yields significantly better results.

3. Results

In this section, we comprehensively investigate the influence of reputation preferences on evolutionary cooperation in spatial multigames from many aspects, which mainly include a preliminary inquiry, the asymmetry of the payoff matrix, reputation adjustment, reputation distribution, a $b - K$ phase diagram and the robustness of the model.

3.1. Impact of reputation preferences

Before formally exploring the influence of reputation preferences on evolutionary cooperation in spatial multigames, we first compare the results for the model with reputation preferences with those obtained by random selection in figure 1, hoping to gain a preliminary understanding of the effectiveness of reputation preferences. Significantly, we point out that the results obtained by random selection are consistent with a previous work [28]. In particular, there is no doubt that a larger b means stronger social dilemmas in the system. Thus, it is difficult for cooperation to be maintained and to spread for larger values of b. In general, although the abundance of cooperators ρ_c continuously declines with an increase of b in both cases, the evolution of cooperation with reputation preferences is always much higher than that gained by random selection. For instance, when $b = 1.2$, ρ_c is about 0.315 in the absence of reputation preferences, while the corresponding value is 0.566 in the situation with reputation preferences; that is, the latter is about 79.7% larger than the former. In fact, there is no direct relationship between reputation preferences and fitness during the process of evolution, but reputation preferences can significantly affect the method of strategy imitation, which is the main reason for promoting cooperation. According to our model, cooperative strategy can increase an individual's reputation, while a defector's reputation will be damaged, and thus the mechanism of reputation preferences increases the likelihood that cooperators will be imitated, which leads to the spread of cooperative strategies in the system.

As we all know, cooperators form tight clusters to fight against an invasion of defectors with the help of spatial reciprocity [19]. Reputation preferences can further enhance spatial reciprocity. Moreover, the trend for a nonlinear decline of ρ_c is more obvious with an increase of b after reputation preferences are introduced into the system. All these preliminary results show that, on the one hand, reputation preferences can dramatically enhance the level of cooperation in spatial multigames; on the other hand, they also make evolutionary dynamics more complex.

3.2. Effects of payoff asymmetry

Compared with the traditional random-selection model, we have preliminarily observed that reputation preferences have a better performance in improving cooperation, which stimulates our enthusiasm for further exploration. In our reputation-preference model of spatial multigames, positive or negative values of δ determine payoff asymmetry, even though the proportions of individuals taking part in the PDG or the SDG are equal. Furthermore, the greater the δ, the higher the payoff asymmetry. In general, the level of cooperation will be strongly affected if there is asymmetry in the system. Thus, we will investigate the impact of payoff asymmetry on evolutionary cooperation in spatial multigames with reputation preferences, although it has been well discussed in previous work [52].

Figure 2 depicts that the abundance of cooperators ρ_c varies with b for several δ. On the whole, regardless of any value of δ, ρ_c nonlinearly and monotonically declines with an increase of b. As individuals participating in the PDG or the SDG are uniformly distributed within the structured populations, it is important to emphasize that the average payoff matrix of the system is equivalent to the traditional weak PDG for any $\delta > 0$ when the numbers of individuals in both types of game are equal, i.e. $\tau = 0.5$. Furthermore, the system is properly the traditional weak PDG when $\delta = 0$. However, for any δ, the results of the proposed model are quite different from those of the traditional weak PDG. For instance, when $\delta = 0$, the value of b_c, indicating the threshold where cooperators completely disappear, is 1.0375 in the traditional weak PDG [22], in which the neighbor is randomly selected as the target for strategic imitation. However, from figure 2, we can find that the corresponding value of b_c is 1.4 in the model with reputation preferences. Moreover, for any $\delta > 0$, we can further visually observe that the threshold b_c increases with an increase of δ, which is attributed to the joint effect of reputation preferences and the payoff asymmetry of multigames. However, one can find an interesting phenomenon, that is, when b is in a specific range, the abundance of cooperators ρ_c hardly changes with an increase of b, especially for $\delta = 0.9$, which seems to be counterintuitive and in contradiction to the fact that ρ_c gradually decreases with an increase of b. Although the exact reasons behind these counterintuitive appearances are not exactly clear to us, we speculate that they may be the result of the combination of reputation preferences and the spatial multi-game environment, which can make evolutionary dynamics much more complex and affect the direction of its evolution of cooperation. All these results indicate that, with the mechanism of

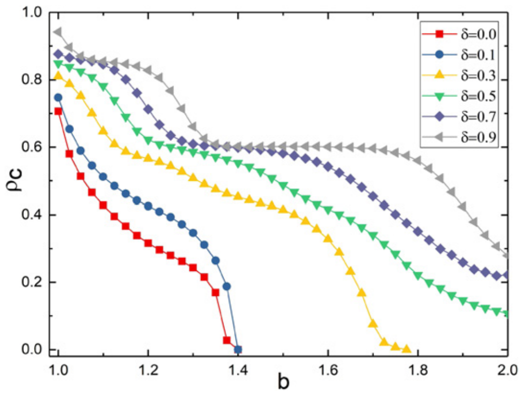

Figure 2. The dependence of the fraction of cooperators ρ_c on the temptation to defect b, as obtained for different values of the δ parameter that determines whether the multi-game environment is used ($\delta > 0$) or not ($\delta = 0$). It can be observed that the higher the value of δ, the better cooperation fares.

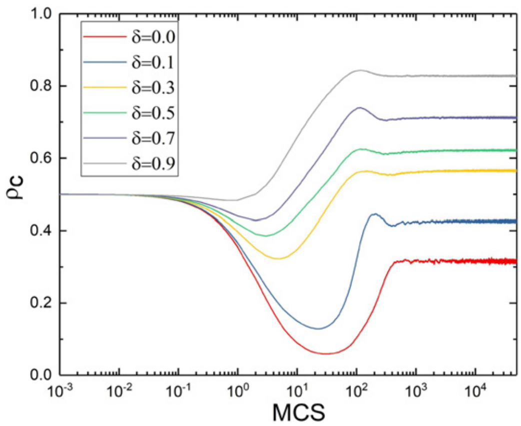

Figure 3. Time series of the fraction of cooperators ρ_c, as obtained for different values of δ that determines whether the multi-game environment is used ($\delta > 0$) or not ($\delta = 0$), for the temptation to defect $b = 1.2$. The red curve ($\delta = 0.0$) corresponds to the traditional weak PDG, while the other cases correspond to $\delta > 0$, thus giving rise to the ever more pronounced multi-game setup. These results support those shown in figure 2.

reputation preferences, not only does payoff asymmetry continue to be effective in promoting the evolution of cooperation, but its effectiveness is further enhanced, and that the evolutionary dynamics are also profoundly changed.

