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Pool expulsion and cooperation in the spatial public goods game

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ABSTRACT

We study the evolution of cooperation in the spatial public goods game with cooperation, defection, and pool expulsion as the three competing strategies. Using the Monte Carlo method, we show that the evolution of pool expulsion and cooperation can be maintained even if the synergistic effects are not high enough to sustain cooperation based on spatial reciprocity alone, and even if the cost of pool expulsion is not negligible. Interestingly, pool expellers are protected against, or even prevail over, defectors as a result of spatial pattern formation, by means of which vacant sites form an active layer around them. Moreover, we observe continuous and discontinuous phase transitions between frozen coexistence, stationary coexistence, absorbing states, and oscillatory states in the phase diagrams. Our results indicate that pool expulsion might play an important role in the resolution of social dilemmas that unfold in groups.

1. Introduction

One of the grand scientific challenges until now concerns the question: How did cooperative behavior evolve and how can it be maintained in social dilemma situations [1]? While the prisoner's dilemma game represents a standard paradigm for studying the evolution of cooperation among selfish individuals, the multiplayer public goods game corresponds to an extension of such a two-player game from pairwise interactions to collective interactions [2–5]. In the social dilemma game of public goods [6,7], players decide simultaneously whether they contribute (i.e., cooperate) or not (i.e., defect) to a common pool. The collecting contributions are then multiplied by an enhancement factor that takes into account synergetic effects of cooperation. The resulting public goods are finally divided equally among all group members irrespective of their strategies. Obviously, rational players should contribute nothing if the costs of investment exceed the return of the public goods game. However, if nobody invests, the group will fail to harvest the benefits resulted from a collaborative investment, and the group as a whole may evolve toward the "tragedy of the commons" [8,9]. It has been reported that evolutionary mechanisms such as reputation [10], reward [11–13] and punishment [14–18] can support contributors to fence off freeriders and thus promote cooperation in the public goods game [19–29].

While the fundamental problem of the evolution of cooperation has attracted a great deal of interest among scientists from biology, economics, mathematics and social sciences, it is also closely relevant to physics due to the fascinating collective behavior that is exhibited by the resulting complex systems that consist of a large number of interacting agents. Particularly, methods of statistical physics have proved valuable for studying phase transition, pattern formation, equilibrium selection, and self-organization in evolutionary games on graphs [30–38]. For example, Szabó and Hauert found that the phase transitions between one-, two-, and three-strategy states either are in the class of directed percolation or show interesting analogies to Ising-type models in the spatial public goods game with voluntary participation [39]. Wakano *et al.* showed that the spatial dynamics of the ecological public goods game lead to static or dynamic processes of pattern formation, including spatial chaos of ever-changing configurations [40]. Matsuzawa and Tanimoto considered a social dilemma structure in diffusible public goods, and revealed a rich diversity of evolutionary dynamics including cooperator dominated, extinct and coexistent states [41].

Here we explore the spatial dynamics of the public goods game with pool expulsion, and contrast the results with those reported previously for peer expulsion in the spatial prisoner's dilemma

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game [42]. In fact, peer and pool expulsion represent two different but complementary kinds of expulsive behavior. Peer expulsion focuses on the perspective of personal traits: It may happen when there exist individual differences (e.g., cooperativeness) [43]. On the contrary, pool expulsion is based on the viewpoint of group membership: It may be executed if members violate group norms (e.g., cooperative norm) [44]. Although large numbers of psychological studies provide numerous proximate causes of pool expulsion [45], there is still lack of theoretical explanation on its ultimate causes from evolutionary perspective. Furthermore, behavioral experiments show that contributions by members in a public goods game under threat of collective expulsion rose to nearly 100% of endowments [46], which indicates clearly a positive relationship between pool expulsion and cooperation. Then another question naturally arising here is how to understand the positive impacts of pool expulsion in the evolution of cooperation in the public goods game. By constructing and studying an evolutionary model of the spatial public goods game with pool expulsion, we show that self-organizing spatiotemporal structures are able to maintain pool expulsion and cooperation viable without the support of any additional mechanisms. As we show in detail, the spatiotemporal dynamics lead to the formation of an active vacant layer around pool expellers, which protects them against the exploitation of defectors. Besides, the phase diagram for a representative value of the enhancement factor also reveals surprisingly rich and interesting behavior from the physics point of view.

2. Public goods game with pool expulsion

In the public goods game with pool expulsion, individuals can choose from three different strategies: defection, cooperation and pool expulsion. In each public goods game with pool expulsion, defectors do not contribute to the common pool but only enjoy the public goods produced by cooperative individuals; Cooperators contribute $c = 1$ to the joint venture but do not bear the extra cost of pool expulsion; Pool expellers not only make contributions to the common pool but also are willing to allocate resources $c_{OE} > 0$ to construct an expulsion pool for the purpose of collectively expelling defectors from present sites to any other vacant sites. Here pool expellers are challenged by dual social dilemmas: (1) The first-order freeriding problem: Cooperative players, who contribute to the common pool, seem to fare worse than those who do not cooperate; (2) The second-order freeriding problem: Pool expulsion seems to be an altruistic act, given that players who cooperate but do not contribute to the expulsion pool are better off than the pool expellers.

