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Evolutionary dynamics of cooperation in the public goods game with pool exclusion strategies

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Abstract Social exclusion is widely used as a control mechanism to promote cooperative behavior in human societies. However, it remains unclear how such control strategies actually influence the evolutionary dynamics of cooperation. In this paper, we introduce two types of control strategies into a population of agents that play the public goods game, namely prosocial pool exclusion and antisocial pool exclusion, and we use the replicator equation to study the resulting evolutionary dynamics for infinite well-mixed populations. We show that the introduction of prosocial pool exclusion can stabilize the coexistence of cooperators and defectors by means of periodic oscillations, but only in the absence of second-order prosocial pool exclusion. When considering also antisocial pool exclusion, we show that the population exhibits a heteroclinic circle, where cooperators can coexist with other strategists. Moreover, when second-order exclusion is taken into account, we find that prosocial pool exclusion is the dominant strategy, regardless of whether the second-order exclusion is prosocial or antisocial. In comparison with punishment, we conclude that prosocial pool exclusion is a more effective control mechanism to curb free-riding.

Keywords Evolutionary dynamics of cooperation · Pool exclusion · Public goods game · Replicator equation

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1 Introduction

Survival of the fittest is the survival rule of nature and human society [1–14]. In order to survive and reproduce, naturally there will be deadly competition among species. However, cooperative or altruistic behavior in human society is a common phenomenon [15–25]. In a competitive environment, how to promote the emergence of cooperation has attracted considerable attention, although a series of mechanisms have been proved to promote cooperation, such as indirect reciprocity, kin selection, reputation, punishment, and reward [26–36]. As an effective control tool to promote cooperation in the real world, costly punishment plays a decisive role in restraining free-riders by reducing their benefit at a cost to the performers. Previous theoretical and experimental studies based on the public goods game (PGG) have shown that individuals are willing to pay costs to punish non-cooperators [37–42]. On the other hand, however, the research community has begun to question the positive effects of costly punishment on cooperation [43–45]. When non-cooperators are allowed to punish cooperators which is called as antisocial punishment [46,47], the appearance of such punishment regime causes cooperation not be promoted and instead natural selection favors substantial levels of antisocial punishment in the system [48,49].

Social exclusion, as a stronger control means than costly punishment, has been widely used to promote the evolution of cooperation [50–55]. A series of theoretical researches found that social exclusion can regulate the number of beneficiaries by preventing free-riders from sharing public goods [51,52]. For example, Sasaki and Uchida focused on the incentive systems in which peer punishment and peer exclusion are, respectively, considered and revealed that such exclusion can overcome two substantial difficulties of costly punishment [51]. Subsequently, Li et al. [52] proposed another different exclusion regime, pool exclusion, to investigate the evolution of social exclusion in finite populations. They assumed that individuals can resort to a central control authority for expelling free-riders and all the excluders equally share the associated exclusion cost, and found that peer excluders can overcome pool excluders if the exclusion costs are small.

Note that pool excluders, similarly to pool punishers [56], should pay a fixed, permanent cost before contributing to the public pool to maintain an institutionalized mechanism for restraining free-riders [53]. Thus, the pool exclusion strategy considered in this work is different from previous one in Ref. [52], where exclusion costs are shared by all excluders. However, it remains unclear how pool exclusion strategy with such scenario influences the evolution of cooperation in infinite populations. On the other hand, almost all the previous studies only consider the prosocial exclusion strategies in a fair and reasonable exclusion rule, and thus completely ignore the influence of antisocial exclusion, i.e., non-cooperators exclude cooperators. Indeed, the existence of antisocial exclusion is inevitable in the real society. For example, social exclusion has been found to stimulate antisocial emotions [57–60]. Thus, if somebody who is beneficial to groups is excluded, what strategy will other individuals choose? Will the introduction of antisocial exclusion strategy result in negative effects on the evolutionary dynamics of cooperation just as the antisocial punishment strategies does [48,49]?

In this paper, we thus introduce two forms of control strategies for pool exclusion, that is, prosocial pool exclusion and antisocial pool exclusion, into a population of agents who play the public goods game, and aim to study the evolutionary dynamics of exclusion strategies in infinite well-mixed populations by using replicator equations [61–66]. Briefly, the main contribution of this paper can be summarized as follows. In our model, we introduce the prosocial pool exclusion strategy proposed in Ref. [53], under which pool excluders apply a permanent effort to maintain the sanctioning institutions, and prove that the introduction of prosocial pool exclusion can transform the public goods game into a rock–paper–scissors game. Furthermore, we consider the characteristic of applying a permanent cost for the antisocial pool exclusion strategy, and further propose the antisocial pool exclusion strategy accordingly. We demonstrate that the coexistence of the four strategies can appear. We also consider the second-order exclusion for prosocial pool exclusion and antisocial exclusion strategies, respectively. We find that the prosocial pool exclusion is the most advantaged strategy no matter whether the type of second-order exclusion is prosocial or antisocial.

The rest of this paper is organized as follows. In Sect. 2, we investigate the evolutionary dynamics of cooperation in the public goods game with the prosocial pool exclusion strategy. In Sect. 3, we further incorporate the antisocial pool exclusion into the population system. In Sect. 4, we consider the second-order exclusion for prosocial pool exclusion strategy. And in Sect. 5, we introduce antisocial pool exclusion strategy and also consider the second-order exclusion for both prosocial and antisocial exclusion strategies. Finally, conclusions are drawn in Sect. 6.

2 Prosocial pool exclusion without second-order exclusion

2.1 Modeling

We assume that in an infinitely large, well-mixed population, $N > 2$ players are chosen randomly and form a group to play a one-shot PGG. In the PGG, a cooperator (C) contributes a fixed cost c to the common pool, whereas a defector (D) does not contribute anything.

The sum of the contributions is multiplied by the synergy factor r ($1 < r < N$) and then equally allocated among all individuals who participate in public goods game irrespective of their contribution to the common pool. If all the individuals contribute to the common pool, then everyone in the group can obtain the largest benefit $c(r-1)$. However, if nobody contributes, the group can get nothing.

Then, we introduce a control strategy, namely the prosocial pool exclusion, into the game. Pool excluders not only contribute c to the game, but also, beforehand, a permanent cost, δ_P, which is a control parameter, to the exclusion pool. In the absence of second-order exclusion, only defectors are excluded by pool excluders, and then, they get nothing. It needs to be pointed out that the excluded defector is different from loner who is just watching the game but can get a constant payoff from PGG [56,64]. Besides, the control effect of the exclusion strategy is not the same as costly punishment, and the difference lies in whether defector can share the public goods.

Accordingly, we have three following strategies in the game: cooperators (C) who contribute to the PGG but not to the exclusion pool, prosocial pool excluders (EC) who contribute to both the PGG and prosocial exclusion pool, and defectors (D) who contribute to neither the PGG nor the exclusion pool. Then, using replicator equations, we study the evolutionary dynamics for the cooperators (C), defectors (D), and prosocial pool excluders (EC), with frequencies x, y, and z, respectively. Here $x, y, z \geq 0$ and $x + y + z = 1$. The expected payoffs of these three strategies can be described by P_I, with $I = C, D$, or EC. Thus, the replicator equations [61] are written as

$$\begin{cases} \dot{x} = x(P_C - \bar{P}), \\ \dot{y} = y(P_D - \bar{P}), \\ \dot{z} = z(P_{EC} - \bar{P}), \end{cases} \tag{1}$$

where $\bar{P} = xP_C + yP_D + zP_{EC}$ represents the average payoff of the entire population.

Remark 1 Here, we assume perfect exclusion; namely, exclusion never fails. Thus, defectors can get nothing from the PGG if there exist prosocial pool excluders in the group. Then, each EC individual can get the payoff $rc - c - \delta_P$, which is positive.

Accordingly, the expected payoff of defectors is given by

$$\begin{aligned} P_D &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C}(1-x-z)^{N-N_C-1} \frac{rcN_C}{N} \\ &= (1-z)^{N-1} \frac{rc}{N}(N-1)\frac{x}{1-z}, \end{aligned} \tag{2}$$

where $\binom{N-1}{N_C} x^{N_C}(1-x-z)^{N-N_C-1}$ describes the probability of finding the $N-1$ coplayers with N_C cooperators, $N - N_C - 1$ defectors, and no excluders.