Then, to further understand the details of the evolutionary cooperation, we depict the time series of ρ_c for several given values of δ in figure 3. As the previous work shows, the full evolutionary time series can be further separated into two parts according to the changing trend of ρ_c, that is, the enduring (END) stage and the expanding (EXP) stage [63]. It can be clearly observed that ρ_c continuously declines during the END stage, regardless of the value of δ, which is due to the fact that there are the

J. Stat. Mech. (2021) 013403

social dilemmas in the system are too strong when $b = 1.2$, leading to the cooperators being forced to undergo the exploitation of defectors. However, the larger the δ, the less obvious the downward trend of ρ_c, which signifies that payoff asymmetry, with the help of reputation preferences, can restrain the declining tendency for cooperation. Nevertheless, after that stage, the evolutionary process enters the EXP stage with an increase of ρ_c, and the disadvantage of the cooperators will be quickly reversed, which means that the cooperators may have broken through the close encirclement of the defectors. Even when $\delta = 0$, the case of traditional weak PDG, a lot of cooperators still exist in the system rather than disappearing over time. Furthermore, when $\delta > 0$, the greater the δ, the higher the level of evolution of cooperation. All these results can be attributed to the role of reputation preferences.

Although we have tentatively concluded that the level of cooperation can be further enhanced with the help of reputation preferences, and the larger the value of δ, the higher the level of cooperation, this does not provide any information about the actual spatial arrangement of strategic types. Thus, to capture more details about the impact of reputation preferences in promoting evolutionary cooperation, it is indispensable to scrutinize the microscopic strategic distribution of cooperation or defection on the square lattice by collecting some representative snapshots at several key time steps, which are shown in figure 4. From left to right, the MCS steps are 0, 50, 200 and 5×10^4, respectively. From top to bottom, the values of δ correspond to 0.1, 0.3, 0.7 and 0.9, respectively. In addition, to capture how cooperative the cluster is, we also calculate the conditional probability q_{CC} according to [69],

$$q_{CC} = p_{CC}/\rho_c \tag{6}$$

where p_{CC} is the density of cooperative links. By comparing ρ_c and q_{CC}, we can get more information on the evolution of cooperative clusters. To be specific, if $q_{CC} > \rho_c$, then a cooperator is more likely to be clustered with other cooperative individuals than randomly distributed, that is, assortment is positive. Furthermore, the more q_{CC} is larger than ρ_c, the more obvious the aggregation effect is. If $\rho_c = q_{CC}$, then a cooperative individual randomly encounters other players, in other words, there is no assortment. Otherwise, if $\rho_c > q_{CC}$, then there is negative assortment.

As we have mentioned before, cooperating individuals (green) and defecting ones (red) are randomly and uniformly distributed on the square lattice, and for any δ, one can find that there are no obvious differences between q_{CC} and ρ_c when MCS = 0. However, for different values of δ, there are clear differences in the evolution of strategies. When $\delta = 0.1$ (the first line in figure 4), due to the fact that there are the strong social dilemmas in the system when $b = 1.2$, defectors are in an advantageous position, so that defectors have unparalleled advantages over cooperators and some cooperative clusters even disappear, which directly reduces the number of cooperators once the system begins to evolve. Fortunately, cooperators can form tight clusters to protect themselves from further attack by defectors, which can be intuitively reflected by the paired values (ρ_c, q_{CC}). One can find that the values of (ρ_c, q_{CC}) are (0.16, 0.60) when MCS = 50, that is, q_{CC} is 3.75 times as much as ρ_c. Afterwards, as time evolves, cooperators can fight back against the defectors and gradually expand their territories until a stable state is reached. Although the abundance of cooperators ρ_c increases, one can find that the