We simulate the spatial public goods game with pool expulsion by randomly placing N individuals on a square lattice of $L \times L$ ($\geq N$) sites with von Neumann neighborhood (i.e., the degree $k = 4$) and periodic boundary conditions. Each site can be either empty or occupied by an individual who is randomly assigned with one of the three strategies with equal probability initially. During the whole evolutionary process, the population density is kept constant and is given by $\rho = N/L^2$. If not all the neighboring sites of an individual are empty, the individual can accumulate payoff by playing the public goods games with pool expulsion that centered on both its neighbors and itself. Otherwise, the individual has no chance to play the game, and thus obtains no payoff. In our model, each time step includes three stages: the game interaction stage, the pool expulsion stage and the strategy update stage. In the *game interaction stage*, all individuals play the public goods game with pool expulsion synchronously. Denoting the number of defectors, cooperators and pool expellers in a group of size $G \in [2, k+1]$ by G_D, G_C and G_{OE} respectively, the payoff of each type of players for a particular public goods game is thus given by

$$\begin{cases} P_D = r(G_C + G_{OE})/G, & \text{(a)} \\ P_C = r(G_C + G_{OE})/G - 1, & \text{(b)} \\ P_{OE} = r(G_C + G_{OE})/G - 1 - c_{OE}, & \text{(c)} \end{cases} \tag{1}$$

where $r \in (1, G)$ denotes the enhancement factor applied to the group investment $G_C + G_{OE}$. Here $r > 1$ takes into account the synergistic effects of public cooperation while $r < G$ ensures the presence of a social dilemma between personal and group interests. Then the whole system enters into the *pool expulsion stage*. In a random sequence manner, each defector is selected exactly once to be expelled to any other vacant sites, if any, on the square lattice, once there is at least one pool expeller in any one of public goods games it participates. Finally, in the *strategy update stage*, all individuals synchronously update their strategies either by imitation or by exploration. With probability $1 - \mu$, a individual (e.g., the one at site i) imitates the strategy of another randomly chosen neighbor (e.g., the one at site j), if any, with a probability given by the Fermi function

$$W\left(P_j - P_i\right) = \frac{1}{1 + \exp\left[-\left(P_j - P_i\right)/K\right]}, \tag{2}$$

where $K = 0.1$ quantifies the uncertainty by strategy adoptions, implying that the strategies of better-performing players are readily adopted, although it is not impossible to adopt the strategy of a player that performs worse. Despite capturing one root of humans or animals changing their strategies: imitation of fitter behaviors, the imitation dynamics ignore the function of innovation: exploration of new strategies different from the ones available in their neighborhoods [47–50]. Hence we introduce exploration dynamics into our model: With probability μ, a individual (e.g., the one at site i) randomly explores any other available strategies. In our study, we mainly focus on the limiting case $\mu \to 0$ (i.e., $\mu = 10^{-5}$ in present work), which ensures successful avoidance of the above evolutionary system being stuck in artifact stationary (or frozen) states where no strategy updating event happens for each individual on the one hand, and efficient investigation of spatial interactions among defection, cooperation and pool expulsion on the other hand [51].

3. Results

The average fractions of all three strategies $\bar{\rho}_X/\rho$ ($X \in D, C, OE$) on the square lattice are determined in the stationary state after a sufficient long relaxation time. Depending on the proximity to phase transition points, the linear system size is varied from $L = 500$ to 5000, and the relaxation time is varied from 10^4 to 3×10^6 time steps to ensure that the statistical error is comparable with the size of the symbols in the figures.

Before presenting the main results, let us briefly analyze the evolutionary outcomes in a well-mixed population with an infinite size. In the absence of a limited interaction range, the mean-field dynamics of the public goods game with pool expulsion can be described by the following differential equation set:

$$\begin{cases} \frac{\partial \rho_D}{\partial t} = -(1-\mu)\left[\rho_D \rho_{OE} \tanh\left(\frac{\bar{P}_{OE} - \bar{P}_D}{2K}\right) \right. \\ \quad \left. + \rho_D \rho_C \tanh\left(\frac{\bar{P}_C - \bar{P}_D}{2K}\right)\right] + \mu\left(\frac{\rho_{OE} + \rho_C}{2} - \rho_D\right), & \text{(a)} \\ \frac{\partial \rho_C}{\partial t} = -(1-\mu)\left[\rho_C \rho_{OE} \tanh\left(\frac{\bar{P}_{OE} - \bar{P}_C}{2K}\right) \right. \\ \quad \left. + \rho_C \rho_D \tanh\left(\frac{\bar{P}_D - \bar{P}_C}{2K}\right)\right] + \mu\left(\frac{\rho_{OE} + \rho_D}{2} - \rho_C\right), & \text{(b)} \\ \frac{\partial \rho_{OE}}{\partial t} = -(1-\mu)\left[\rho_{OE} \rho_C \tanh\left(\frac{\bar{P}_C - \bar{P}_{OE}}{2K}\right) \right. \\ \quad \left. + \rho_{OE} \rho_D \tanh\left(\frac{\bar{P}_D - \bar{P}_{OE}}{2K}\right)\right] + \mu\left(\frac{\rho_C + \rho_D}{2} - \rho_{OE}\right), & \text{(c)} \end{cases} \tag{3}$$