And the expected payoff of cooperators can be given by

$$\begin{aligned} P_C = &\sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C}(1-x-z)^{N-N_C-1} \quad \frac{rc(N_C+1)}{N} \\ &+ \sum_{N_{EC}=1}^{N-1} \sum_{N_C=0}^{N-1-N_{EC}} \binom{N-1}{N_C}\binom{N-1-N_C}{N_{EC}} x^{N_C} z^{N_{EC}} \\ &\times (1-x-z)^{N-N_C-N_{EC}-1} rc - c \\ = &(1-z)^{N-1} \frac{rc(1-z+Nx-x)}{N(1-z)} \\ &+ rc[1-(1-z)^{N-1}] - c, \end{aligned} \tag{3}$$

where the first term on the right side represents the benefit of cooperators in the group without prosocial pool excluders, the second term denotes the benefit of cooperators when there exist prosocial pool excluders, and N_{EC} denotes the number of EC individuals in the group.

After taking some calculations, the average payoffs for cooperators, defectors, and prosocial pool excluders can be, respectively, written as

$$P_C = rc - c - (1-z)^{N-1} \frac{rc(N-1)y}{N(1-z)}, \tag{4}$$

$$P_D = (1-z)^{N-1} \frac{rc(N-1)x}{N(1-z)}, \tag{5}$$

$$P_{EC} = rc - c - \delta_P, \tag{6}$$

where $(1-z)^{N-1}$ denotes the probability that there is no excluder in the $N-1$ coplayers.

Remark 2 Because of $z = 1 - x - y$, the equations of system (1) can be rewritten as

$$\begin{cases} \dot{x} = x[(1-x)(P_C - P_{EC}) - y(P_D - P_{EC})], \\ \dot{y} = y[(1-y)(P_D - P_{EC}) - x(P_C - P_{EC})], \end{cases} \tag{7}$$

where

$$P_C - P_{EC} = \delta_P - (x+y)^{N-1} \frac{rcy(N-1)}{N(x+y)}, \tag{8}$$

$$P_D - P_{EC} = (x+y)^{N-1}\frac{rcx(N-1)}{N(x+y)} - rc + c + \delta_P. \quad (9)$$

Solving $P_C = P_D$ results in

$$z = 1 - \left[\frac{N(r-1)}{r(N-1)}\right]^{\frac{1}{N-1}}. \quad (10)$$

Similarly, by solving $P_C = P_{EC}$, we have

$$y = \frac{\delta_P\left[\frac{N(r-1)}{r(N-1)}\right]^{\frac{1}{N-1}}}{(r-1)c}. \quad (11)$$

Theorem 1 *For $1 < r < N$ and $\delta_P < rc - c$, system (1) or (7) has a unique interior equilibrium point $(x, y, z) = ([\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}} - \frac{\delta_P[\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}}}{(r-1)c}, \frac{\delta_P[\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}}}{(r-1)c}, 1-[\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}})$. In addition, there are three vertex fixed points, namely $(x, y, z) = (1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.*

Proof See Remark 2.

Then, we investigate the dynamics on each edge of the simplex S_3. On the edge $C - D$, we have $z = 0$, resulting in $\dot{y} = y(1-y)(P_D - P_C) = y(1-y)(c - rc/N) > 0$. Thus, the direction of the dynamics goes from C to D.

On the edge $D-EC$: since $x = 0$ and $y+z = 1$, we have $\dot{z} = z(1-z)(P_{EC}-P_D) = z(1-z)(rc-c-\delta_P) > 0$. Thus, the direction of the dynamics goes from D to EC.

On the edge $C-EC$: since $y = 0$ and $x + z = 1$, we have $\dot{x} = x(1-x)(P_C - P_{EC}) = x(1-x)\delta_P > 0$. Thus, the direction of the dynamics goes from EC to C. □

2.2 The stabilities of equilibria

Here we set that

$$f(x, y) = x[(1-x)(P_C - P_{EC}) - y(P_D - P_{EC})],$$
$$g(x, y) = y[(1-y)(P_D - P_{EC}) - x(P_C - P_{EC})].$$

Then, the Jacobian matrix of the system is

$$J = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} & \frac{\partial f(x,y)}{\partial y} \\ \frac{\partial g(x,y)}{\partial x} & \frac{\partial g(x,y)}{\partial y} \end{bmatrix}, \quad (12)$$

where

$$\begin{cases} \frac{\partial f(x,y)}{\partial x} = [(1-x)(P_C - P_{EC}) - y(P_D - P_{EC})] \\ \quad + x[-(P_C - P_{EC}) + (1-x)\frac{\partial}{\partial x}(P_C - P_{EC}) \\ \quad - y\frac{\partial}{\partial x}(P_D - P_{EC})], \\ \frac{\partial f(x,y)}{\partial y} = x\Big[(1-x)\frac{\partial}{\partial y}(P_C - P_{EC}) \\ \quad - (P_D - P_{EC}) - y\frac{\partial}{\partial y}(P_D - P_{EC})\Big], \\ \frac{\partial g(x,y)}{\partial x} = y\Big[(1-y)\frac{\partial}{\partial x}(P_D - P_{EC}) \\ \quad - (P_C - P_{EC}) - x\frac{\partial}{\partial x}(P_C - P_{EC})\Big], \\ \frac{\partial g(x,y)}{\partial y} = [(1-y)(P_D - P_{EC}) - x(P_C - P_{EC})] \\ \quad + y[-(P_D - P_{EC}) + (1-y)\frac{\partial}{\partial y}(P_D - P_{EC}) \\ \quad - x\frac{\partial}{\partial y}(P_C - P_{EC})\Big]. \end{cases} \quad (13)$$

Theorem 2 *In the condition of Theorem 1, all the three vertex equilibria are saddle points, and the interior equilibrium point is neutrally stable, surrounded by closed orbits.*

Proof (1) For $(x, y, z) = (0, 0, 1)$, the Jacobian is

$$J(0,0,1) = \begin{bmatrix} \delta_P & 0 \\ 0 & -rc + c + \delta_P \end{bmatrix}, \quad (14)$$

and thus, the fixed equilibrium is unstable.

(2) For $(x, y, z) = (1, 0, 0)$, the Jacobian is

$$J(1,0,0) = \begin{bmatrix} -\delta_P & -(\delta_P + c - rc/N) \\ 0 & c - rc/N \end{bmatrix} \quad (15)$$

thus, the fixed equilibrium is unstable.

(3) For $(x, y, z) = (0, 1, 0)$, the Jacobian is

$$J(0,1,0) = \begin{bmatrix} rc/N - c & 0 \\ -(rc/N - rc + \delta_P) & rc - c - \delta_P \end{bmatrix}, \quad (16)$$

and thus, the fixed equilibrium is unstable.

(4) For $(x, y, z) = ([\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}} - \frac{\delta_P[\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}}}{(r-1)c}, \frac{\delta_P[\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}}}{(r-1)c}, 1 - [\frac{N(r-1)}{r(N-1)}]^{\frac{1}{N-1}})$, we define the

equilibrium point as (x^*, y^*, z^*) hereafter, and the elements in the Jacobian matrix are written as

$$\begin{cases} \frac{\partial f}{\partial x}(x^*, y^*) = x^*\left[(1-x^*)\frac{\partial}{\partial x}(P_C - P_{EC}) - y^*\frac{\partial}{\partial x}(P_D - P_{EC})\right], \\ \frac{\partial f}{\partial y}(x^*, y^*) = x^*\left[(1-x^*)\frac{\partial}{\partial y}(P_C - P_{EC}) - y^*\frac{\partial}{\partial y}(P_D - P_{EC})\right], \\ \frac{\partial g}{\partial x}(x^*, y^*) = y^*\left[(1-y^*)\frac{\partial}{\partial x}(P_D - P_{EC}) - x^*\frac{\partial}{\partial x}(P_C - P_{EC})\right], \\ \frac{\partial g}{\partial y}(x^*, y^*) = y^*\left[(1-y^*)\frac{\partial}{\partial y}(P_D - P_{EC}) - x^*\frac{\partial}{\partial y}(P_C - P_{EC})\right], \end{cases} \tag{17}$$

where

$$\begin{cases} \frac{\partial}{\partial x}(P_C - P_{EC}) = -(x^*+y^*)^{N-3}rcy^*\frac{(N-1)(N-2)}{N}, \\ \frac{\partial}{\partial y}(P_C - P_{EC}) = -(x^*+y^*)^{N-3}rc(N-1)\frac{(N-1)y^*+x^*}{N}, \\ \frac{\partial}{\partial x}(P_D - P_{EC}) = (x^*+y^*)^{N-3}\frac{rc(N-1)}{N}[(N-1)x^*+y^*], \\ \frac{\partial}{\partial y}(P_D - P_{EC}) = (x^*+y^*)^{N-3}\frac{rcx^*(N-1)(N-2)}{N}. \end{cases}$$