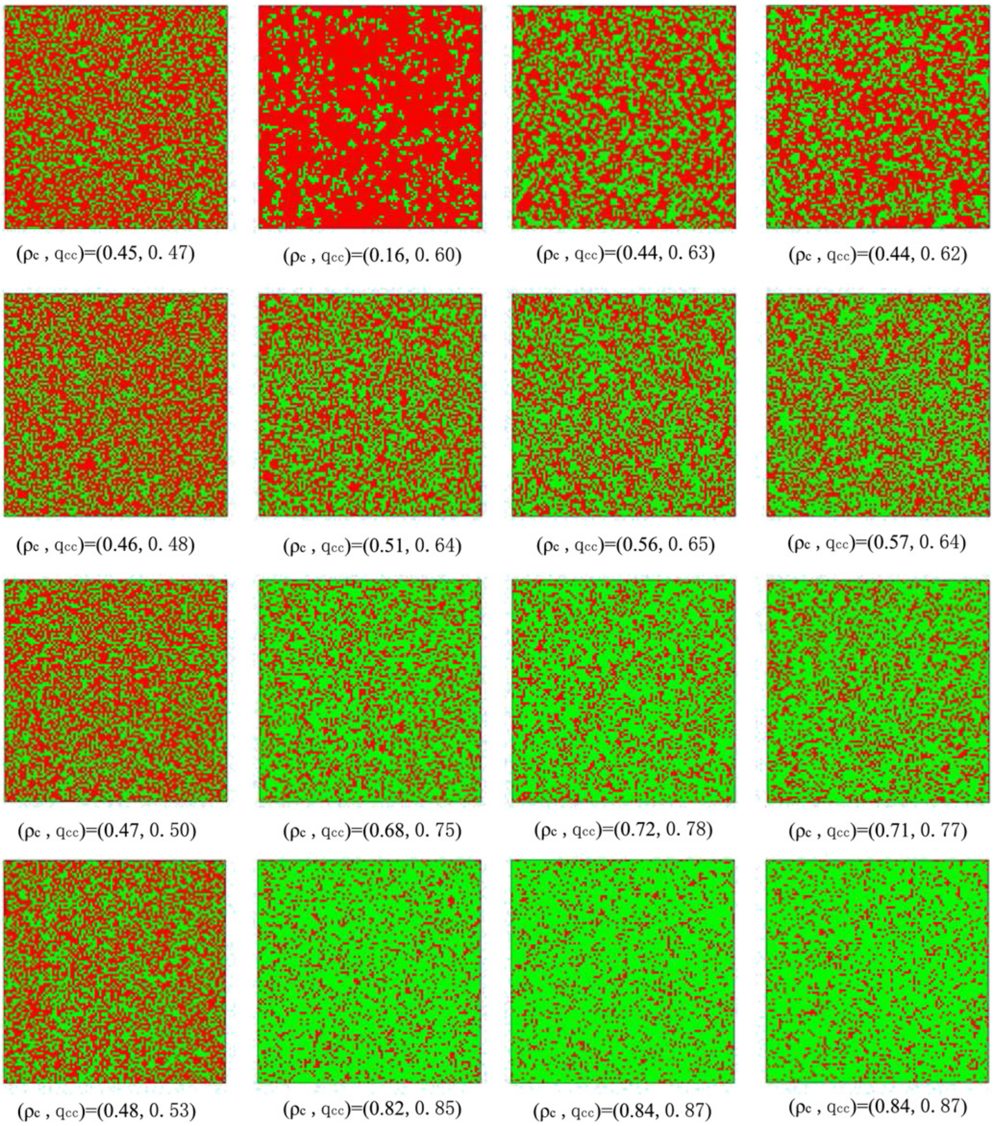

Figure 4. Representative snapshots of the spatial distributions of cooperators (green) and defectors (red), as obtained for different values of δ when $b = 1.2$. From the top to the bottom the values of δ are 0.1, 0.3, 0.7 and 0.9, respectively. The number of MCS in the columns from left to right are 0, 50, 200, and 5×10^4, respectively. Moreover, to get more information on the evolution of cooperative clusters, we also calculate the conditional probability q_{CC} and further compare q_{CC} and ρ_{c}. If $q_{\mathrm{CC}} > \rho_{\mathrm{c}}$, then a cooperative individual is more likely to be clustered with other cooperators than randomly distributed, i.e. assortment is positive.

comparative advantage between q_{CC} and ρ_{c} has been significantly decreased, and q_{CC} is only 1.4 times as much as ρ_{c} when $\mathrm{MCS} = 5 \times 10^4$, which indicates that cooperators try their best to break through the encirclement, while defectors fight hard, and finally they are evenly matched. Furthermore, with an increase of δ, even if the abundance of cooperators ρ_c is increased bit by bit, the difference between q_{CC} and ρ_c gets narrower and narrower. For instance, when the system is in a stable state, the values of (ρ_c, q_{CC}) are (0.57, 0.64) for $\delta = 0.3$, and the corresponding values are (0.71, 0.77) for $\delta = 0.7$, while the same values are (0.84, 0.87) for $\delta = 0.9$. All these phenomena may be attributed to the following reasons: cooperators can survive and spread through network reciprocity, and moreover, network reciprocity becomes stronger and stronger with an increase of δ. In detail, pairs of cooperators (C–C) can be mutually supported for steady payoffs while the relationship between defectors (D–D) is fragile due to the lack of mutual payoffs. In addition, for any individual, their reputation will be gradually enhanced (diminished) by cooperation (defection). Therefore, with the help of reputation preferences, the cooperators on the C–D boundary will be likely to be selected as strategic imitators, which further facilitates the spread of cooperation, that is, reputation preferences further enhance network reciprocity. However, the system has very strong social dilemmas when $b = 1.2$, and thus for any value of δ, although there are more and more cooperators emerging with the increase of δ, there are still some stubborn defectors being scattered among the sea of cooperators, which prevents the cooperators from gathering further. On the one hand, although reputation preferences and payoff asymmetry can enhance network reciprocity to some extent, the strong social dilemmas make it impossible for cooperators to completely crush defectors. On the other hand, the optimal strategy is still defection (D), even when an individual participating in the SDG encounters a PDG-type neighbor adopting the cooperative strategy, (C).

3.3. role of reputation adjustment

In previous works on reputation [47–49], it is usually assumed that reputation adjustment $\Delta = 1$. Therefore, it is significant to investigate the role of reputation adjustment in improving evolutionary cooperation. Considering that the reputation adjustment caused by one's strategic behavior may be discontinuous, but not jump, we set $\Delta \leqslant 10$. In particular, for the sake of studying the role of reputation preferences on anti-intellectual societies, the value of Δ could be less than 0. The abundances of cooperators ρ_c varying with b for several considered values of Δ are plotted in figure 5. When $\Delta = 0$, the current system degenerates into the quenched-reputation model, which signifies that all individuals' reputations are unaffected by their behavior. The level of cooperation is lower than that under any case of $\Delta > 0$, which further verifies that reputation preferences can promote cooperation from another aspect. It is known that pro-social behavior is a product of our evolution as a species, as well as our unique capacity to internalize norms of social behavior [64]. Yet there are inevitable antisocial phenomena caused by impaired recognition and lack of cognitive skills. Thus, it is necessary to investigate the role of reputation preferences in the situation of anti-intellectual societies [54, 55], that is, where prosocial cooperative behavior is negatively evaluated, while antisocial defector behavior is positively evaluated. Herein, when $\Delta = -2$, this corresponds to the case of antisocial behavior. At this moment, ρ_c is overall not only less than that for the case of $\Delta = 0$ but also lower than that in the situation of the random selection of neighbors in figure 1, which means that anti-intellectual societies can hinder the spread of cooperation. The results warn us that antisocial behaviors do hinder the evolution

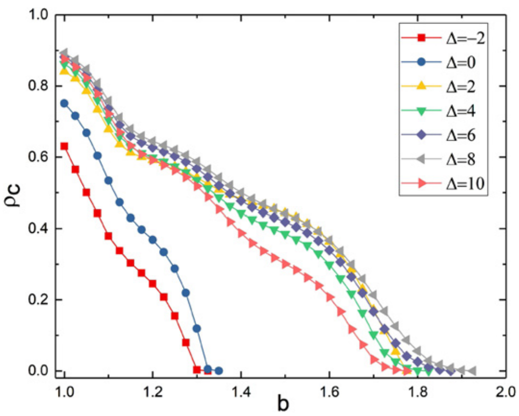

Figure 5. The dependence of the fraction of cooperators ρ_c on the temptation to defect b, as obtained for different values of Δ that determines the step in the reputation dynamics. It can be observed that there is a non-monotonous dependence of cooperation on the value of Δ. There is an intermediate value of Δ, at which cooperation fares best.

of cooperation, and we should actively evaluate prosocial behavior to facilitate the proliferation and spread of cooperation. Based on these observations, we conclude that reputation adjustment also plays a vital role in promoting cooperation when reputation preferences are present.