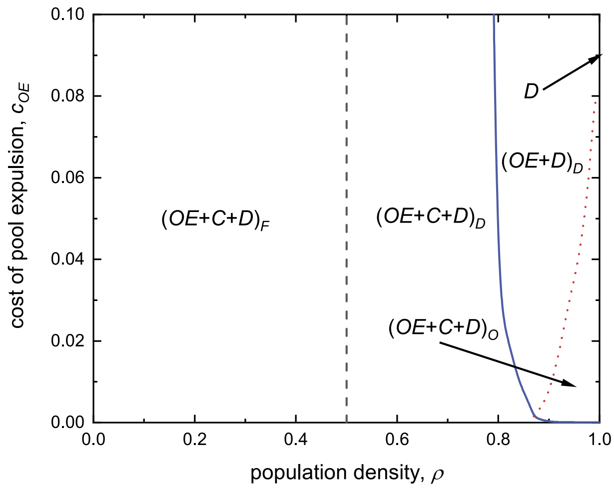

Fig. 1. (Color online.) Full ρ - c_{OE} phase diagram of the spatial public goods game with pool expulsion for the enhancement factor $r = 2.5$, where cooperators cannot survive solely due to spatial reciprocity [30]. Solid blue lines denote continuous phase transitions, while dotted red lines denote discontinuous phase transitions. Note that the dashed black line at $\rho = 0.5$ represents the phase boundary between the dynamical state [i.e., $(OE + C + D)_D$] and the frozen state [i.e., $(OE + C + D)_F$]. Herein, due to the presence of exploration events (i.e., $\mu = 10^{-5}$), we say one strategy (e.g., $X \in D, C, OE$) dies out if and only if $\rho_X = 0$ for at least one time step during the whole evolutionary process as well as its mean faction in the stationary state $\bar{\rho}_X < 1/N$. Here the mean fractions of all three strategies $\bar{\rho}_X$ are evaluated over 10^4 time steps once the spatial system evolves into the stationary state. Two representative cross sections of this phase diagram is presented in Fig. 2.

where ρ_X $(X \in D, C, OE)$ represents the density of players (i.e., ρ_D of defectors, ρ_C of cooperators and $\rho_{OE} = \rho - \rho_D - \rho_C$ of pool expellers), and $\bar{P}_X$ $(X \in D, C, OE)$ denotes the average payoff for players (i.e., $\bar{P}_D$ for defectors, $\bar{P}_C$ for cooperators and $\bar{P}_{OE}$ for pool expellers):

$$\begin{cases} \bar{P}_D = \frac{r}{G}\frac{\rho_{OE}\rho_D+\rho\rho_C}{\rho(\rho-\rho_{OE})}(G-1), & \text{(a)} \\ \bar{P}_C = \frac{r}{G}\left[\frac{\rho_{OE}\rho_D+\rho\rho_C}{\rho(\rho-\rho_{OE})}(G-1)+1\right]-1, & \text{(b)} \\ \bar{P}_{OE} = \frac{r}{G}\left[\frac{\rho_{OE}\rho_D+\rho\rho_C}{\rho(\rho-\rho_{OE})}(G-1)+1\right]-1-c_{OE}. & \text{(c)} \end{cases} \tag{4}$$

According to Eq. (4), ρ_C and ρ_{OE} tend to zero in the limit $\mu \to 0$ for arbitrary value of K as $\bar{P}_D > \bar{P}_C > \bar{P}_{OE}$. In short, both cooperators and pool expellers become extinct in the system with an infinite range of interaction.

In what follows, we aim to study spatial dynamics of the public goods game with pool expulsion including phase separation of the spatial public goods game with pool expulsion as well as illumination of pattern formation mechanisms.

3.1. Phase diagram

Representative phase diagram for the enhancement factor $r = 2.5$, where cooperators can no longer survive solely due to spatial reciprocity (see App. A), is presented in Fig. 1. At such a low value of r, pool expellers are able to sustain or even prevail in the diluted lattice (i.e., $\rho < 1$) even if the cost of pool expulsion $c_{OE} > 0$. If the cost of pool expulsion $0.08 > c_{OE} > 0.0015$ is moderate, the absorbing state D at population density $\rho = 1$ first gives way to the oscillatory phase $(OE + C + D)_O$, subsequently to the dynamical and mixed state $(OE + D)_D$ via a discontinuous phase transition, then to the dynamical and mixed state $(OE + C + D)_D$ by a continuous phase transition and finally to the mixed and frozen state $(OE + C + D)_F$ as the population density ρ decreases from 1 to 0. Moreover, while the sufficiently low cost of pool expulsion $0.0015 > c_{OE} > 0$ leads to the disappear of the $(OE + D)_D$ phase, the adequately high cost of pool expulsion $c_{OE} > 0.08$ results in the absence of the $(OE + C + D)_O$ phase in comparison with the case for the moderate cost of pool expulsion.