Then, we define that $p = \frac{\partial f}{\partial x}(x^*, y^*)\frac{\partial g}{\partial y}(x^*, y^*) - \frac{\partial f}{\partial y}(x^*, y^*)\frac{\partial g}{\partial x}(x^*, y^*)$ and $q = \frac{\partial f}{\partial x}(x^*, y^*) + \frac{\partial g}{\partial y}(x^*, y^*)$. Thus, we have

$$\begin{aligned} p &= \frac{\partial f}{\partial x}(x^*, y^*)\frac{\partial g}{\partial y}(x^*, y^*) - \frac{\partial f}{\partial y}(x^*, y^*)\frac{\partial g}{\partial x}(x^*, y^*) \\ &= x^*y^*(1-x^*-y^*)\left[\frac{\partial}{\partial x}(P_C - P_{EC})\frac{\partial}{\partial y}(P_D - P_{EC}) - \frac{\partial}{\partial y}(P_C - P_{EC})\frac{\partial}{\partial x}(P_D - P_{EC})\right] \\ &= x^*y^*(1-x^*-y^*)(x^*+y^*)^{2N-4}r^2c^2(N-1)^3/N^2 > 0, \end{aligned}$$

and

$$\begin{aligned} q &= \frac{\partial f}{\partial x}(x^*, y^*) + \frac{\partial g}{\partial y}(x^*, y^*) \\ &= x^*(1-x^*)\frac{\partial}{\partial x}(P_C - P_{EC}) + y^*(1-y^*)\frac{\partial}{\partial y}(P_D - P_{EC}) \\ &\quad - x^*y^*\left[\frac{\partial}{\partial x}(P_D - P_{EC}) + \frac{\partial}{\partial y}(P_C - P_{EC})\right] \\ &= x^*y^*(x^*+y^*)^{N-3}rc(N-1)(N-2)[(y^*-x^*)+(1-y^*)-(1-x^*)]/N \\ &= 0. \end{aligned}$$

We have $q^2 - 4p < 0$ and $q = 0$; therefore, the eigenvalues of the Jacobian matrix corresponding to the interior equilibrium point are pure imaginary. The dynamics analysis of the interior of S_3 and the stability of interior fixed point can see the following subsection.

□

2.3 The Hamiltonian system

Theorem 3 *System (1) or (7) is a conservative Hamiltonian system.*

Proof We introduce a new variable $\varepsilon = \frac{x}{x+y}$, which represents the fraction of cooperators among members who do not contribute to the exclusion pool. This yields

$$\dot{\varepsilon} = \frac{xy}{(x+y)^2}(P_C - P_D) = -\varepsilon(1-\varepsilon)(P_D - P_C). \tag{18}$$

On the other hand, $\dot{z} = z(P_{EC} - \bar{P})$, where

$$\begin{aligned} \bar{P} &= xP_C + yP_D + zP_{EC} \\ &= x(P_C - P_D) + (1-z)(P_D - P_{EC}) + P_{EC}. \end{aligned} \tag{19}$$

Thus, we have

$$\dot{z} = z[x(P_D - P_C) - (1-z)(P_D - P_{EC})], \tag{20}$$

where

$$P_D - P_C = (1-z)^{N-1}\frac{rc(N-1)}{N} - rc + c, \tag{21}$$

$$P_D - P_{EC} = (1-z)^{N-1}\frac{rc(N-1)x}{N(1-z)} - rc + c + \delta_P. \tag{22}$$

Accordingly, the equation system becomes

$$\begin{cases} \dot{\varepsilon} = -\varepsilon(1-\varepsilon)[(1-z)^{N-1}\frac{rc(N-1)}{N} - rc + c], \\ \dot{z} = z(1-z)[rc - c - \delta_P - \varepsilon(rc-c)]. \end{cases} \tag{23}$$

The separability of the factors allows us to write

$$\frac{\mathrm{d}z}{\mathrm{d}\varepsilon} = \frac{z(1-z)}{(1-z)^{N-1}\frac{rc(N-1)}{N} - rc + c} \frac{rc - c - \delta_P - \varepsilon(rc-c)}{-\varepsilon(1-\varepsilon)},$$

such that

$$\int \frac{(1-z)^{N-1}\frac{rc(N-1)}{N} - rc + c}{z(1-z)}\mathrm{d}z = \int \frac{rc - c - \delta_P - \varepsilon(rc-c)}{-\varepsilon(1-\varepsilon)}\mathrm{d}\varepsilon. \tag{24}$$

The integral of the right-hand side is

$$(rc-c-\delta_P)[\log(1-\varepsilon) - \log(\varepsilon)] - (rc-c)\log(1-\varepsilon) = -\delta_P\log(1-\varepsilon) - (rc-c-\delta_P)\log(\varepsilon).$$

The integral of the left-hand side is

$$(rc-c)[\log(1-z) - \log(z)] + \frac{rc(N-1)}{N}\int \frac{(1-z)^{N-2}}{z}\mathrm{d}z,$$

where

$$\int \frac{(1-z)^{N-2}}{z}\mathrm{d}z = \int \frac{1 + \sum_{k=1}^{N-2}\binom{N-2}{k}(-z)^k}{z}\mathrm{d}z = \sum_{k=1}^{N-2}\binom{N-2}{k}(-1)^k\frac{z^k}{k} + \log(z) + Const. \tag{25}$$

In this way, we identify the constant of motion

$$H(\varepsilon, z) = -\delta_P\log(1-\varepsilon) - (rc-c-\delta_P)\log(\varepsilon) + (rc-c)[\log(1-z) - \log(z)] + \sum_{k=1}^{N-2}\binom{N-2}{k}(-1)^k\frac{z^k}{k} + \log(z). \tag{26}$$

Therefore, we have

$$\dot{H} = \frac{\partial H}{\partial \varepsilon}\dot{\varepsilon} + \frac{\partial H}{\partial z}\dot{z} = 0. \tag{27}$$

Thus, the system is conservative and all constant level sets of H correspond to closed curves. Besides, the interior fixed point is neutrally stable surrounded by those closed and periodic orbits (see [62,67] for details), which indicates that the introduction of the prosocial pool exclusion strategy can stabilize the coexistence of cooperators and defectors by forming periodic oscillations in the absence of second-order prosocial pool exclusion. □

2.4 Numerical example

Example 1 As seen in Theorems 1 and 2, for $r < N$ and $rc - c - \delta_P > 0$ the dynamics of system (1) or (7) have been presented in simplex S_3 (see Fig. 1b). We show that there are four fixed points, and three of them are the vertex equilibrium points, that is, all C ($x = 1$), all D ($y = 1$), and all EC ($z = 1$), corresponding to three vertices of the simplex S_3, respectively. Besides, there exists an interior equilibrium. The result suggests that when defectors can be excluded by prosocial pool excluders, C, D, and EC could coexist by forming a rock–paper–scissors circle [68,69]. As shown in Fig. 1a, time series of the frequencies of these three strategies reveal that the population follows periodic oscillations among C, D, and EC. And they correspond to the close circle shown in Fig. 1b.

