Evolution of emotions on networks leads to the evolution of cooperation in social dilemmas

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We show that the resolution of social dilemmas in random graphs and scale-free networks is facilitated by imitating not the strategy of better-performing players but, rather, their emotions. We assume sympathy and envy to be the two emotions that determine the strategy of each player in any given interaction, and we define them as the probabilities of cooperating with players having a lower and a higher payoff, respectively. Starting with a population where all possible combinations of the two emotions are available, the evolutionary process leads to a spontaneous fixation to a single emotional profile that is eventually adopted by all players. However, this emotional profile depends not only on the payoffs but also on the heterogeneity of the interaction network. Homogeneous networks, such as lattices and regular random graphs, lead to fixations that are characterized by high sympathy and high envy, while heterogeneous networks lead to low or modest sympathy but also low envy. Our results thus suggest that public emotions and the propensity to cooperate at large depend, and are in fact determined by, the properties of the interaction network.

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I. INTRODUCTION

Evolutionary games [1] have recently received ample attention in the physics community, as it became obvious that methods of statistical physics can be used successfully also to study interactions that are more complex than just those between particles [2]. Broadly classified as statistical physics of social dynamics [3], these studies aim to elevate our understanding of collective phenomena in society on a level that is akin to the understanding we have about interacting particle systems. Within the theoretical framework of evolutionary games, the evolution of cooperation [4] is probably the most interesting collective phenomenon to study. Several evolutionary games constitute so-called social dilemmas [5], the most prominent of which is the prisoner's dilemma game, in which understanding the evolution of cooperation is still a grand challenge. Regardless of game particularities, a social dilemma implies that the collective well-being is at odds with individual success. An individual is therefore tempted to act so as to maximize his or her own profit but, at the same time, neglecting negative consequences this has for the society as a whole. A frequently quoted consequence of such selfish actions is the "tragedy of the commons" [6]. While cooperation is regarded as the strategy leading away from the threatening social decline, it is puzzling why individuals would choose to sacrifice some fraction of personal benefits for the well-being of society.

According to Nowak [7], five rules promote the evolution of cooperation. These are kin selection, direct and indirect reciprocity, network reciprocity, and group selection. Recent reviews [8–11] clearly attest to the fact that physics-inspired research has helped refine many of these concepts. In particular, evolutionary games in networks, spurred on by the seminal discovery of spatial reciprocity [12] and, subsequently, by the discovery that scale-free networks strongly facilitate the evolution of cooperation [13,14], are still receiving ample attention to this day [15–34]. One of the most recent contributions to the subject concerns the assignment of cognitive skills to individuals that engage in evolutionary games in networks [35,38–41]. The earliest forerunners to these advances can be considered strategies such as "tit for tat" [42] and Pavlov [43], many of which were already proposed during the seminal experiments performed by Axelrod [44] and which assume that individuals have cognitive skills that exceed those granted to them in the framework of classical game theory. It has recently been shown, for example, that incipient cognition solves several open questions related to network reciprocity and that cognitive strategies are particularly fit to take advantage of the ability of heterogeneous networks to promote the evolution of cooperation [39].

Here we build on our previous work [35], where we have presented the idea that not strategies but, rather, emotions could be the subject of imitation during the evolutionary process. It is worth noting that the transmissive nature of positive and negative emotional states was already observed in [36], where it was concluded that humans really do adjust their emotions depending on their contacts in a social network. Moreover, the connection between intuition and willingness to cooperate was also tested in human experiments [37]. It therefore is of interest to determine how the topology of the interaction network affects the spreading of emotions, which may in turn determine the level of cooperation. In the context of games on lattices, we have shown that imitating emotions such as goodwill and envy from the more successful players reinstalls imitation as a tour de force for resolving social dilemmas, even for games where the Nash equilibrium is a mixed phase. We have also argued that envy is an important inhibitor of cooperative behavior. We now revisit the snowdrift, stag-hunt, and prisoner's dilemma games in random graphs and scale-free networks, with the aim of determining the role of interaction heterogeneity within this framework. We focus