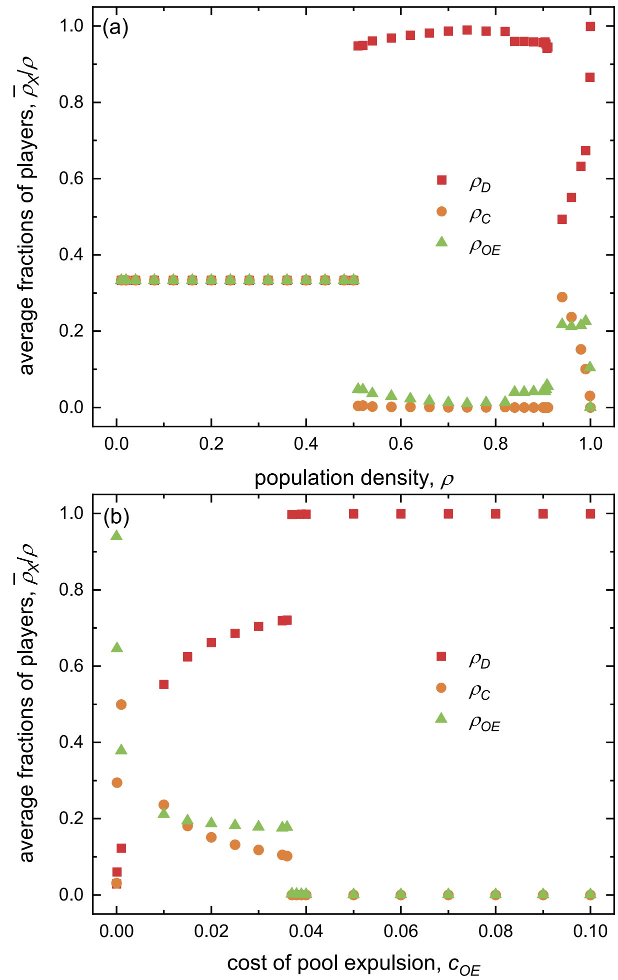

Fig. 2. (Color online.) Cross sections of the phase diagram depicted in Fig. 1, as obtained for the enhancement factor $r = 2.5$. Depicted are stationary fractions of the three competing strategies in dependence on (a) the population density ρ when the cost of pool expulsion $c_{OE} = 0.01$ and (b) the cost of pool expulsion c_{OE} when the population density $\rho = 0.96$. We note that there are two different types of phase transition in Fig. 2(a): the continuous phase transition from $(OE + C + D)_D$ to $(OE + D)_D$ and the discontinuous phase transition from $(OE + C + D)_O$ to $(OE + D)_D$. Here the stationary fractions of all three strategies $\bar{\rho}_X$ $(X \in D, C, OE)$ are averaged over 10^4 time steps after the evolutionary system enters into the steady state.

Fig. 2 shows two characteristic cross sections of the phase diagram presented in Fig. 1. The process depicted in Fig. 2(a) is considerably interesting. When the population density $\rho \in (0, 0.5]$, the average fractions of the three strategies are all equal to 1/3. Further increment of the population density ρ leads to the decrease of the average fractions of strategies C and OE, which results from the increasing contacts among players. Note that the sparse interaction structure in this case disables cooperators and pool expellers to form compact clusters, and thus makes the mechanism of spatial reciprocity absent [52]. Once the second-order freeriders are eliminated from the system, pool expellers can resist against defectors and even spread at the cost of defectors, which in turn introduces the emergence of cooperators. Finally, defectors wipe out cooperators and pool expellers if the population density $\rho = 1$. More interestingly, there also reveals from Fig. 2(a) two qualitatively different manners for the $(OE + C + D)_D$ dynamical coexistence phase as well as the $(OE + C + D)_O$ oscillatory coexistence phase to give way to the $(OE+D)_D$ phase. For the phase transition from $(OE+C+D)_D$ to $(OE+D)_D$, the average fraction of cooperators $\bar{\rho}_C/\rho$ decays gradually to zero. For the phase transition from $(OE+C+D)_O$ to $(OE+D)_D$, however, the termination of the $(OE+C+D)_O$ phase is due to the sudden vanish of cooperators. Fig. 2(b) shows the average fractions of the three strategies as a function of the cost of pool expulsion c_{OE} for the population density $\rho = 0.96$, which reveals a relatively straightforward process. Here, the average fraction of strategy D increases because of the increment of the cost of pool expulsion c_{OE}. For the cost of pool expulsion $c_{OE} \to 0$, pool expellers win over both defectors and cooperators, and thus prevail in the population. If the cost of pool expulsion $c_{OE} > 0.037$, on the other hand, cooperators disappear in a discontinuous manner, and defectors dominate the spatial system.

3.2. Pattern formation

For the spatial public goods game with pool expulsion, it is important to notice that there exists a maximal population density $\rho_T = 0.5$, in the case of which the spatial configurations are able to satisfy the condition that no individuals are located in their respective neighborhood. Below such a critical threshold of the population density $\rho \le \rho_T$, the above evolutionary system with the presence of mutation will eventually converge into separating states wherein all individuals are separated from each other by vacant sites. Once evolving into separating states (e.g., at time step t_{tr}), the whole population is solely governed by random exploration of strategies:

$$\begin{cases} \frac{\partial \rho_D}{\partial (t-t_{tr})} = \frac{\mu}{2}\left(\rho - 3\rho_D\right), & \text{(a)} \\ \frac{\partial \rho_C}{\partial (t-t_{tr})} = \frac{\mu}{2}\left(\rho - 3\rho_C\right), & \text{(b)} \\ \frac{\partial \rho_{OE}}{\partial (t-t_{tr})} = \frac{\mu}{2}\left(\rho - 3\rho_{OE}\right), & \text{(b)} \end{cases} \quad (5)$$