3 Antisocial pool exclusion without second-order exclusion

3.1 Modeling

We further introduce a new control strategy, antisocial pool exclusion strategy (ED), into the model in Sect. 2. Cooperator will also contribute c to the common pool while defectors contribute nothing. The ED individuals do not contribute to the PGG but invest δ_A, which works as a control parameter, to the antisocial exclusion pool, and the EC players not only contribute c to the PGG but also invest δ_P to the prosocial exclusion pool. In the absence of second-order exclusion, we assume that they exclude cooperators as well as prosocial excluders since the latter also contributes to the common pool. As previously, in the absence of second-order exclusion, prosocial pool excluders exclude those who do not contribute to the PGG including defectors and ED individuals. Thus, there are four strategies

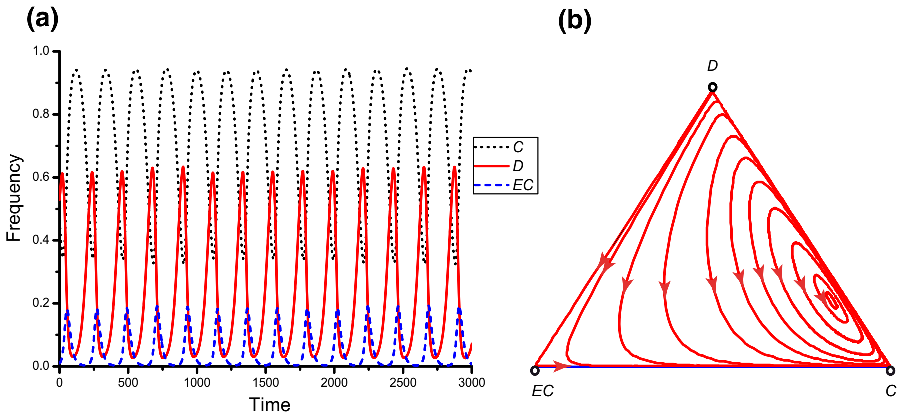

Fig. 1 Evolutionary dynamics of prosocial pool exclusion without second-order exclusion. **a** Time series of the frequencies of three strategies C (cooperators, black dot line), D (defectors, red solid line), and EC (excluding cooperators, blue dash dot line). The population system displays periodic oscillations among C, D, and EC. **b** The dynamics of the system with cooperators, defectors, and prosocial pool excluders in the simplex S_3. The triangle represents the state space $S_3 = \{(x, y, z) : x, y, z \geq 0, x + y + z = 1\}$, where x, y, z denote the frequencies of cooperators, defectors, and excluding cooperators, respectively. The open circles represent unstable equilibria, and the three corners are saddle points. There exists a fixed point in the S_3, and it is neutrally stable surrounded by closed orbits. Parameters: $N = 5$, $r = 3$, $c = 1$, and $\delta_p = 0.5$. (Color figure online)

in the system, namely C, D, EC, and ED, respectively. We denote by x, y, z, and w the frequencies of these strategies, respectively. Thus, x, y, z, $w \geq 0$ and $x+y+z+w = 1$. We also use replicator equations to depict the evolutionary dynamics of the system as

$$\begin{cases} \dot{x} = x(P_C - \bar{P}), \\ \dot{y} = y(P_D - \bar{P}), \\ \dot{z} = z(P_{EC} - \bar{P}), \\ \dot{w} = w(P_{ED} - \bar{P}), \end{cases} \tag{28}$$

where $\bar{P} = xP_C + yP_D + zP_{EC} + wP_{ED}$ describes the average payoff of the population.

Next, we calculate the expected payoffs for each strategy. As previously, we consider the perfect exclusion. Thus, defectors can get nothing from the PGG if the number of EC individuals is nonzero, and this is the same to cooperators when there exist ED individuals in the group.

Accordingly, the expected payoff of defectors can be given as

$$\begin{aligned} P_D = &\sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w-z)^{N-N_C-1} \\ &\frac{rcN_C}{N} + \sum_{N_C=0}^{N-1} \sum_{N_{ED}=1}^{N-N_C-1} \binom{N-1}{N_C} \\ &\binom{N-N_C-1}{N_{ED}} x^{N_C} w^{N_{ED}} \\ &\times (1-x-w-z)^{N-N_{ED}-N_C-1} \\ &rcN_c/(N-N_C) \\ = &(1-w-z)^{N-1} \frac{rcx(N-1)}{N(1-w-z)} \\ &+rc[(1-z)^{N-1}x(1-w-x-z) \\ &+wx^N - x(1-w-z)^{N-1} \\ &(1-x-z)]/[(1-x-z)(1-w-x-z)], \end{aligned}$$

where the first term on the right side denotes the payoff of defectors when there are no ED individuals, while the second term represents the obtained payoff when there are ED individuals. And $(1 - w - z)^{N-1}$ represents the probability that there are no excluders in the $N - 1$ coplayers.

Similarly, cooperators will be excluded if there exist ED individuals. Thus, in a group without ED, the expected payoff of cooperators is given by

$$\begin{aligned} P_C = &\sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w-z)^{N-N_C-1} \\ &\frac{rc(N_C+1)}{N} + \sum_{N_{EC}=1}^{N-1} \sum_{N_C=0}^{N-1-N_{EC}} \binom{N-1}{N_{EC}} \\ &\binom{N-1-N_{EC}}{N_C} z^{N_{EC}} x^{N_C} \\ &\times (1-x-w-z)^{N-N_{EC}-N_C-1} rc - c \end{aligned}$$

$$= (1-w-z)^{N-1}\left[\frac{rcx(N-1)}{N(1-w-z)} + \frac{rc}{N}\right] + rc[(1-w)^{N-1} - (1-w-z)^{N-1}] - c,$$

where the first term on the right side represents the payoff of cooperators when the number of EC individuals is zero, while the second term denotes the expected payoff of cooperators when there exist EC individuals in the group.

Since EC individuals can only get the payoff from the game when the number of ED individuals is zero, we have

$$P_{EC} = \sum_{N_{EC}=0}^{N-1}\sum_{N_C=0}^{N-1-N_{EC}} \binom{N-1}{N_{EC}}\binom{N-1-N_{EC}}{N_C} x^{N_C} \times z^{N_{EC}}(1-x-w-z)^{N-N_C-N_{EC}-1} rc - c - \delta_P = (1-w)^{N-1} rc - c - \delta_P.$$

Similarly, we can give the payoff expression for ED individuals as

$$P_{ED} = \sum_{N_C=0}^{N-1}\sum_{N_{ED}=0}^{N-N_C-1} \binom{N-1}{N_C}\binom{N-N_C-1}{N_{ED}} x^{N_C} w^{N_{ED}} \times (1-x-w-z)^{N-N_{ED}-N_C-1}\frac{rcN_c}{N-N_C} - \delta_A = \frac{rcx[(1-z)^{N-1} - x^{N-1}]}{1-x-z} - \delta_A.$$

3.2 Equilibria and their stabilities

In order to study the fixed points in the system and do the stability analysis, we define that

$$\begin{cases} k(x,y,z) = x[(1-x)(P_C - P_{ED}) - y(P_D - P_{ED}) - z(P_{EC} - P_{ED})], \\ m(x,y,z) = y[(1-y)(P_D - P_{ED}) - x(P_C - P_{ED}) - z(P_{EC} - P_{ED})], \\ h(x,y,z) = z[(1-z)(P_{EC} - P_{ED}) - y(P_D - P_{ED}) - x(P_C - P_{ED})]. \end{cases}$$

Accordingly, the Jacobian matrix of the system is written as

$$J = \begin{bmatrix} \frac{\partial k(x,y,z)}{\partial x} & \frac{\partial k(x,y,z)}{\partial y} & \frac{\partial k(x,y,z)}{\partial z} \\ \frac{\partial m(x,y,z)}{\partial x} & \frac{\partial m(x,y,z)}{\partial y} & \frac{\partial m(x,y,z)}{\partial z} \\ \frac{\partial h(x,y,z)}{\partial x} & \frac{\partial h(x,y,z)}{\partial y} & \frac{\partial h(x,y,z)}{\partial z} \end{bmatrix}. \quad (29)$$

Theorem 4 *The evolutionary dynamics of four strategies are described in the simplex* $S_4 = \{(x,y,z,w) : x,y,z,w \geq 0, x+y+z+w = 1\}$, *where the four vertexes, namely* D $(y=1)$, ED $(w=1)$, EC $(z=1)$, *and* C $(x=1)$, *are unstable equilibria. Besides, there exists a boundary equilibrium* $(0, 0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1-[\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$ *on the edge* EC–ED, *which is unstable when* $r < N$ *and* $rc - c - \delta_P > 0$ *are satisfied.*

Proof (1) For $(x,y,z,w) = (0,0,0,1)$, the Jacobian is

$$J = \begin{bmatrix} \delta_A - c & 0 & 0 \\ 0 & \delta_A & 0 \\ 0 & 0 & \delta_A - c - \delta_P \end{bmatrix},$$

and thus, this fixed point is unstable.

(2) For $(x,y,z,w) = (1,0,0,0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-2) + c - \delta_A & a_{12} & a_{13} \\ 0 & c - rc/N & 0 \\ 0 & 0 & -\delta_P \end{bmatrix},$$

where $a_{12} = rc(N-1)^2/N - \delta_A$ and $a_{13} = rc(N-2) + c + \delta_P - \delta_A$; thus, this fixed point is unstable.