Furthermore, some modest values of Δ, leading to the best ideal scenario for cooperation, may exist in the system, as seen in figure 5. To quantify the influence of reputation adjustment Δ on improving cooperation more accurately, we investigate the variation of cooperator abundance ρ_c with reputation adjustment Δ for a given b in figure 6. Although with an increase of Δ, the change of ρ_c is not so obvious, we can still intuitively observe that the evolution of cooperation is at the highest level when $\Delta = 7$. Through the above analyses, under the mechanism of reputation preferences, we have captured the evolution of cooperation being affected by both payoff asymmetry and reputation adjustment. Thus, to fully understand the impact of reputation preferences on cooperation in spatial multigames, figure 7 gives a color-map encoding of ρ_c on the $\sigma - \tau$ parameter plane for several given values of b and Δ. By comparing the case of $\Delta = 1$ with $\Delta = 7$, it is distinctly found that the levels of cooperation in the latter cases are much higher than in the former for any given value of b, which further verifies our previous discussion regarding figure 6. Moreover, for any fixed τ, the larger the δ, the higher the evolution of cooperation, which will not be repeated here as it has been analyzed at length in the above discussion. Also, for any fixed δ, the larger the value of τ, the lower the evolution of cooperation. It is not difficult to understand these phenomena. The larger τ means that more individuals are involved in the PDG, which indicates that there are stronger social dilemmas in the system. Therefore, under reputation preferences, the phenomenon that multigames represent, depending on δ and τ, can affect the level of cooperation, which is significantly beyond that warranted by network reciprocity alone, and the level of cooperation can be further enhanced if Δ is appropriately increased.

J. Stat. Mech. (2021) 013403

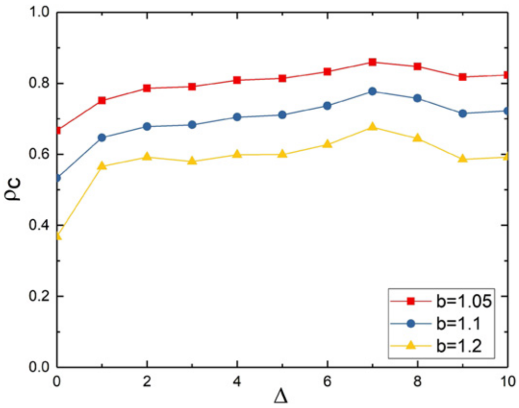

Figure 6. The dependence of the fraction of cooperators ρ_c on the step in the reputation dynamics Δ, as obtained for different values of the temptation to defect b. It can be observed that there is indeed a non-monotonous dependence of cooperation on the value of Δ, although weakly expressed.

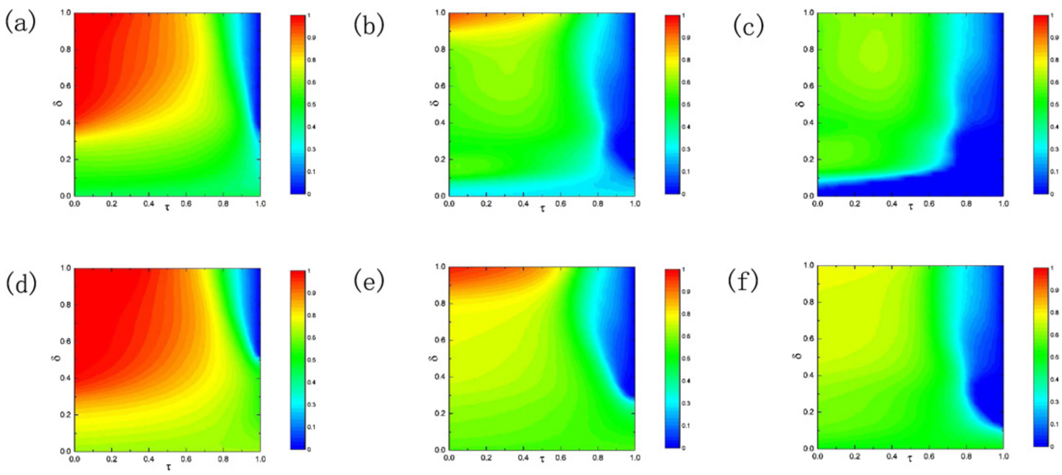

Figure 7. Color-coded fraction of cooperators ρ_c on the $\delta - \tau$ (τ is the fraction of individuals assigned to a different game of the multi-game setup) parameter plane, as obtained for different values of the temptation to defect b and the step in the reputation dynamics Δ. (a)–(c) Obtained for $\Delta = 1$, while (d)–(f) were obtained for $\Delta = 7$. The corresponding values of b are 1.1, 1.3 and 1.4 in the columns from left to right, respectively.

It must be pointed out that it is a fact that the effects of memory decay cannot be ignored. Herein, we introduce the memory-decay effect into the model of reputation preferences, which can be described as [47],

$$R_x(t+1) = \begin{cases} \beta * R_x(t) + \Delta, & s_x = C \\ \beta * R_x(t) - \Delta, & s_x = D \end{cases} \tag{7}$$

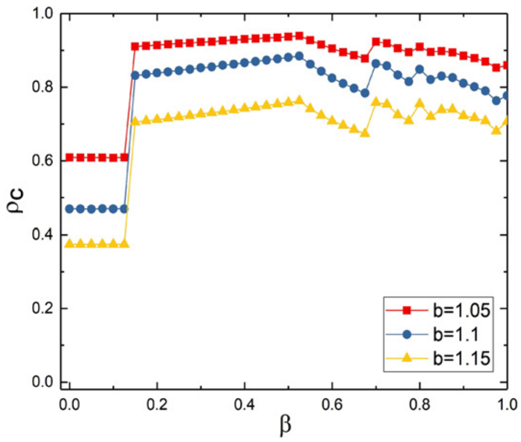

Figure 8. The dependence of the fraction of cooperators ρ_c on the memory decay rate β, as obtained for different values of the temptation to defect b and $\Delta = 7$. It can be observed that there is a non-monotonous dependence of cooperation on the value of β.

where β is an adjustable parameter indicating the rate of memory decay. Evidently, for $\beta \to 0$, this means that an individual's reputation fully depends on the current game result, but for $\beta \to 1$, it returns to our primary model. To explore its impact, we present the relevant results in figure 8. It is expressly noted that the evolution of cooperation goes through a non-monotonous change. To be specific, when β is less than about 0.12, the abundance of cooperators hardly changes, which means that individuals' reputation may mainly be affected by the last round. However, if β exceeds this value, as β increases, the abundance of cooperators ρ_c suddenly leaps forward and then reaches an optimal level at about $\beta = 0.5$. After that value, ρ_c shifts to a gradual decline amid turbulence. Hence, we can confirm that there is a modest value of β that gives rise to an optimal level of cooperation. To have a more comprehensive comprehending of β, we further present the color-map coding of ρ_c on the $b - \beta$ parameter plane for several given values of δ in figure 9. It can clearly be found that there are similar phenomena of a sudden increase of cooperation with an increase of β for any condition of δ. In addition, the moderate value of β inducing an optimal level of cooperation exists in all panels.