which leads to the following solutions of above ordinary differential equations:

$$\begin{cases} \rho_D\left(t-t_{tr}\right) = \frac{\rho}{3} - \frac{\rho - 3\rho_D(t_{tr})}{3e^{3\mu(t-t_{tr})/2}} & \text{if } t \ge t_{tr}, \quad \text{(a)} \\ \rho_C\left(t-t_{tr}\right) = \frac{\rho}{3} - \frac{\rho - 3\rho_C(t_{tr})}{3e^{3\mu(t-t_{tr})/2}} & \text{if } t \ge t_{tr}, \quad \text{(b)} \\ \rho_{OE}\left(t-t_{tr}\right) = \frac{\rho}{3} - \frac{\rho - 3\rho_{OE}(t_{tr})}{3e^{3\mu(t-t_{tr})/2}} & \text{if } t \ge t_{tr}, \quad \text{(c)} \end{cases} \quad (6)$$

where $\rho_D(t)$, $\rho_C(t)$ and $\rho_{OE}(t)$ denotes the density of defectors, cooperators and pool expellers at time step t, respectively. From above ordinary differential equations, we can obtain the stationary state of the spatial system in the limit $t \to \infty$:

$$\rho_D = \rho_C = \rho_{OE} = \frac{\rho}{3}. \quad (7)$$

Note that the analytical results above are robust against the variation of the enhancement factor r and the cost of pool expulsion c as long as both the exploration rate $\mu > 0$ and the population density $\rho \le 0.5$ are satisfied. Particularly, if the population density $\rho = 0.5$, the structured population as a whole produces the spatial ordering by displaying a stable checkerboard pattern as every site is in exactly opposite states ('occupied' versus 'empty') in comparison with its all neighbors (see Fig. 3). A similar phenomenon is firstly explored in magnetic models where many interesting spin-glass phenomena may arise, and is also well investigated in spatial models of evolutionary games where each player can choose the opposite strategy of all neighbors [53].

Lastly, it still requires to clarify the mechanism of pattern formations for the $(OE+C+D)_O$ oscillatory coexistence phase. Fig. 4(a) shows the fraction of each strategy as a function of time

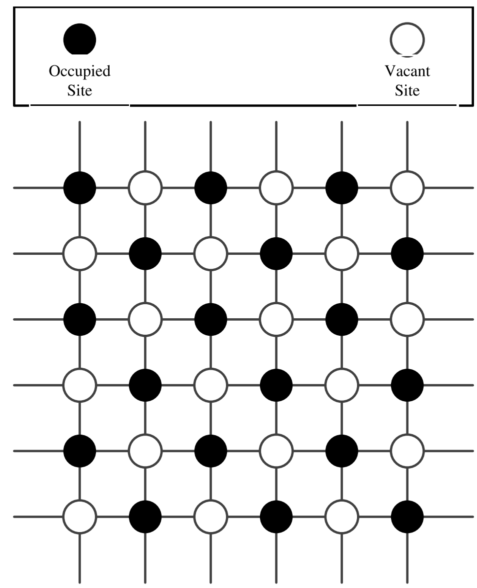

Fig. 3. The configuration of spatial ordering in the square lattice for the population density $\rho = \rho_T = 0.5$. In the schematic square lattice of size 6×6 above, a black filled circle denotes a site that is occupied by a player who is either a defector, a cooperator or a pool expeller, and a white hollow circle represents a site that is just empty. At such a critical population density, the spatial system performs a stable checkerboard pattern with 'occupied' and 'empty' as the two opposite states of sites in the stationary state.

for the population density $\rho = 0.918$ and the cost of pool expulsion $c_{OE} = 0.01$ when the enhancement factor $r = 2.5$. At the start of the evolution, the randomly mixed initial state is particularly beneficial for the exploitation of cooperators and pool expellers by defectors. Accordingly, the fraction of defectors ρ_D/ρ rises rapidly, while the fractions of cooperators ρ_C/ρ and pool expellers ρ_{OE}/ρ fall [see Fig. 4(a)]. Thanks to spatial reciprocity [30] as well as the evolutionary mechanism of pool expulsion, pool expellers are able to form a number of spatial clusters, which are partially isolated from defectors by vacant sites, in a self-organizing manner to resist the invasion of defectors (see Fig. 5). Cooperators, on the other hand, cannot win the direct competition with defectors solely by virtue of spatial reciprocity at such a low enhancement factor, i.e., $r = 2.5$, and thus merely maintain in the vicinity of pool expellers' clusters by not contributing to the expulsion pool (see Fig. 5). After the initial relaxation time, the end of which is indicated by an arrow in Fig. 4(a), the system reaches the equilibrium state in a oscillatory way. Fig. 4(b) shows a 'cut-out' for a full cycle (i.e., from time $t = 950$ to $t = 2050$) of fluctuation from Fig. 4(a). When the second-order freeriders are almost wiped out by defectors from the population, pool expellers surrounding by thin active vacant layers become more effective to compete with defectors, and thus the clusters of pool expellers begin to expand uniformly [see Fig. 4(b) as well as Figs. 6(a) and 6(b)]. However, during the expansion process of pool expellers' clusters, cooperators begin to invade the spatial territories of pool expellers from interior though the fraction of pool expellers is still increasing [see Fig. 4(b) as well as Figs. 6(b) and 6(c)]. Once the clusters of cooperators reach the boundaries of the islands between defectors and pool expellers, traveling waves dominate the spatial system: Defectors can spread toward the spatial domains of cooperators, and cooperators in turn invade the bulk of the spatial domains of pool expellers [see Fig. 4(b) as well as Figs. 6(d) and 6(e)]. Based on this process, defectors may control a significant portion of the spatial system for a long period of time until a new cycle of fluctuation begins [see Figs. 4(a) and 4(b) as well as Fig. 6(f)].