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on sympathy and envy as the two key emotions determining the emotional profile of each player, and we define them simply as the probability of cooperating with less and more successful opponents, respectively. Strategies thus become link specific rather than player specific, whereby the level of cooperation in the population can be determined by the average number of times players choose to cooperate. Interestingly, in agreement with a recent experiment, we find that network reciprocity plays a negligible role [45]. The outcome in regular random graphs is the same as reported previously for the square lattice, leading to the conclusion that the ability of cooperators to aggregate into spatially compact clusters is irrelevant. Only when degree heterogeneity is introduced to interaction networks do we find that the evolution of emotional profiles changes. As we show, homogeneous networks lead to fixations that are characterized by high sympathy and high envy, while heterogeneous networks lead to low or modest sympathy and low envy. Network heterogeneity thus alleviates a key impediment to higher levels of cooperation in lattices and regular networks, namely, envy, and by doing so opens the possibility of much more cooperative states even under extremely adverse conditions. From a different point of view, it can be argued that some topological features of interaction networks in fact determine the emotional profiles of players, and they do so in such a way that cooperation is the most frequently chosen strategy.

The remainder of this paper is organized as follows. First, we describe the mathematical model, in particular, the protocol for the imitation of emotional profiles as well as the definition of social dilemmas in networks. Next we present the main results, and finally, we summarize and discuss their implications.

II. MATHEMATICAL MODEL

The traditional setup of an evolutionary game assumes N players occupying vertices of an interaction network. Moreover, each player x, having a pure strategy, cooperates ($s_x = C$) or defects ($s_x = D$) with all neighbors independently of their strategy and payoff. Here, instead of pure strategies, we introduce an emotional profile $(\alpha_x,\beta_x) \in [0.1]$ to each player, which characterizes the willingness to cooperate with a neighbor dependent on the other player's success, which is quantified by the payoff value. More precisely, if the corresponding payoff values are p_x and p_y for players x and y, respectively, then α_x determines the probability that player x will cooperate with player y in the case of $p_x > p_y$. Conversely, when $p_x < p_y$, the parameter β_x is the probability that player x will cooperate with player y. In the rare case of equality ($p_x = p_y$), the corresponding probability is the average of α_x and β_x.

In this way the (α_x,β_x) pair thus determines how a given player x will behave when facing a less or a more successful opponent y. As described in Sec. I, the pair determines each player's sympathy and envy. If $\alpha_x = 1$, we say that the player is completely sympathetic. Alternative interpretations such as goodwill and charity are also viable, given that player x will always cooperate with less successful opponents. Similarly, if $\beta = 1$, we say that the player is not envious. Despite the fact that the opponents are more successful, the player will always

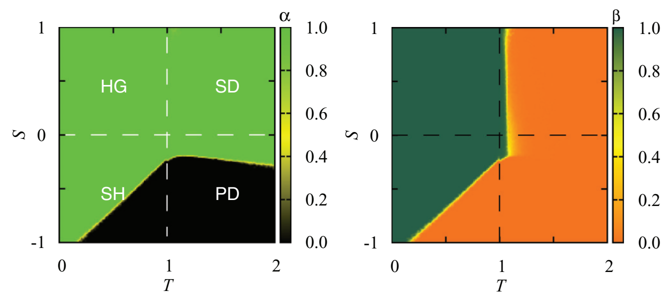

FIG. 1. (Color online) Map depicting the final values of α (left) and β (right) on the T-S parameter plane, as obtained in a regular random graph. The results are strikingly similar to those obtained on a square lattice (see [35]).

cooperate with them. As by α_x, alternatives such as servility and proneness to brownnose or "butter up" appear to fit as well. It is important to note that a player x may simultaneously cooperate with and defect towards neighbors y and y' if their payoffs are very different. Furthermore, player x may adopt different strategies even if $p_y \approx p_{y'}$ due to the probabilistic nature of an emotional profile.

When two players engage in a round of an evolutionary game, we assume that mutual cooperation yields the reward R, mutual defection leads to punishment P, and the mixed choice gives the cooperator the sucker's payoff S and the defector the temptation T. Within this traditional setup we have the prisoner's dilemma (PD) game if $T > R > P > S$, the snowdrift game (SG) if $T > R > S > P$, and the stag-hunt (SH) game if $R > T > P > S$, thus covering all three major social dilemma types where players can choose between cooperation and defection. Following common practice [8], we set $R = 1$ and $P = 0$, thus leaving the remaining two payoffs to occupy $-1 \leqslant S \leqslant 1$ and $0 \leqslant T \leqslant 2$, as depicted schematically in Fig. 1(a).