Under reputation preferences, the influence of reputation adjustment on the evolutionary cooperation in spatial multigames is due to the premise that the reputation value of an individual cannot suddenly change. However, one may wonder what happens to evolutionary cooperation if the previous assumption is broken. For this purpose, we not only enlarge the reputation adjustment, but also amplify the maximum reputation, R_x, and change the initial distribution of reputation, which is presented in figure 10. To be specific, the result of panel (a) is obtained in the model that we set up. As control groups, all individuals' reputations R_x uniformly and randomly fall into the interval $[R_{\min}, R_{\max}]$ at the initial stage and $R_{\max} = 1000$ in panel (b), while the reputations of all individuals are set to be 1, i.e. $R(0) = 1$ at the initial stage and $R_{\max} = 100$ in panel (c). By examining the results in figure 10, it can be clearly found that ρ_c fluctuates

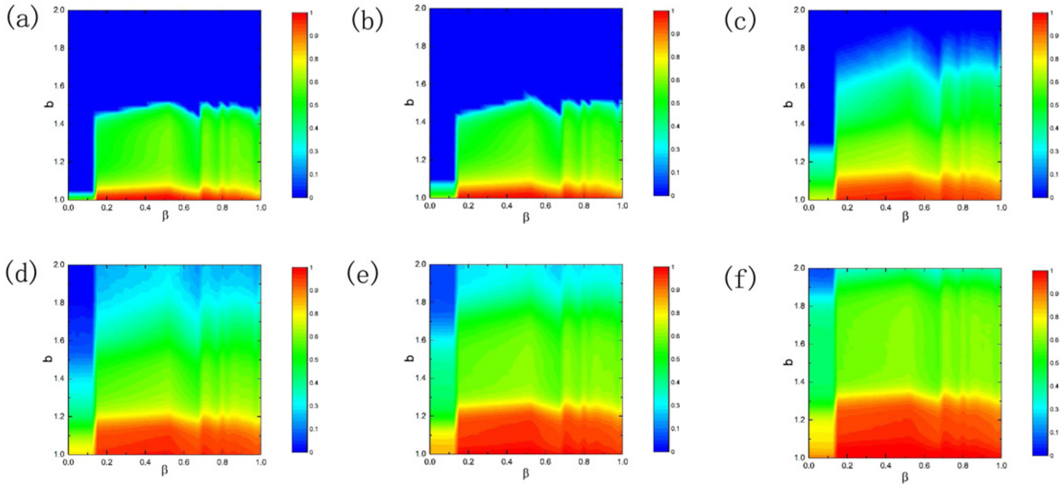

Figure 9. Color-coded fraction of cooperators ρ_c on the $b-\beta$ parameter plane, as obtained for different values of δ for the step in the reputation dynamics equaling $\Delta = 7$. (a)–(c) Obtained for $\Delta = 1$, while (d)–(f) were obtained for $\Delta = 7$. (a)–(f) The values of δ correspond to 0, 0.1, 0.3, 0.5, 0.7 and 0.9, respectively.

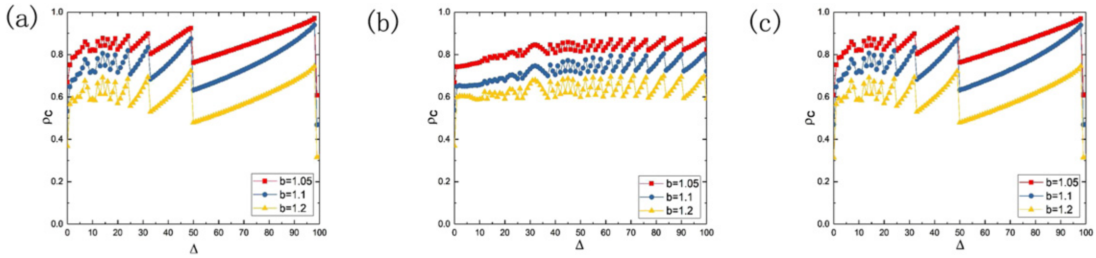

Figure 10. The dependence of the fraction of cooperators ρ_c on the step in the reputation dynamics Δ, as obtained for different values of the temptation to defect b. All the individual reputations R_x uniformly and randomly fall into the interval $[R_{\min}, R_{\max}]$ at the initial stage in (a) and (b), while the reputations of all individuals are set to 1, i.e. $R(0) = 1$ at the initial stage. According to the previous setting, we set $R_{\min} = 1$. Furthermore, $R_{\max}$ is set to 100, 1000, and 100 in (a)–(c), respectively.

aperiodically with an increase of Δ in all sub-figures. According to the proposed model, an individual's reputation can be dynamically adjusted within the given interval. Thus, under the mechanism of reputation preferences, individuals may misjudge the behavior of their neighbors based on reputation preferences if the reputation adjustment is too large. Moreover, the results in figure 10(a) are similar to the ones in figure 10(c), but different from the ones in figure 10(b), which illustrates that the difference in the trend of oscillation is only related to the maximum value of reputation, $R_{\max}$, but has no connection with the initial distribution.