(3) For $(x,y,z,w) = (0,1,0,0)$, the Jacobian is

$$J = \begin{bmatrix} \frac{rc}{N} - c & 0 & 0 \\ c - \delta_A - rc/N & -\delta_A & -(rc - c - \delta_P + \delta_A) \\ 0 & 0 & rc - c - \delta_P \end{bmatrix},$$

and thus, this fixed point is unstable since $rc - c - \delta_P > 0$.

(4) For $(x,y,z,w) = (0,0,1,0)$, the Jacobian is

$$J = \begin{bmatrix} \delta_P & 0 & 0 \\ 0 & -(rc - c - \delta_P) & 0 \\ -(rc - c + \delta_A) & -\delta_A & -(rc - c - \delta_P + \delta_A) \end{bmatrix};$$

thus, this fixed point is unstable.

(5) For $(x,y,z,w) = (0, 0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1 - [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$

$$J = \begin{bmatrix} \delta_P & 0 & 0 \\ 0 & \delta_A & 0 \\ a_{31} & a_{32} & (N-1)(1-z)(c + \delta_P - \delta_A) \end{bmatrix},$$

where $a_{31} = rc[(N-1)z^{N-1}(1-z) - z(1-z)^{N-1}] - z\delta_P$ and $a_{32} = (1-z)(N-1)rcz^{N-1} - z\delta_A$; thus, this fixed point is unstable. □

In addition, it is difficult to theoretically determine the interior equilibrium and its stability, and we do some numerical calculations for the representative sets of model parameters in what follows.

3.3 Numerical example

Example 2 As shown in Fig. 2, the four vertices C, D, EC, and ED, respectively, correspond to the four homogeneous states in which the population consists of all the cooperators ($x = 1$), all the defectors ($y = 1$), all the prosocial excluders ($z = 1$), or all the antisocial excluders ($w = 1$), and each homogeneous state is unstable (see Theorem 4). On the edge EC–ED, there exists an unstable boundary equilibrium when $0 < c + \delta_P - \delta_A < rc$ (Fig. 2c) (also see Appendix A for theoretical analysis about the boundary faces of S_4). In addition, we find that the population system can exhibit a heteroclinic circle where cooperative strategy can coexist with other types of strategies (Fig. 2a, b).

4 Prosocial pool exclusion with second-order exclusion

4.1 Modeling

Based on the model in Sect. 2, we consider an extended model including second-order exclusion under which cooperators (second-order free-riders) will also be excluded by prosocial excluders. In this scenario, the expected payoff for cooperators is given by

$$P_C = \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C}(1-x-z)^{N-N_C-1} \frac{rc(N_C+1)}{N} - c = (1-z)^{N-1}\frac{rc(1-z+Nx-x)}{N(1-z)} - c, \quad (30)$$

and the expected payoff for defectors is the same to Eq. (2). But the expected payoff of EC individuals is changed to

$$P_{EC} = \sum_{N_C=0}^{N-1}\sum_{N_{EC}=0}^{N-1-N_C}\binom{N-1}{N_C}\binom{N-1-N_C}{N_{EC}} x^{N_C} z^{N_{EC}} \times (1-x-z)^{N-N_C-N_{EC}-1}\frac{rc(N_C+N_{EC}+1)}{N_{EC}+1} - c - \delta_P = \frac{rcx[1-(1-z)^{N-1}]}{z} + rc - c - \delta_P. \quad (31)$$

Accordingly, the dynamical system becomes to

$$\begin{cases} \dot{x} = x[(1-x)(P_C - P_{EC}) - y(P_D - P_{EC})], \\ \dot{y} = y[(1-y)(P_D - P_{EC}) - x(P_C - P_{EC})], \end{cases} \quad (32)$$

where

$$P_C - P_{EC} = (x+y)^{N-1}\frac{rc[x(N-1)/(x+y)+1]}{N} - \frac{rcx[1-(x+y)^{N-1}]}{1-x-y} - rc + \delta_P, \quad (33)$$

$$P_D - P_{EC} = (x+y)^{N-1}\frac{rc(N-1)x}{N(x+y)} - \frac{rcx[1-(x+y)^{N-1}]}{1-x-y} - rc + c + \delta_P. \quad (34)$$

By solving $P_C = P_D$, we have $(x+y)^{N-1}\frac{rc}{N} = c$. Obviously, there is no interior equilibrium point for $r < N$.

Then, we explore the dynamics on each edge of the face $C - D - EC$. On the edge $C{-}EC$, we have $y = 0$ resulting in $\dot{z} = z(1-z)(P_{EC} - P_C) = z(1-z)[\frac{rc(1-x^{N-1})}{1-x} - \delta_P]$. Since $\frac{rc(1-x^{N-1})}{1-x}$ increases with increasing x, the evolutionary direction goes from C to EC when $\delta_P < rc$. The dynamics on other two edges $C - D$ and $D{-}EC$ are same to Sect. 2.

4.2 Equilibria and their stabilities

Based on the above analysis, we know that there exists only three vertex equilibrium points. In this subsection, we explore the stability of the three fixed points.

Theorem 5 *The dynamical system only has three vertex equilibrium points. In the conditions of $r < N$ and $\delta_P < rc - c$, the vertex EC ($z = 1$) is stable, while the other two vertices ($x = 1$ and $y = 1$) are both unstable.*

Proof (1) For $(x, y, z) = (0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} \delta_P - rc & 0 \\ 0 & -rc + c + \delta_P \end{bmatrix};$$

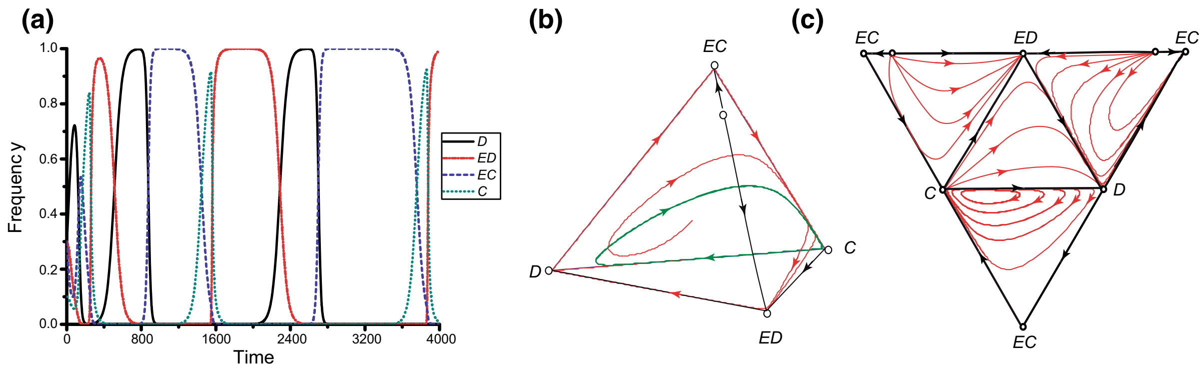

Fig. 2 Evolutionary dynamics of cooperators (C), defectors (D), excluding cooperators (EC), and excluding defectors (ED) in the absence of second-order exclusion. In **a** we show the time series of the frequencies of four strategies, namely C (green dot line), D (black line), EC (blue dash line), and ED (red dash dot line). **b** The evolutionary trajectories in simplex S_4, where open circles represent unstable fixed points. The four strategies can coexist in the group. **c** The replicator dynamics on the boundary faces of the simplex S_4. These four strategies can coexist in the simplex S_4 by exhibiting a heteroclinic circle. Initial conditions are: $(x, y, z, w) = (0.1, 0.3, 0.3, 0.3)$. Other parameters are $N = 5$, $r = 3$, $c = 1$, $\delta_P = 0.5$, and $\delta_A = 0.5$. (Color figure online)

thus, the fixed point is stable since $\delta_P < rc$ and $rc - c - \delta_P > 0$.

(2) For $(x, y, z) = (1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-1)-\delta_P & rc(N-1)-\delta_P-c+rc/N \\ 0 & c-rc/N \end{bmatrix};$$

thus, the fixed point is unstable since $rc(N-1) - \delta_P > 0$ and $r < N$.