To begin, each player x is assigned a random (α_x,β_x) pair and a payoff from the reachable $[kS,kT]$ interval, where k denotes the average degree of players. Subsequently, every payoff value is updated by considering the proper neighborhoods of a player and the actual emotional parameters. Importantly, after the accumulation of new payoffs, player y cannot imitate a pure strategy from player x, but only its emotional profile, i.e., the α_x and/or β_x value. Imitation is decided so that a randomly selected player x first acquires its payoff p_x by playing the game with all its k_x neighbors, as defined by the interaction network. Note that k_x is thus the degree of player x. Next, one randomly chosen partner of x, denoted y, also acquires its payoff p_y by playing the game with all its k_y neighbors. Player y then attempts to imitate the emotional profile of players x with the probability $q = 1/\{1 + \exp[(p_y - p_x)/K]\}$, where K determines the level of uncertainty by strategy adoptions [8]. The latter can be attributed to errors in judgment due to mistakes and external influences that affect the evaluation of the opponent. Without loss of generality we set $K = 0.5$, implying that better-performing players are readily imitated, but it is not impossible to adopt the strategy of a player performing worse. Importantly, since the emotional profile consists of two parameters, two random numbers are drawn to enable independent imitation of α_x and β_x. This is vital to avoid potential artificial propagations of freak (extremely successful) (α_x,β_x) pairs. Technically, 100×100 (α_x,β_x) pairs were available at the start of the evolutionary process. Finally, after each imitation the payoff of player y is updated using its new emotional profile, whereby each full Monte Carlo step involves all players having a chance to adopt the emotional profile from one of their neighbors once on average.

Prior to presenting the result of this model, it is important to note that there will almost always be a fixation of (α_x,β_x) pairs; i.e., irrespective of T and S, only a single pair will eventually spread across the whole population. Once fixation occurs the evolutionary process stops. The characteristic probability of encountering cooperative behavior in the population, which is equivalent to the stationary fraction of cooperators f_C in the traditional version of the game, can then be determined by means of averaging over the final states that emerge from different initial conditions. Exceptions to single (α,β) pair fixations are likely to occur for strongly heterogeneous networks, where more than one (α_x,β_x) pair can survive around strong hubs. This effect is more pronounced in the harmony game (HG) quadrant but becomes negligible in the prisoner's dilemma parametrization of the game. In case more than a single (α_x,β_x) pair does survive, we present in what follows the average over several independent realizations. For Monte Carlo simulations, we have used $N = 5000$–$40\,000$ players and up to 10^7 full steps, and we have averaged over 100–500 independent runs.

III. RESULTS

We start by presenting results obtained in a regular random graph [46] with $k = 4$, as it is a natural extension of a simple square lattice population which we have considered before, in [35]. Importantly, while the degree distribution remains uniform, other topological features, like the presence of shortcuts and the emergence of a nonzero clustering coefficient, change significantly. Previous work on games using pure strategies emphasized that these details may play a significant role via the evolution of cooperation in social dilemmas [47–49]. Figure 1 is a color map encoding the fixation values of α (left) and β (right) on the T-S parameter plane. From the presented results it follows that if the governing social dilemma is of the snowdrift type, players will always (never) cooperate with their neighbors provided their payoff is lower (higher). In the prisoner's dilemma quadrant, we can observe either complete dominance of defection, regardless of the status of the opponents, or the same situation as in the snowdrift quadrant, provided S is not too negative. For the stag-hunt and the harmony games the outcome is practically identical to that obtained by means of the traditional version of the two games. In general, however, both color profiles differ only insignificantly from those ones we reported in [35] (see Figs. 2 and 3 there) for a square lattice. This leads to the conclusion that the structure of interactions does not play a prominent role as long as the degree of all players is uniform.