J. Stat. Mech. (2021) 013403

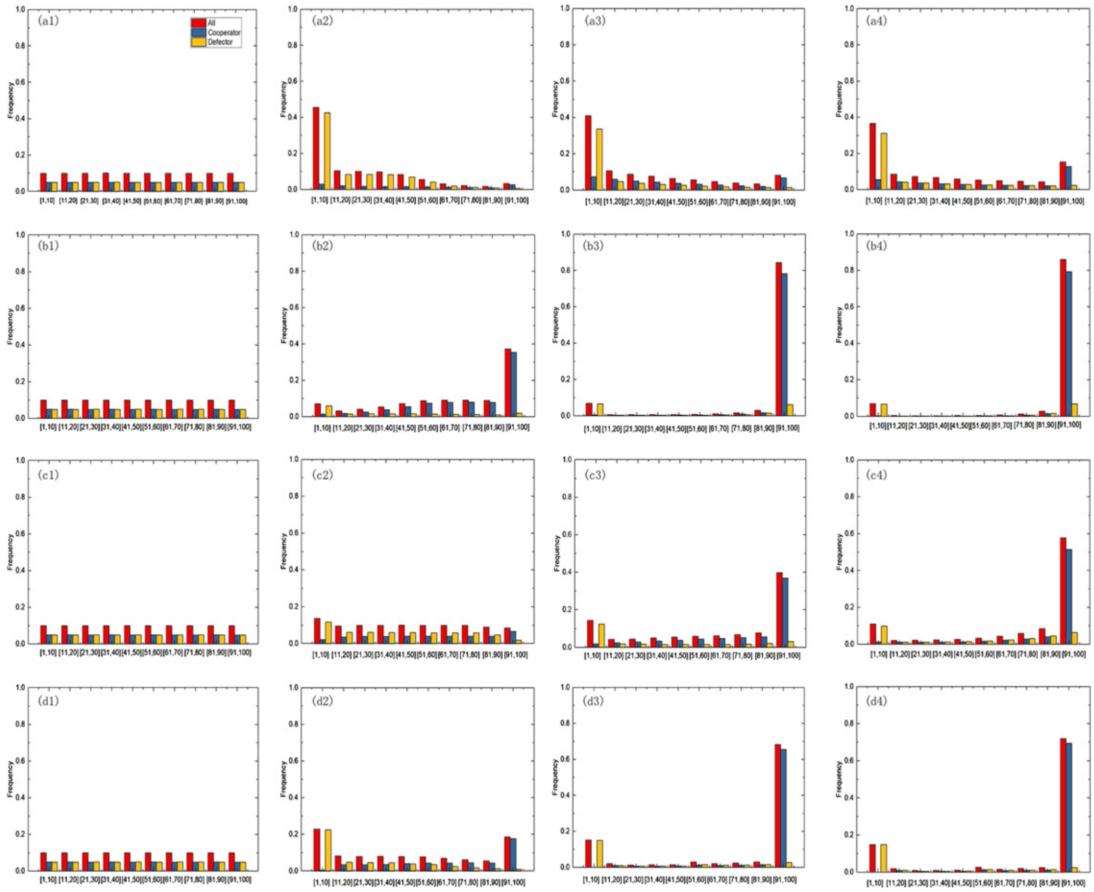

Figure 11. Statistics of the reputation distribution within different subintervals, as obtained for several pivotal evolutionary steps under different conditions. (a1)–(a4), (b1)–(b4) Obtained with $\delta = 0.1$ and $\delta = 0.9$ at $\Delta = 1$ and $b = 1.2$. The number of MCSs in the columns from left to right are 0, 50, 200, and 5×10^4, respectively. (c1)–(c4), (d1)–(d4) Obtained for $\Delta = 2$ and $\Delta = 8$ at $\delta = 0.3$ and $b = 1.1$. The number of MCSs in the columns from left to right are 0, 5, 50, and 5×10^4, respectively.

3.4. Reputation distribution and the $b - K$ phase diagram

From the aforementioned discussions, it can be distinctly confirmed that the cooperative patterns are profoundly changed after introducing reputation preferences into the system, i.e. information about reputation plays a vital role in promoting evolutionary cooperation in spatial multigames. Accordingly, a statistical analysis of the reputation distribution provides an insight into the relationship between reputation distribution and the level of evolutionary cooperation. We display the distribution of reputation values of the total population in figure 11, where the entire interval $[R_{\min}, R_{\max}]$ can be further equally split into ten subintervals. From figure 11, for any given case, due to the fact that every individual randomly adopts a strategy of cooperation or defection strategy and all individuals' reputations are randomly and uniformly distributed within every subinterval at the initial stage, individuals within various subintervals almost hold

J. Stat. Mech. (2021) 013403

the same proportions in spite of cooperators or defectors. To be specific, the abundance of individuals is about 0.1, the abundance of cooperators is about 0.05 and the abundance of defectors is about 0.05 in every subinterval at the initial stage. However, for different given conditions, the evolution of reputation distribution is quite different over time. For instance, we can capture this phenomenon by comparing the case of $\delta = 0.1$ with $\delta = 0.9$ for the fixed value of $\Delta = 1$. For $\delta = 0.1$, when MCS = 50, a large number of individuals gather in subintervals of low reputation and are almost defectors, most of which are in the sub-interval of [1, 10], as illustrated in (figure 11(a2). Meanwhile, the abundance of cooperators ρ_c is in a trough (see figure 3). As evolution proceeds, the dominance of low-reputation individuals does not fundamentally change, although the proportion of high-reputation individuals gradually increases, which corresponds to the low level of cooperation. However, the situation is completely different for $\delta = 0.9$. Once the system begins to evolve, individuals converge to the sub-interval of high reputation; in particular, most of them are cooperators, which corresponds to a sustained improvement in the evolution of cooperation. A similar situation can also be observed for the cases of $\Delta = 2$ and $\Delta = 8$ for the fixed value of $\delta = 0.3$. Thus, we can conclude that the level of cooperation strongly goes hand in hand with the distribution of reputation. Meanwhile, low-reputation individuals are almost defectors while high-reputation ones are almost cooperators.