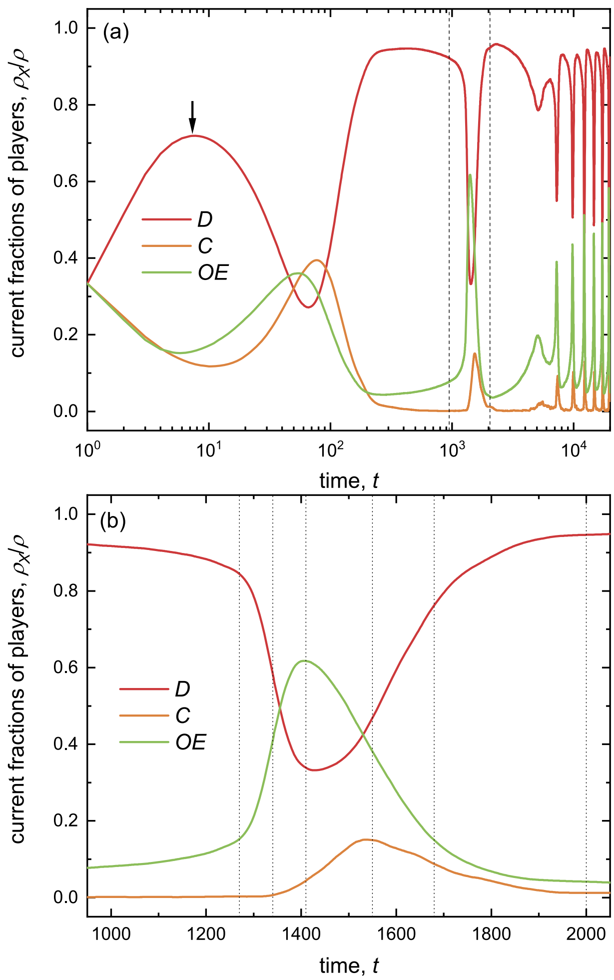

Fig. 4. (Color online.) Time courses depicting the evolutionary process for the oscillatory phase $(OE+C+D)_O$ occurring at the population density $\rho = 0.918$ and the cost of pool expulsion $c_{OE} = 0.01$ when the enhancement factor $r = 2.5$. (a) Evolution of the distribution of strategies starting with a random initial state. The vertical dashed lines mark a full cycle of time from $t = 950$ to $t = 2050$, between which the densities of defectors ρ_D/ρ, cooperators ρ_C/ρ and pool expellers ρ_{OE}/ρ undergo rise and fall once. Note that the horizontal axis is logarithmic. (b) Enlargement of the rectangle area shown by the vertical dashed lines in Fig. 4(a).

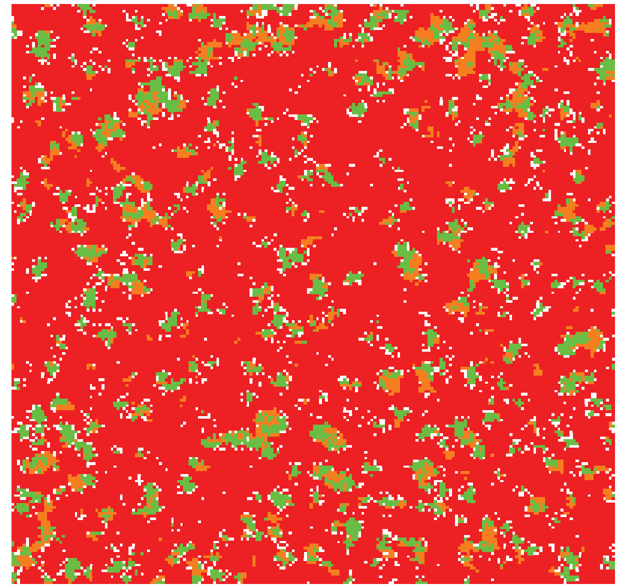

Fig. 5. (Color online.) Spatial distribution of strategies on a 200×200 portion of a larger 800×800 square lattice for the same parameters as those used in Fig. 4. The snapshot was taken at the end of the initial relaxation time, which is indicated by an arrow in Fig. 4(a). Defectors, cooperators, pool expellers and vacant sites are depicted in red, orange, green and white, respectively. In this snapshot, pool expellers survive in the square lattice by expelling their defective neighbors as well as forming spatial clusters of small sizes while cooperators merely maintain in the vicinity of pool expellers' clusters by not contributing to the expulsion pool.