(3) For $(x, y, z) = (0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc/N - c & 0 \\ rc - \delta_P - rc/N & rc - c - \delta_P \end{bmatrix};$$

thus, the fixed point is unstable since $rc - c - \delta_P > 0$. □

4.3 Numerical example

Example 3 As shown in Fig. 3, in the presence of second-order prosocial exclusion, the population system will end up with a homogeneous state of all EC individuals, which is consistent with Theorem 5. Indeed, in this scenario the $C - D - EC$ cycle can be broken easily in the cooperative state. Without second-order exclusion, prosocial pool excluders are invaded by cooperators, but prosocial pool excluders who exclude cooperators are protected from such an invasion. The competitions between other strategies are same as Fig. 1, that is, cooperators are invaded by defectors, and excluders invade defectors.

5 Antisocial pool exclusion with second-order exclusion

5.1 Modeling

In this section, we investigate the dynamics of the four strategies in the scenario of second-order exclusion where excluders fully exclude their respective non-excluding types. That is, prosocial excluders exclude cooperators, defectors, and antisocial excluders, while antisocial excluders exclude cooperators, prosocial excluders, and defectors. Thus, when some EC and ED players are selected simultaneously in the same group, all participants are supposed to be excluded from the group and no one receives the benefits of the PGG.

Accordingly, the expected payoffs of defectors, cooperators, prosocial excluders, and antisocial excluders can be, respectively, given as

$$P_D = \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1 - x - w - z)^{N-N_C-1} \frac{rcN_C}{N} = (1 - w - z)^{N-1} \frac{rcx(N-1)}{N(1 - w - z)}, \quad (35)$$

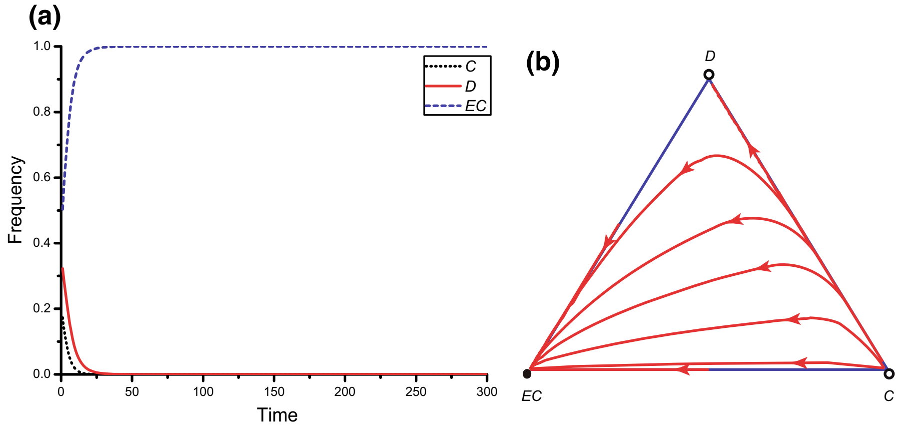

Fig. 3 Evolutionary dynamics of prosocial pool exclusion with second-order exclusion. **a** Time courses of the frequencies of three strategies C (cooperators, black dot line), D (defectors, red solid line), and EC (excluding cooperators, blue dash dot line). **b** Evolutionary trajectories in $C-D-EC$ simplex. When considering second-order exclusion, the trajectory converges to the homogeneous state of all prosocial pool excluders. Parameters are same as Fig. 1. (Color figure online)

$$P_C = \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C}(1-x-w-z)^{N-N_C-1} \frac{rc(N_C+1)}{N} - c = (1-w-z)^{N-1}[\frac{rcx(N-1)}{N(1-w-z)} + \frac{rc}{N}] - c, \quad (36)$$

$$P_{EC} = \sum_{N_{EC}=0}^{N-1} \sum_{N_C=0}^{N-1-N_{EC}} \binom{N-1}{N_{EC}}\binom{N-1-N_{EC}}{N_C} \times x^{N_C} z^{N_{EC}} (1-x-w-z)^{N-N_C-N_{EC}-1} \times \frac{rc(N_C+N_{EC}+1)}{N_{EC}+1} - c - \delta_P = rc(1-w)^{N-1}\frac{x+z}{z} - (1-w-z)^{N-1}\frac{rcx}{z} - c - \delta_P, \quad (37)$$

and

$$P_{ED} = \sum_{N_{ED}=0}^{N-1} \sum_{N_D=0}^{N-1-N_{ED}} \binom{N-1}{N_{ED}}\binom{N-1-N_{ED}}{N_D} \times x^{N-N_{ED}-N_D-1}(1-x-y-z)^{N_{ED}} y^{N_D} \times \frac{rc(N-N_{ED}-N_D-1)}{N_{ED}+1} - \delta_A = \frac{rcx[(1-z)^{N-1}-(x+y)^{N-1}]}{1-x-y-z} - \delta_A. \quad (38)$$

5.2 Equilibria and their stabilities

We know that $P_C < P_D$; thus, there is no interior equilibrium in the simplex S_4. The dynamics on each edge and each face of S_4 are presented in Appendix B. Then, we investigate the stability of the four vertex equilibria and the boundary fixed points.

Theorem 6 *In the condition of $r < N$ and $\delta_P < rc - c$, the vertex EC ($z = 1$) is stable, while the other three vertex fixed points ($x = 1$, $y = 1$, and $w = 1$) and the boundary fixed point ($(x, y, z, w) = (0, 0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1-[\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$) are unstable.*

Proof (1) For $(x, y, z, w) = (0, 0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} \delta_A - c & 0 & 0 \\ 0 & \delta_A & 0 \\ 0 & 0 & \delta_A - c - \delta_P \end{bmatrix};$$

thus, this fixed point is unstable.

(2) For $(x, y, z, w) = (1, 0, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-2) + c - \delta_A & a_{12} & a_{13} \\ 0 & c - \frac{rc}{N} & 0 \\ 0 & 0 & rc(N-1) - \delta_P \end{bmatrix},$$

where $a_{12} = rc(N-1)(1-\frac{1}{N}) - \delta_A$ and $a_{13} = c - rc + \delta_P - \delta_A$; thus, this fixed point is unstable.

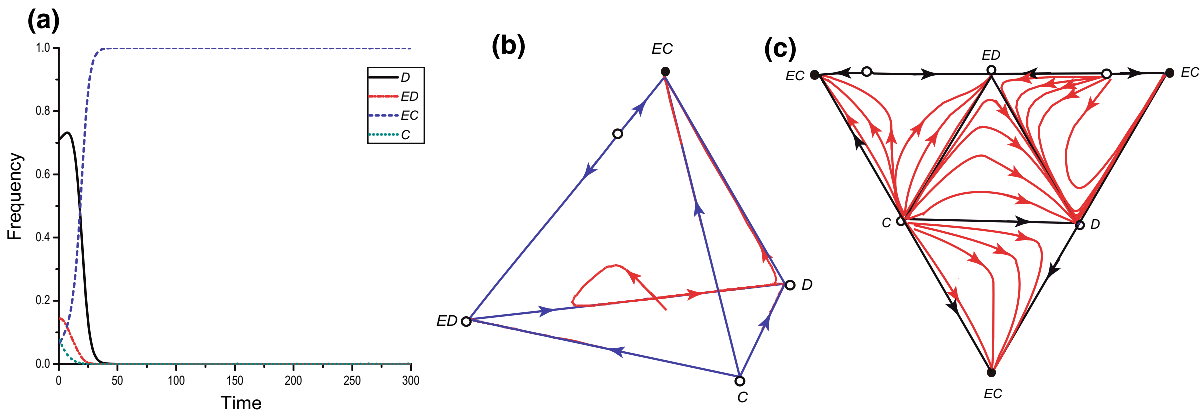

Fig. 4 Evolutionary dynamics of cooperators, defectors, excluding cooperators, and excluding defectors in the presence of second-order exclusion. **a** The time series of frequencies of four strategies C (green dot line), D (black line), EC (blue dash line), and ED (red dash dot line). **b** The evolutionary trajectories in the $C-D-ED-EC$ simplex, where filled circles represent stable fixed points and open circles represent unstable fixed points. **c** The replicator dynamics on the boundary faces of the simplex S_4. Parameters are $N=5$, $r=3$, $c=1$, $\delta_P=0.5$, and $\delta_A=0.5$. (Color figure online)

(3) For $(x, y, z, w) = (0, 1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} \frac{rc}{N} - c & 0 & 0 \\ c - \delta_A - \frac{rc}{N} & -\delta_A & -(rc - c - \delta_P + \delta_A) \\ 0 & 0 & rc - c - \delta_P \end{bmatrix};$$

thus, this fixed point is unstable.