It is well known that the parameter K characterizes the irrationality of an individual in the process of strategic imitation. For $K \to 0$, individuals will imitate the strategy of the selected neighbor with full certainly. For $K \to \infty$, all information will be lost, so that individuals adopt strategies by means of a tossed coin [14]. The effect of K on evolutionary cooperation has been discussed in depth in the PDG and the PGG [39, 60, 65]. However, as far as we know, there are no previous works involved in investigating the impact of K on strategic imitation in spatial multigames. To scrutinize the influence of irrationality on strategic learning, the full $b - K$ phase diagrams under different scenarios are illustrated in figure 12. For comparison, the cases of the PDG without reputation preferences (figure 12(a)) and with reputation preferences (figures 12(d) and (g)) and the conditions of the multigames without reputation preferences (figures 12(b) and (c)) as control groups are also presented. There are three representative states, such as pure-C, coexistence of C and D, and pure-D, that emerge in all of these phase diagrams, in which the red line and the blue curve are the boundaries of these states. It should be pointed out that the range of b can be relaxed, i.e. $b < 1$ and $b > 2$. We first compare the phase diagrams without reputation preferences in the first row of figure 12. From figure 12(a), we can clearly find a bell-like border separating the pure D and the C–D coexistence phases, except for the monotonous increasing boundary between the pure-C and C–D coexistence phases, indicating that there is an optimal level of irrationality ($0.2 < K < 0.3$) improving cooperation, which has been well reported in [66]. This phenomenon can be only be observed for the interaction topologies lacking overlapping triangles [67], which can be explained as an evolutionary resonance. In the environment of multigames without reputation preferences, the phase diagrams in figures 12(b) and (c) show that the pure-C and C–D coexistence areas are enlarged, which means that the multi-game environment can promote cooperation. Interestingly, once reputation preferences are introduced into the system, the

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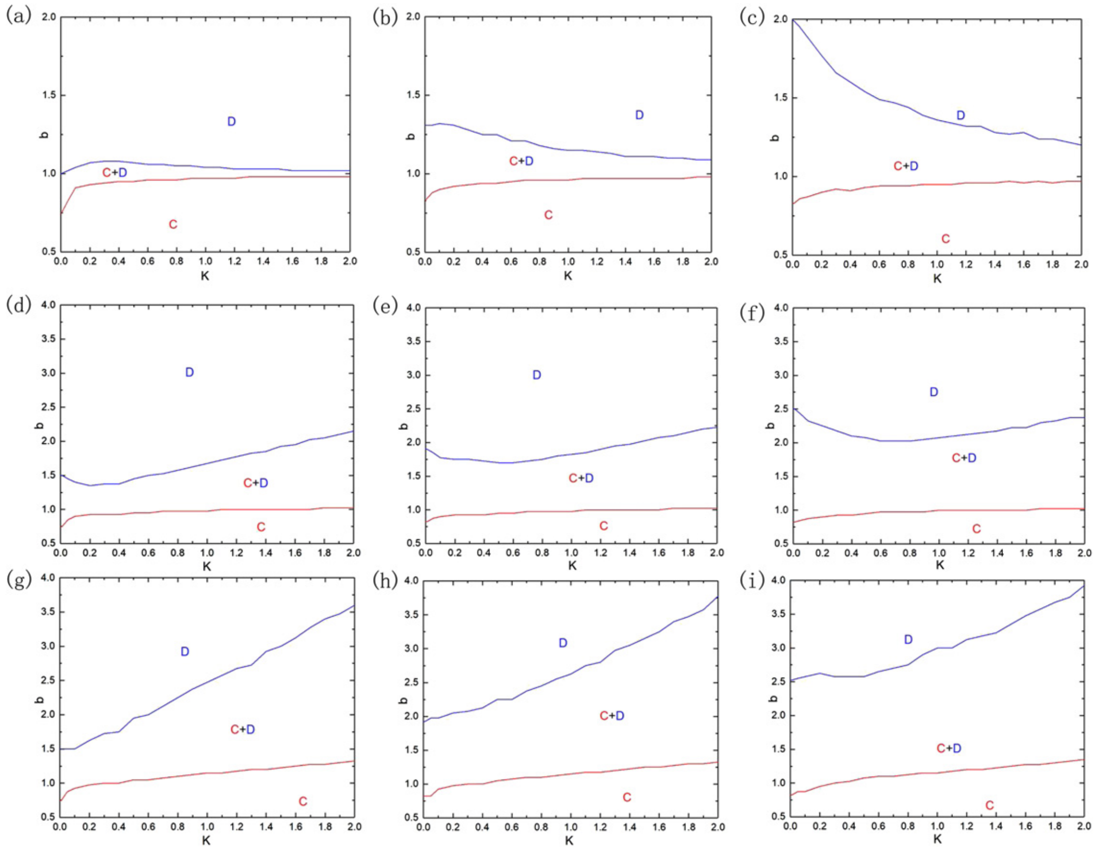

Figure 12. Full $b - K$ phase diagrams. (a)–(c) Obtained without reputation preferences. (d)–(f), (e)–(i) Obtained with $\Delta = 1$ and $\Delta = 7$, respectively. For all cases, we refer to the main text for details.

phase diagrams can be drastically changed even though the trend of the boundary between pure-C and C–D coexistence does not significantly change, as illustrated in the second and third rows in figure 12. On the one hand, the area of pure-C and C–D coexistence has expanded; on the other hand, the trend of the boundary of pure-D and C–D coexistence is obviously different from the case of random choice of strategic imitation. To be specific, when $\Delta = 1$ (the second row), one can find that phase diagrams appear as an inverted bell-shaped $D \leftrightarrow C + D$ transition line, as illustrated in figures 12(d)–(f), which are similar to a previous work, [39]. The simulation results indicate that reputation preferences eradicate the existence of an optimal temperature K, which means that there is a worst rather than an optimal K for the evolution of cooperation both for the PDG and for the multigames; furthermore, the phenomenon can be further enlarged if we appropriately augment the reputation adjustment for $\Delta = 7$ (the third row), as presented in figures 12(g)–(i). All these results indicate that the interaction network can be effectively changed to some extent after the introduction of reputation preferences into the system. It is known that a square lattice is obviously short of overlapping triangles and thus ensures the observation of an optimal temperature K. However, reducing the likelihood of who will act as the strategy source seems to