4. Discussion

The impact of pool expulsion has been studied in the spatial public goods game with defection, cooperation, and pool expulsion as the three competing strategies. We have found that pool expellers are able to survive or even prevail across the whole $\rho - c_{OE}$ parameter region even if the synergistic effects of cooperation are low to the point that spatial reciprocity alone fails to sustain it, and if thus the second-order freeriders are present in the population, as well as if the expulsion behavior is costly. Detailed analysis of the pattern formation process reveals that the evolutionary advantage of pool expulsion over defection, when the population density ρ is considerably large, comes from: (1) formation of compact clusters by spatial reciprocity; (2) formation of active vacant layers between defectors and pool expellers by pool expulsion. It turns out that pool expellers in this case can even dominate the population only within a strongly limited region of parameters, i.e., the cost of pool expulsion $c \to 0$. Furthermore, if the cost of pool expulsion c becomes slightly larger, the spatial system evolves into a self-organizing spatial temporal pattern where all three strategies coexist in a oscillatory manner. When the population density ρ is small, however, pool expellers are able to survive in the sparse lattice solely by virtue of pool expulsion. Particularly, if the population density $\rho \leq 0.5$, the whole population is governed by exploration dynamics [47], which leads to the equal abundances of all three strategies. Interestingly, when the population density $\rho = 0.5$, the spatial system produces checkerboard patterns with 'occupied' and 'empty' as the two opposite states. Although the contributors, i.e., cooperators and pool expellers, account for a large majority of the population in this case, it does not mean that the individual interest or the social welfare can benefit from such a cooperative outcome. In contrast, due to the isolation of individuals from each other in the stationary state, nobody is able to play the public goods game in the square lattice. Therefore, both the individual payoff and the overall wealth of the entire society are equal to zero, which is a situation similar to the "tragedy of the commons" where defectors prevail over the whole population. In order to understand such a nontrivial phenomenon, it is instructive to characterize the evolutionary process in two stages, wherein the evolutionary system is driven by the coupling interactions between the imitation and the exploration dynamics in the initial stage, and is solely governed by the exploration dynamics in the subsequent stage. In the first stage, defectors invade into the spatial territories of both cooperators and pool expellers due to the diluted effects of the population. The evolutionary system with the presence of mutation reaches the end of the first stage for this process once it evolves into the separating state where individuals are isolated from each other. In the second stage, the exploration dynamics finally result in the equal fractions of all three strategies.

In contrast with the efficiency of peer expulsion in maintaining socially advantageous states [42], we have found that pool expulsion is less effective because cooperators can always invade the spatial territories of pool expellers as long as the cost of pool expulsion is positive. This also partially leads to the spontaneous emergence of self-organizing spatiotemporal patterns governed by oscillatory dynamics among defection, cooperation, and pool expulsion. On the contrary, cooperators and peer expellers are able to separate from each other and fight independently against de-