This leads us to suspect that the heterogeneity of interactions might play a pivotal role. We therefore depart from the regular random graph and move to random graphs with different degree distributions, as depicted in Fig. 2. We consider four types of random graphs with Gaussian-distributed degrees, yet

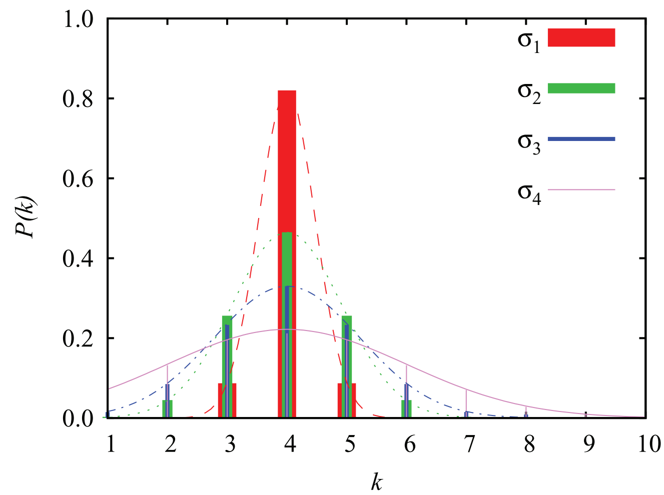

FIG. 2. (Color online) Degree distributions of random graphs with a gradually increasing variance of degree ($\sigma_1 < \sigma_2 < \sigma 3 < \sigma_4$). For easier reference the envelopes of discrete degree distributions are depicted as well.

with increasing variance. According to the legend to Fig. 2, the random graph with σ_1 is thus the least heterogeneous (only degrees $k = 3$, 4, and 5 are possible), while the random graph with σ_4 is the most heterogeneous. Gradually increasing the variance from σ_1 to σ_4 thus enables us to monitor directly the consequences of heterogeneity stemming from the interaction network.

Color maps encoding the fixation values of α and β for the four random graphs are depicted in Fig. 3. By following the plots from left to right, it can be observed that as the heterogeneity of the interaction network increases, the fixation of the profiles of both α and β change. Focusing on the snowdrift and the prisoner's dilemma quadrants, there is a gradual shift from high-α, low-β emotional profiles to low α and high β values as the heterogeneity increases. Accordingly, taking into account also results presented in Fig. 1, we conclude that homogeneous interaction networks promote emotions like sympathy and envy ($\alpha \to 1$ and $\beta \to 0$), while heterogeneous interaction networks prefer indifference and servility ($\alpha \to 0$ and $\beta \to 1$). It is worth emphasizing that these emotional profiles emerge completely spontaneously based on payoff-driven imitation. The change is thus brought about exclusively by the heterogeneity of the interaction network.

It is possible to take a step farther in terms of the heterogeneity of the interaction network by considering scale-free networks. We therefore make use of the standard model proposed by Barabási and Albert [50]. Results presented in Fig. 4 further support our arguments, as the region of low and moderate α values extends farther into the snowdrift quadrant, while at the same time low β values vanish more and more from both the snowdrift and the prisoner's dilemma quadrant. As before, the harmony game and the stag-hunt quadrants remain relatively unaffected, which corroborates the fact that the proposed shift from the imitation of strategies to the imitation of emotional profiles affect predominantly the social dilemma games. It is also worth remembering that on scale-free networks the fixation may not be unique because different hubs