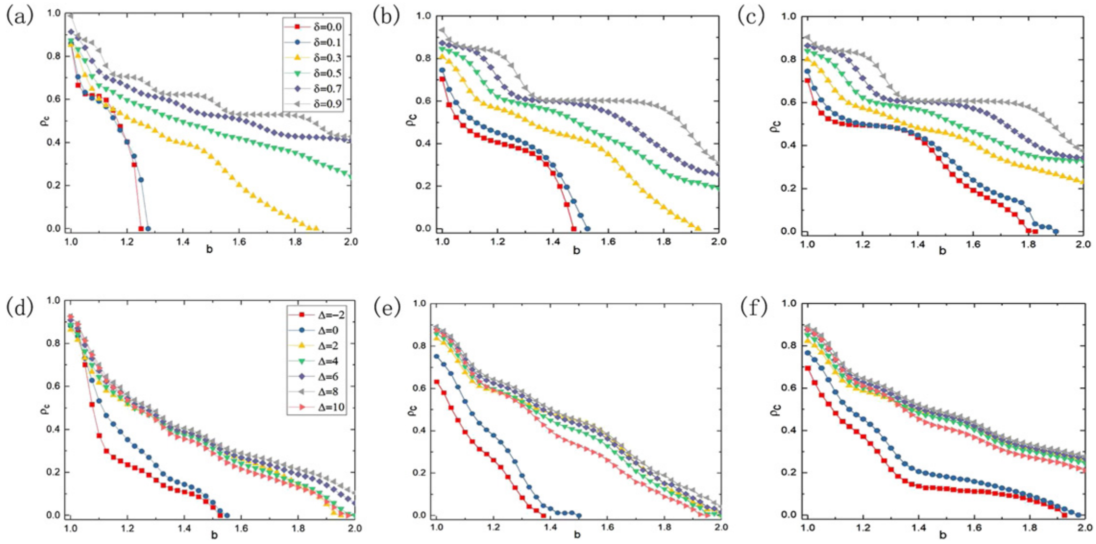

Figure 13. The dependence of the fraction of cooperators ρ_c on the temptation to defect b, as obtained from a square lattice with a Moore neighborhood (a), (d), small-world (b), (e) and random regular graphs (c), (f). It should be noted that $\langle k \rangle$ is 4 in the small-world and random regular graphs. (a)–(c) Results for different values of δ at a fixed $\Delta = 1$, while (d)–(f) show results for different values of Δ at a fixed $\delta = 0.3$. It can be observed that the main results reported above are robust to variations in the structure of the interaction network.

effectively strengthen the association among essentially unrelated triplets, thereby ruling out the same observation. It is noted that a similar observation can also be found in the PGG [60], since joint membership in large groups alters effective interaction networks and thus the effects of indeterminacy on evolutionary cooperation [67]. Taken together, this proves that the introduction of reputation preferences can alter the irrationality of individuals, further increasing the probability that cooperative neighbors are selected to imitate, and ultimately improving evolutionary cooperation.

3.5. Robustness of the main results

Finally, validating the robustness of the reputation preferences on the other potential network topologies is indispensable. Herein, we further investigate how the influence of reputation preferences affects the evolution of cooperation on a square lattice with a Moore neighborhood, a small-world network and a random regular graph in figure 13. It should be noted that k is 4 in the small-world and random regular graphs. For any given network topology, the related results are qualitatively analogous to the observations in figure 2 or figure 5. When we enlarge the payoff asymmetry by increasing the value of δ, cooperation can be gradually improved, which can be discovered in figures 13(a)–(c). Moreover, the larger the δ, the higher the evolutionary cooperation. When we increase the reputation adjustment Δ appropriately over a small scale, we also find that there is an optimal value of Δ supporting the best level of cooperation, which is illustrated in figures 13(d)–(f). These results suggest that reputation preferences are a broadly fruitful

J. Stat. Mech. (2021) 013403

way to sustain and promote cooperation in spatial multigames, regardless of the form of the potential network structure.

4. Conclusions

In brief, a novel model of reputation preferences has been proposed to probe the key question regarding the emergence, maintenance, and spread of cooperation in spatial multigames. In detail, except for gaining payoffs as fitness, each individual's reputation was dynamically adjusted according to his adopted strategy during the evolutionary process, which led to heterogenous influence. Then, by selecting one of his neighbors y according to reputation preferences, the focal individual x changed his current strategy with the Fermi rule for the next round. Through plenty of numerical simulations, we have analyzed the role of reputation preferences in improving evolutionary cooperation in spatial multigames from diversiform views. Under reputation preferences, the promotion effect of payoff asymmetry caused by δ on cooperation was further strengthened. In particular, the greater the value of δ, the stronger the payoff asymmetry, which induces a higher abundance of cooperators ρ_c. Additionally, when we increase the reputation adjustment Δ appropriately over a small scale, the evolution of cooperation can be further enhanced. Moreover, the memory-decay effect cannot be ignored, and there exists a modest value of forgetfulness, β leading to an optimal level of cooperation. Furthermore, we also explain the reason why the reputation adjustment cannot be too large. If we broaden the value of reputation adjustment Δ, the abundance of cooperators ρ_c presents aperiodic oscillation with an increase of Δ. By comparing different conditions for the distribution of reputation, there is no doubt that the difference of the phenomenon is only related to the reputation maximum $R_{\max}$, and has no connection with the initial distribution. In addition, although individuals' strategies and their reputations are randomly and uniformly distributed at the initial stage, it is clearly shown that the cooperative behavior can be highly correlated with the distribution of reputation once the system enters the evolutionary stage. Meanwhile, low-reputation individuals are almost defectors, while high-reputation ones are almost cooperators. Through a full $b - K$ phase diagram, we can confirm that the introduction of reputation preferences can change the irrationality of the individual. Furthermore, the robustness of the proposed model is also verified using other networks.

In terms of future work, given that group interactions make the game more complicated, it would be significant to investigate the impact of the model on the PGG. In addition, for the sake of analysis, we assume that individuals can always accurately identify their opponents' strategies and that there are no execution errors during the process of evolution. However, on the one hand, such information about the opponents' strategies is sometimes wrong [68, 70] or inaccessible [71]; on the other hand, individuals may fail to cooperate (or defect) even when they intend to adopt a strategy of cooperation (or defection) [72]. Thus, it makes sense to include the information failure or execution error in the mechanism of reputation preferences in the future. Furthermore,

J. Stat. Mech. (2021) 013403

it may also be useful to combine the vacant space with reputation to study the evolution of cooperation. Meanwhile, in our proposed model, we declare that an individual's reputation is assessed and communicated to all other individuals by a third-party institution, and thus an individual's reputation has the same score for any other neighbors around him. In the real world, an individual's reputation can be evaluated differently by his neighbors, which will make different neighbors have heterogeneous evaluations of his reputation. This phenomenon is also worth studying in the future, especially considering the case of reputation preferences.

Although there are also some limitations in our simplest model, the current results are especially inspiring, since reputation heterogeneity is ubiquitous in reality. Henceforth, our model can be expected to offer some valuable clues for future works to resolve the tragedy of the commons prevalent in realistic environments.

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