(4) For $(x, y, z, w) = (0, 0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} -rc + \delta_P & 0 & 0 \\ 0 & -(rc - c - \delta_P) & 0 \\ c - \delta_A & -\delta_A & -(rc - c - \delta_P + \delta_A) \end{bmatrix};$$

thus, this fixed point is stable.

(5) For $(x, y, z, w) = (0, 0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1 - [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$, the Jacobian is

$$J = \begin{bmatrix} \delta_A - c & 0 & 0 \\ 0 & \delta_A & 0 \\ a_{31} & a_{32} & rc(N-1)z^{N-1}(1-z) \end{bmatrix},$$

where $a_{31} = rcz(1-z)[Nz^{N-2} - (1-z)^{N-2}] - z(\delta_A - c)$ and $a_{32} = rc(N-1)(1-z)z^{N-1} - z\delta_A$; thus, this fixed point is unstable. □

5.3 Numerical example

Example 4 As shown in Fig. 4, when second-order exclusion for prosocial and antisocial exclusion is considered, EC is the only stable fixed point. Figure 4 also suggests that the excluding cooperator strategy is the most advantaged control strategy, which can dominate the whole population. Accordingly, the other three vertices and the boundary fixed point are unstable, and there are no other equilibria in the interior of S_4.

6 Conclusions

In this study, we have introduced two forms of control strategies for pool exclusion, that is, prosocial pool exclusion and antisocial pool exclusion, into a population of agents who play the public goods game, and then study the evolutionary dynamics of cooperation by means of replicator equations with the control strategies of pool exclusion. Our study is different from previous studies, which can be reflected in the following two aspects. First, we consider that pool excluders contributed a constant cost to a central control authority to expel individuals. Second, we incorporate the antisocial exclusion into the public goods game for the first time and study its control effects on public cooperation in infinite well-mixed populations.

We have shown that prosocial pool excluders can stabilize the coexistence of cooperators and defectors by forming periodic oscillations in the absence of second-order prosocial pool exclusion. However, in the presence of second-order exclusion, the periodic oscillations disappear, and the population system which consists of these three strategies will end up with a homogeneous state in which all play EC.

Next we have explored the evolution of antisocial pool exclusion and found that the population system can exhibit a heteroclinic circle where cooperative strategy can coexist with other types of strategies in the absence of second-order exclusion. Lastly, we have explored the competition among these four strategies when considering second-order exclusion. We find that excluding cooperators will occupy the whole population, regardless of the initial conditions.

In closing, we stress that pool exclusion strategies allow cooperation to persist even when antisocial pool exclusion is taken into account. This is in stark contrast to the effects induced by antisocial punishment. More specifically, in the framework of prosocial pool punishment, full defection is the only global stable state [70]. While when antisocial pool punishment is further considered, both theoretical studies and behavioral experiments demonstrate that selection favors substantial levels of antisocial punishment [48,49]. Although the presence of antisocial exclusion can still inhibit cooperation, the destructive effect of antisocial exclusion on cooperation is weaker than that of antisocial punishment. Therefore, we can conclude that prosocial pool exclusion is always a more effective control mechanism to curb free-riders than punishment.

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Compliance with ethical standards

Conflict of interest The authors declare that no competing interest exist.

Appendixes

Appendix A

In order to explore the evolutionary dynamics of S_4, which we discuss in Sect. 3, we first study the dynamics of each edge of S_4.

On the edge $D{-}ED$ ($x + z = 0$ and $y + w = 1$), we have $\dot{y} = y(1-y)(P_D - P_{ED}) = y(1-y)\delta_A > 0$, and thus, the direction of the dynamics goes from ED to D.

On the edge $C{-}EC$ ($y + w = 0$ and $x + z = 1$), we have $\dot{x} = x(1-x)(P_C - P_{EC}) = x(1-x)\delta_P > 0$; therefore, the direction of the evolution goes from EC to C.

On the edge $C - D$ ($z + w = 0$ and $x + y = 1$), we have $\dot{y} = y(1-y)(P_D - P_C) = -y(1-y)(\frac{rc}{N} - c) > 0$; therefore, the direction of the evolution goes from C to D.

On the edge $C{-}ED$ ($z + y = 0$ and $x + w = 1$), we have $\dot{w} = w(1 - w)(P_{ED} - P_C) = w(1 - w)\{\frac{rc[1-w-(1-w)^{N-1}]}{w} - \delta_A + c\}$; therefore, when the antisocial exclusion cost is less than the cost of cooperation, namely $\delta_A < c$, the direction of the evolution goes from C to ED.

On the edge $D{-}EC$ ($x + w = 0$ and $y + z = 1$), we have $\dot{z} = z(1-z)(P_{EC} - P_D) = z(1-z)(rc - c - \delta_P) > 0$, and thus, the direction of the dynamics goes from D to EC.

On the edge $ED{-}EC$ ($x + y = 0$ and $w + z = 1$), we have $\dot{w} = w(1-w)(P_{ED} - P_{EC}) = w(1-w)[-\delta_A + c + \delta_P - (1-w)^{N-1} rc]$; thus, there exist an equilibrium, namely $w = 1 - [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}$.

Next, we investigate the evolutionary dynamics of each face. We have discussed the dynamics on the face $C - D{-}EC$ in Sect. 2, and the three strategies can coexist in the population.

On the face $C - D{-}ED$ ($z = 0$ and $x + y + w = 1$), we have

$$\begin{aligned}
P_C &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w)^{N-N_C-1} \\
&\quad \times \frac{rc(N_C+1)}{N} - c \\
&= (1-w)^{N-1} \frac{rc}{N} \frac{1-w+(N-1)x}{1-w} - c, \\
P_{ED} &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x)^{N-N_C-1} \frac{rcN_C}{N-N_C} - \delta_A \\
&= \frac{rcx(1-x^{N-1})}{1-x} - \delta_A, \\
P_D &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w)^{N-N_C-1} \frac{rcN_C}{N} \\
&\quad + \sum_{N_C=0}^{N-1} \sum_{N_{ED}=1}^{N-N_C-1} \binom{N-1}{N_C} \binom{N-N_C-1}{N_{ED}} x^{N_C} w^{N_{ED}} \\
&\quad \times (1-x-w)^{N-N_{ED}-N_C-1} \frac{rcN_c}{N-N_C} \\
&= (1-w)^{N-1} \frac{rc}{N} \frac{(N-1)x}{1-w}
\end{aligned}$$

$$+\frac{rcx[(1-w-x)+wx^{N-1}-(1-w)^{N-1}(1-x)]}{(1-x)(1-w-x)}.$$

We know that $P_C < P_D$ is satisfied, and thus, there is no interior fixed point. In addition, the stability of three boundary points can be described as follow.

(1) For $(x, y, w) = (0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} -c+\delta_A & 0 \\ 0 & \delta_A \end{bmatrix};$$

thus, the fixed point is unstable.

(2) For $(x, y, w) = (1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-2)+c-\delta_A & rc(N-2)-\delta_A+\frac{rc}{N} \\ 0 & c-\frac{rc}{N} \end{bmatrix};$$

thus, the fixed point is unstable.

(3) For $(x, y, w) = (0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} \frac{rc}{N}-c & 0 \\ -\frac{rc}{N}+c-\delta_A & -\delta_A \end{bmatrix};$$

thus, the fixed point is stable.

On the face $D{-}EC{-}ED$ ($x = 0$ and $y+z+w = 1$), the expected payoffs for these three strategies can be given by

$$\begin{aligned} P_{EC} &= \sum_{N_{EC}=0}^{N-1} \binom{N-1}{N_{EC}} z^{N_{EC}} (1-z-w)^{N-N_{EC}-1} rc \\ &\quad - c - \delta_P, \\ &= (1-w)^{N-1} rc - c - \delta_P && (39) \\ P_D &= 0, && (40) \\ P_{ED} &= -\delta_A. && (41) \end{aligned}$$

Based on the above payoff expressions, we know that there is no interior fixed point. Furthermore, the stability of three boundary equilibria can be given as follows.

(1) For $(y, z, w) = (0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} \delta_A & 0 \\ 0 & \delta_A - c - \delta_P \end{bmatrix};$$

thus, this equilibrium is unstable.

(2) For $(y, z, w) = (0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} -rc+c+\delta_P & 0 \\ -\delta_A & -rc+c+\delta_P-\delta_A \end{bmatrix};$$

thus, this equilibrium is stable.