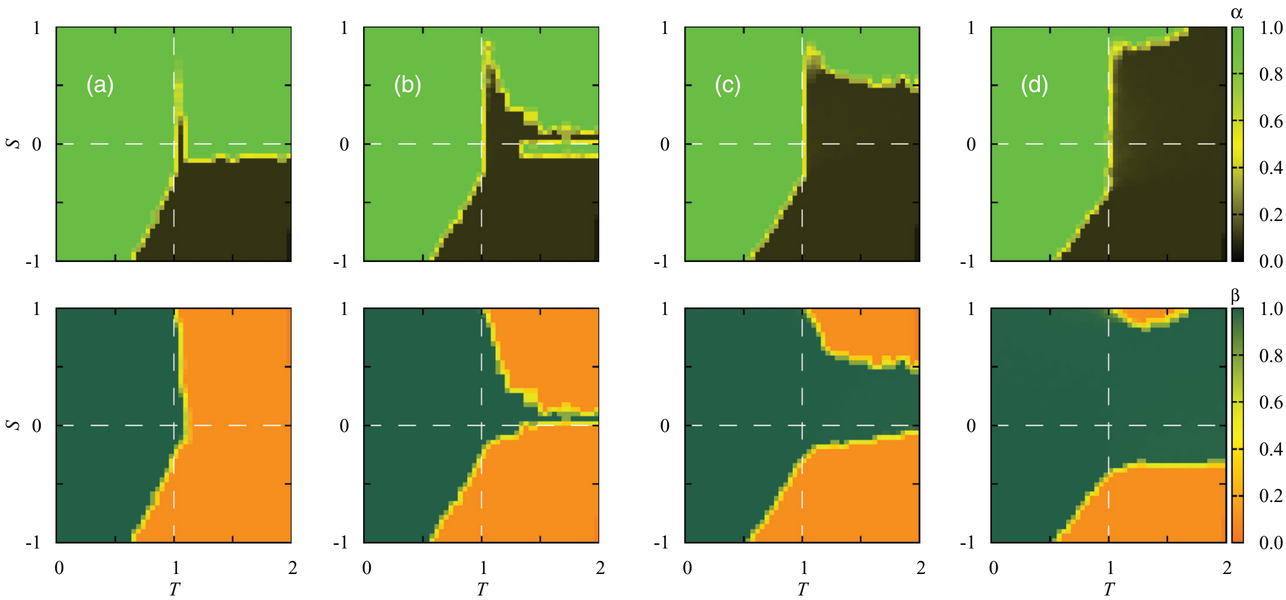

FIG. 3. (Color online) Maps depicting the final values of α (top) and β (bottom) on the T-S parameter plane, as obtained in random graphs with a degree distribution as depicted in Fig. 2. From left to right the variance increases from σ_1 to σ_2 to σ_3 to σ_4 and, hence, increases also the network heterogeneity. It can be observed that the higher the heterogeneity, the more the high-α, low-β emotional profiles give way to profiles that are characterized by low α and high β values. The transition is particularly pronounced in the snowdrift (SD) and the prisoner's dilemma (PD) quadrant, while for the stag-hunt (SH) and the harmony game (HG) the outcome remains little affected.

can sustain their own microenvironment independently from the other hubs. We therefore depict an average over several independent realizations to arrive at representative results.

In order to obtain an understanding of the preference for low α and high β values, as exerted by heterogeneous interaction networks, it is of interest to examine the time evolution of α and β values, as depicted in Fig. 5. The figure shows the probability of any given (α,β) pair in the population at different times increasing from top left to bottom right. It can be observed that high-α/high-β combinations die out first. These players cooperate with both their more and their less successful opponents, and they do so with a high probability. In agreement with well-known results concerning the evolution of cooperation in spatial social dilemmas [8], the bulk of cooperators is always the first to die out. Only after their arrangement into suitable compact domains can the cooperators take advantage of network reciprocity and prevail against defectors. In our case, however, this does not happen; i.e., the "always cooperate" players never recover. Instead, the evolution proceeds by eliminating also all pairs which contain moderate and high α values until, finally, the only surviving low-α profiles are left to compete. However,

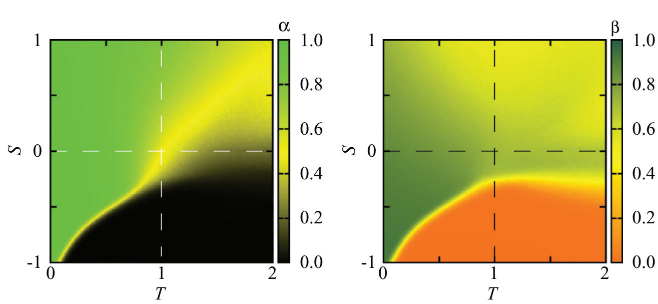

FIG. 4. (Color online) Map depicting the final values of α (left) and β (right) on the T-S parameter plane, as obtained in a scale-free network. The dominance of low and moderate α values and high β values is even more pronounced than for the random graph with degree distribution σ_4 (compare with the two rightmost panels in Fig. 3). This further strengthens the conclusion that, unlike homogeneous networks, strong heterogeneity strongly favors the fixation of emotional profiles that are characterized by low α and high β values.