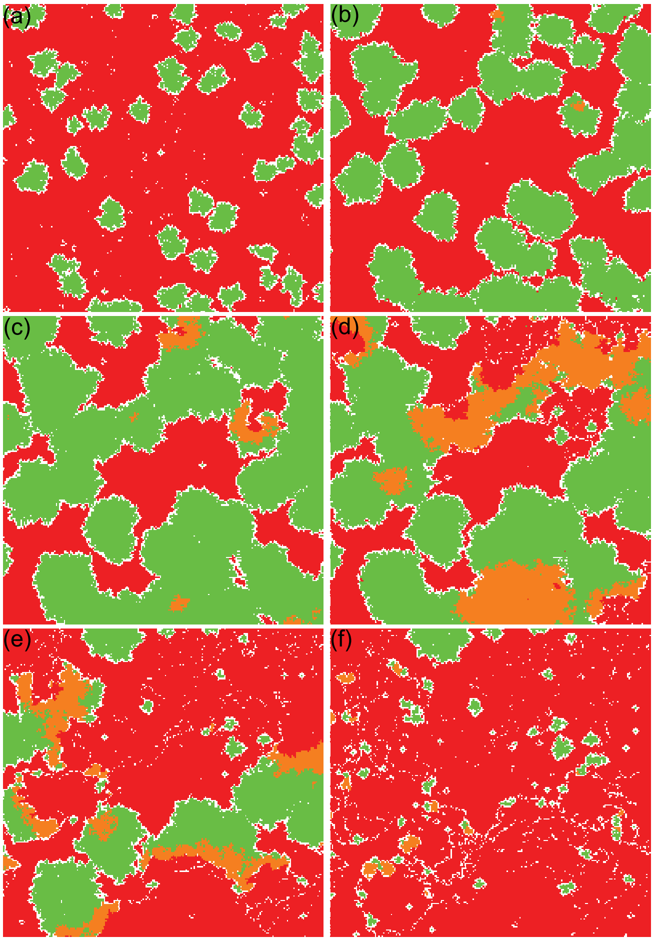

Fig. 6. (Color online.) Spatiotemporal evolution of the strategies at the equilibrium state for the same parameters as those used in Fig. 4. The snapshots were taken at time steps: (a) $t = 1270$; (b) $t = 1340$; (c) $t = 1410$; (d) $t = 1550$; (e) $t = 1680$ and (f) $t = 2000$ [see the dotted lines in Fig. 4(b) marking when the snapshots of the spatial patterns were recorded in Fig. 6]. Here shows a 200×200 portion of a larger 800×800 square lattice. The color code is the same as that used in Fig. 5. From time step 1270 to 1410, the clusters of pool expellers surrounded by thin active layers of vacant sites expand towards the spatial territories of defectors, and gradually occupy the larger parts of the spatial lattice in comparison with defectors do. From time step 1340 to 1550, the cooperators' clusters begin to emerge and expand inside the pool expellers' clusters until reach the boundaries of spatial clusters between defectors and pool expellers. From time step 1550 to 2000, the spatial clusters of defectors begin to retake their lost territories by invading the spatial territories of cooperators while the cooperators' clusters continue to intrude the spatial territories of pool expellers until the nearly extinction of cooperators.

fectors. Since peer expellers do it more successfully, they can outcompete cooperators via both direct and indirect domain competition. As a result, the oscillatory state between defection, cooperation, and peer expulsion cannot be observed [42]. Punishment represents a form of reactive strategy that entails paying a cost for somebody else to incur a cost [7,14]. In pool punishment, punishers are willing to contribute into a common pool in advance, from which resources are then taken to sanction defectors [54–56]. If we classify this kind of costly punishment as an active one, we can consider pool expulsion in our model as an inactive form of punishment: Pool expellers tend to contribute in advance to a common pool, which is used to 'punish' defectors in a way that pool expellers terminate future interactions with defectors. On the other hand, pool expellers in our model are also similar to pool excluders, who share costs to spontaneously form an institution so as to carry out the rejection of the freeriders and preclude them from enjoying the benefits [57,58]. Note that the institutions constructed by pool excluders in their model require the cognitive ability that

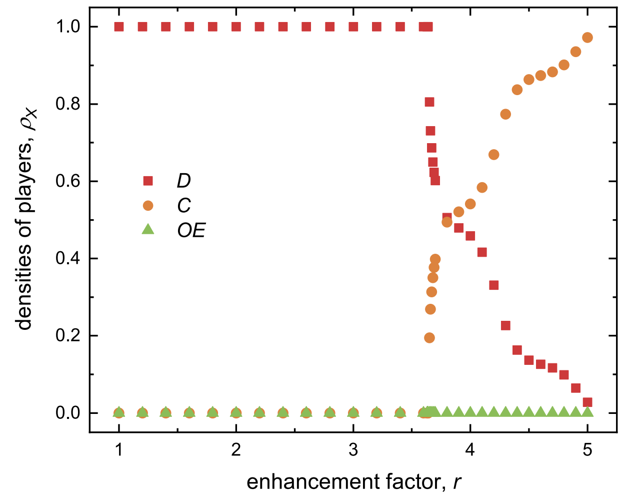

Fig. 7. (Color online.) Densities of players ρ_X $(X \in D, C, OE)$ as a function of the enhancement factor r for the population density $\rho = 1.0$ and the cost of pool expulsion $c_{OE} > 0$. The transition point at $r_{th} = 3.637\,(9)$ separates the parameter region of the enhancement factor $r \in (1, 5)$ into two distinct phases: the absorbing phase D and the dynamical coexistence phase $(C + D)_D$. Simulation results were obtained for $L \in [500, 1000]$ and $t \in \left[10^5, 10^6\right]$ time steps, depending on the proximity to the transition point $r = r_{th}$.

can be used to identify freeriders before the public goods are allocated to group members. In this case, pool excluders can emerge or stabilize themselves even in well-mixed populations if only the net gain of pool excluders from the public goods game is larger than the shared cost of pool expulsion. The institutions built by pool expellers, however, merely have the cognitive ability to recognize freeriders after the public goods game, which thus leads to their extinction in the mean-field limit.

We hope that this research will prove inspirational for further explorations of the fascinating links between physics and society [59], in particular how methods of physics, and the general approach that is characteristic for physics, can lead to a better understanding of human societies.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. The no vacant sites case

Computational results by Monte Carlo simulation presented in Fig. 7 show the stationary fractions ρ_D, ρ_C and ρ_{OE} for various values of the enhancement factor r when there are no vacant sites on the square lattice, i.e., when the population density $\rho = 1.0$. On the saturated square lattice (i.e., population density $\rho = 1.0$), there are no vacant sites available for pool expellers to banish defectors in their public pools. Pool expellers, like cooperators, are able to persist in the structured population by virtue of spatial reciprocity. However, as the cost of pool expulsion $c_{OE} > 0$, the evolutionary performance of pool expellers is inferior to that of cooperators. Therefore, pool expellers vanish in the structured population across the whole applicable range of the enhancement factor r (see Fig. 7). On the other hand, cooperators are able to survive by forming compact clusters if the enhancement factor r is larger than a threshold $r_{th} = 3.637\,(9)$ [60]. Below such a threshold, cooperators become extinct (i.e., $\rho_C = 0$) (see Fig. 7), wherein the transition from the mixed to the homogeneous state pertains to the directed percolation university class for spatial systems of two dimensions since the density of cooperators $\rho_C \propto (r - r_{tr})^{\beta}$ by $\beta = 0.55\,(1)$ [61–63].

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