(3) For $(y, z, w) = (1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} -\delta_A & -(rc-c-\delta_P+\delta_A) \\ 0 & rc-c-\delta_P \end{bmatrix};$$

thus, this equilibrium is unstable.

(4) For $(y, z, w) = (0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1-[\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$, the Jacobian is

$$J = \begin{bmatrix} \delta_A & 0 \\ a_{21} & (1-z)(N-1)z^{N-1}rc \end{bmatrix},$$

where $a_{21} = (1-z)(N-1)z^{N-1}rc - z\delta_A$; thus, this equilibrium is unstable.

On the face $ED{-}C{-}EC$ ($y = 0$ and $x+z+w = 1$), the expected payoffs for these three strategies can be described by

$$\begin{aligned} P_{EC} &= \sum_{N_{EC}=0}^{N-1} \binom{N-1}{N_{EC}} z^{N_{EC}} (1-z-w)^{N-N_{EC}-1} rc \\ &\quad - c - \delta_P \\ &= (1-w)^{N-1} rc - c - \delta_P, && (42) \\ P_C &= (1-w)^{N-1} rc - c, && (43) \\ P_{ED} &= \sum_{N_{ED}=0}^{N-1} \binom{N-1}{N_{ED}} x^{N-N_{ED}-1} (1-z-x)^{N_{ED}} \\ &\quad \times \frac{rc(N-N_{ED}-1)}{N_{ED}+1} - \delta_A \\ &= \frac{rcx[(1-z)^{N-1}-x^{N-1}]}{1-x-z} - \delta_A, && (44) \end{aligned}$$

Based on the payoff expressions, we can get that $P_C > P_{EC}$. Therefore, there is no interior fixed point in S_3. And the stability of three boundary fixed points can be described as follows.

(1) For $(x, z, w) = (0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} \delta_A - c & 0 \\ 0 & \delta_A - c - \delta_P \end{bmatrix};$$

thus, this equilibrium is stable,

(2) For $(x, z, w) = (0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} \delta_P & 0 \\ -(rc - c + \delta_A) & -(rc - c - \delta_P + \delta_A) \end{bmatrix};$$

thus, this equilibrium is unstable.

(3) for $(x, z, w) = (1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-2) + c - \delta_A & a_{12} \\ 0 & -\delta_P \end{bmatrix},$$

where $a_{12} = rc(N-2) + c + \delta_P - \delta_A$, and thus, this equilibrium is unstable.

(4) For $(x, z, w) = (0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1 - [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$, the Jacobian is

$$J = \begin{bmatrix} \delta_P & 0 \\ a_{21} & rc(N-1)(1-z)z^{N-1} \end{bmatrix},$$

where $a_{21} = rcz(1-z)[(N-1)z^{N-2} - (1-z)^{N-2}] - z\delta_P$; thus, this equilibrium is unstable.

Appendix B

Then, we explore the evolutionary dynamics of S_4, which we discuss in Sect. 5.

On the edge $C{-}EC$ ($y + w = 0$ and $x + z = 1$), we have $\dot{z} = z(1-z)(P_{EC} - P_C) = z(1-z)\{\frac{rc[1-(1-z)^{N-1}]}{z} - \delta_P\}$ since $\frac{1-(1-z)^{N-1}}{z}$ decreases with increasing z. Thus, there is not interior equilibrium for $\delta_P < rc$. As a result, the direction of the evolution goes from C to EC. The dynamics of other edges of simplex S_4 are same to those in Sect. 3.

Next, we investigate the evolutionary dynamics on each face. We have discussed the situation on the face $C-D{-}EC$ in Sect. 4. Then, on the face $C-D{-}ED$ ($z = 0$ and $x + y + w = 1$), the expected payoffs for these three strategies can be given as

$$\begin{aligned} P_C &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w)^{N-N_C-1} \\ &\quad \times \frac{rc(N_C+1)}{N} - c \\ &= (1-w)^{N-1} \frac{rc[(N-1)x + (1-w)]}{N(1-w)} - c, \end{aligned} \quad (45)$$

$$\begin{aligned} P_D &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w)^{N-N_C-1} \frac{rcN_C}{N} \\ &= (1-w)^{N-1} \frac{rc(N-1)x}{N(1-w)}, \textit{ and} \end{aligned} \quad (46)$$

$$\begin{aligned} P_{ED} &= \sum_{N_{ED}=0}^{N-1} \sum_{N_D=0}^{N-1-N_{ED}} \binom{N-1}{N_{ED}} \binom{N-1-N_{ED}}{N_D} \\ &\quad \times x^{N-N_{ED}-N_D-1} (1-x-y)^{N_{ED}} y^{N_D} \\ &\quad \times \frac{rc(N-N_{ED}-N_D-1)}{N_{ED}+1} - \delta_A \\ &= \frac{rcx[1-(x+y)^{N-1}]}{1-x-y} - \delta_A. \end{aligned} \quad (47)$$

Since $P_D > P_C$, there is no interior point. And the stability of three boundary points can be described as follows

(1) For $(x, y, w) = (0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} -c + \delta_A & 0 \\ 0 & \delta_A \end{bmatrix};$$

thus, this fixed point is unstable.

(2) For $(x, y, w) = (1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-2) + c - \delta_A & rc(N-2) - \delta_A + \frac{rc}{N} \\ 0 & c - \frac{rc}{N} \end{bmatrix};$$

thus, the fixed point is unstable.

(3) For $(x, y, w) = (0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} \frac{rc}{N} - c & 0 \\ -\frac{rc}{N} + c - \delta_A & -\delta_A \end{bmatrix};$$

thus, this fixed point is stable.

The dynamic on the face $D{-}EC{-}ED$ is the same to those in the situation without second-order exclusion. On the face $C{-}EC{-}ED$ ($y = 0$ and $x + z + w = 1$), the expected payoffs of these three strategies can be given by

$$P_C = (1-z-w)^{N-1} rc - c, \quad (48)$$

$$\begin{aligned} P_{EC} &= \sum_{N_C=0}^{N-1} \binom{N-1}{N_C} x^{N_C} (1-x-w)^{N-N_C-1} \\ &\quad \frac{rcN}{N-N_C} - c - \delta_P, \\ &= \frac{rc[(1-w)^N - (1-w-z)^N]}{z} - c - \delta_P, \end{aligned} \quad (49)$$

$$P_{ED} = \sum_{N_{ED}=0}^{N-1} \binom{N-1}{N_{ED}} (1-z-w)^{N-N_{ED}-1} w^{N_{ED}} \times \frac{rc(N-N_{ED}-1)}{N_{ED}+1} - \delta_A, = \frac{rcx[(1-z)^{N-1} - x^{N-1}]}{w} - \delta_A. \quad (50)$$

Solving $P_C = P_{ED}$ yields $w = \frac{rcx(1-z)[(1-z)^{N-2} - x^{N-2}]}{\delta_A - c}$; therefore, there is not interior fixed point for $\delta_A < c$. In addition, the stability of these four boundary points can be described as follows.

(1) For $(x, z, w) = (0, 0, 1)$, the Jacobian is

$$J = \begin{bmatrix} -c + \delta_A & 0 \\ 0 & \delta_A - c - \delta_P \end{bmatrix};$$

thus, this fixed point is stable.

(2) For $(x, z, w) = (1, 0, 0)$, the Jacobian is

$$J = \begin{bmatrix} rc(N-2) + c - \delta_A & -(rc - c + \delta_A - \delta_P) \\ 0 & rc(N-1) - \delta_P \end{bmatrix};$$

thus, this fixed point is unstable.

(3) For $(x, z, w) = (0, 1, 0)$, the Jacobian is

$$J = \begin{bmatrix} -rc + \delta_P & 0 \\ c - \delta_A & -(rc - c - \delta_P + \delta_A) \end{bmatrix};$$

thus, this fixed point is stable.

(4) For $(x, z, w) = (0, [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}}, 1 - [\frac{c+\delta_P-\delta_A}{rc}]^{\frac{1}{N-1}})$, the Jacobian is

$$J = \begin{bmatrix} \delta_A - c & 0 \\ a_{21} & rcz^{N-1}(1-z)(N-1) \end{bmatrix},$$

where $a_{21} = rcz(1-z)[Nz^{N-2} - (1-z)^{N-2}] - z(\delta_A - c)$; thus, this equilibrium is unstable.

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