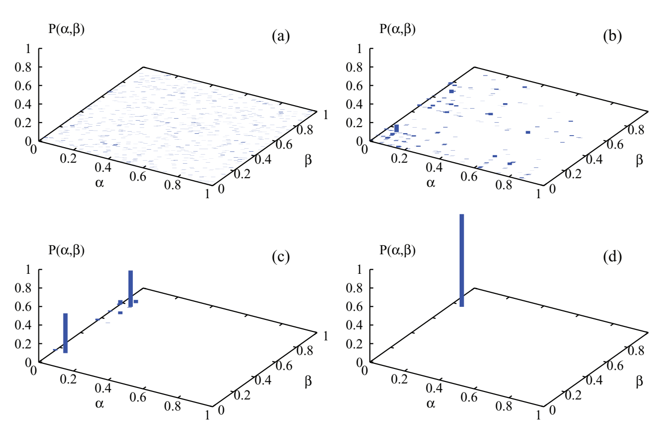

FIG. 5. (Color online) Time evolution of fixation of α and β, as obtained for $T = 1.5$ and $S = -0.1$ in the scale-free network. From top left to bottom right we have the temporary distribution of (α_x,β_x) pairs at 1, 100, 1000, and 100 000 full Monte Carlo steps using $N = 5000$ players. It can be observed that high values of α are the first to vanish. Then gradually the remaining low β values also give way to the complete dominance of low-α and high-β emotional profiles.

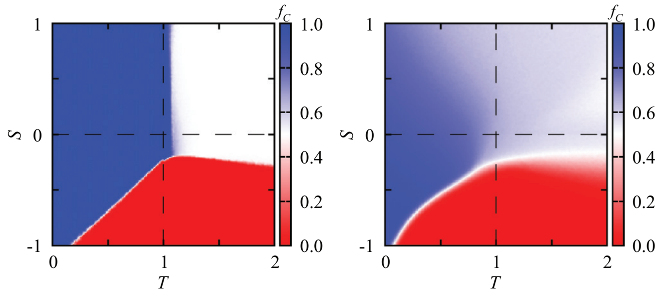

FIG. 6. (Color online) Map depicting the final probability of cooperative behavior f_C on the T-S parameter plane, as obtained in the regular random graph (left) and the scale-free network (right). Since the probability of cooperating should be seen to be equal to the stationary fraction of cooperators in the traditional version of the game, a comparison of presented results (compare with Fig. 1 in [35]) reveals that replacing the imitation of strategies with the imitation of emotional profiles strongly promotes the evolution of cooperation—even more so if the interaction network is strongly heterogeneous.

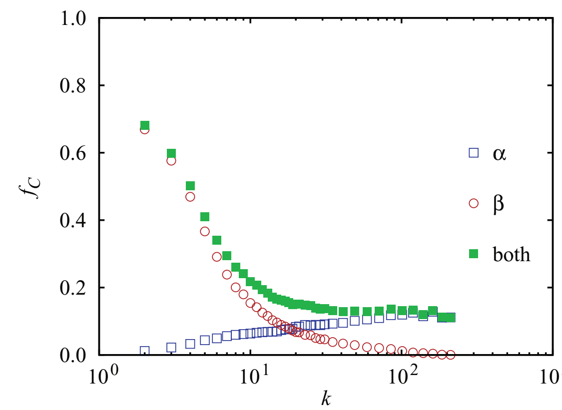

FIG. 7. (Color online) Average willingness to cooperate dependent on degree, as obtained for $T = 1.5$ and $S = -0.1$ in the scale-free network. Fully in agreement with the dominant low-α and high-β emotional profile, it can be observed that hubs very rarely cooperate, while masses almost always do so. The depicted result is an average over 500 independent runs at $N = 5000$ after 10^7 Monte Carlo steps. The average cooperation level is ≈0.6.

preserving at least some form of cooperation may yield an evolutionary advantage, and thus ultimately the low-α/high-β emotional profile emerges as the only one remaining. Notably, the described scenario is characteristic only for heterogeneous networks. For homogeneous networks the differences between players are more subtle, and indeed it is not at all obvious that cooperating with more successful neighbors would confer an evolutionary advantage. Accordingly, high-β profiles are not viable and die out. Cooperation can thrive only atn the expense of high α values, as reported in [35].

Importantly, though, given an appropriately heterogeneous interaction network, the low-α/high-β emotional profile can be very beneficial for the global cooperation level. To support this statement, we present in Fig. 6 the average frequency of cooperation as obtained in the regular random graph (left) and the scale-free network (right). Note that the former, in general, represents homogeneous graphs. The comparison reveals that a much higher cooperation level can be sustained, especially in the snowdrift quadrant, if the dominating emotion is neither sympathy nor envy. To confirm this further, we have manually imposed a high-α/low-β emotional profile in the scale-free network. While this profile is optimal for homogeneous networks (compare also Fig. 6 (left) with Fig. 4 in [35]), the outcome in heterogeneous networks is disappointing, yielding no more than the modest cooperation level of $f_C \approx 0.30$–0.35 in the most challenging snowdrift and prisoner's dilemma regions. This imposes another interesting conclusion, namely, if the emotional profiles of players can evolve freely as dictated by payoff-driven imitation microscopic dynamics, then the topology "selects" the optimal profile in order to produce the highest attainable cooperation level.

Finally, it is instructive to explore how the low-α/high-β emotional profile actually works in scale-free networks. A visualization is possible by measuring separately the average willingness to cooperate for players who have different degrees. Since the payoff of every player is obtained from the pairwise interaction constituted by each individual link, a higher degree k therefore, in general, leads to a higher payoff and also a higher "social prestige." As Fig. 7 illustrates, players with a low degree will predominantly cooperate with their opponents that have a higher degree and thus most likely a higher payoff. In other words, they can use the "β part" of their emotional profile. This act of cooperation, however, is unilateral because the hubs rarely compensate it. Due to the low values of α, cooperation with less successful players is strongly suppressed. What is more, while players with a higher degree also cooperate with more successful opponents (they have the same emotional profile and hence the same high β), this action is very rare given that there are simply not many who would be superior. It is sad but still true that the hubs with the highest degree very rarely cooperate in the stationary state. Despite this rather unfriendly behavior of the "leaders," the average cooperation level is still acceptable and, in fact, remarkably high even under adverse conditions (e.g., $T = 1.5$ and $S = -0.1$), but this is exclusively because the inferior players do their best and virtually always cooperate with their superiors.

IV. DISCUSSION

We have shown that high levels of cooperation can evolve among self-interested individuals if, instead of strategies, they adopt simple emotional profiles from their neighbors. Since the imitation was governed solely by the payoffs of players, we have made no additional assumptions concerning the microscopic dynamics. The latter has been governed by the traditional "follow the more successful" rule, which we have implemented with some leeway due to the Fermi function. Starting from an initial configuration with all possible emotional profiles, we have determined the one that remains after sufficiently long relaxation (only in the harmony game quadrant, if staged in heterogeneous networks, may the fixation be nonunique). We have found that the fixation depends not only on the parametrization of the game but, even more so, on the topology of the interaction network. More precisely, the topology-induced heterogeneity of players has been identified as the most important property. When players were staged in a network where their degree was equal, then, independently of other topological properties of the network, the fixation occurred on emotional profiles characterized by high α and low β values in the interesting payoff region. In agreement with the definition of α and β, these are players characterized by high sympathy but also high envy. This profile is also in agreement with the one reported in [35] for the square lattice. In heterogeneous networks, however, the fixation is most likely to be on low or moderate values of α and high values of β. Accordingly, we have a prevalence of low sympathy (charity, goodwill) for those who are doing worse, but also little envy (high servility, proneness to brownnose or "suck up") of those who are doing better. Noteworthily, although we have not presented actual results, the application of payoffs normalized by the degree of players gives the same results as observed in homogeneous networks. This observation is in agreement with our preliminary expectation because it is well established that the scale-free topology introduces a strong heterogeneity among players, but also that this effect is effectively diminished by applying normalized payoffs or degree-sensitive cost [51–55]. Accordingly, in the latter case players become "equal," which results in the selection of the emotional profile we have recorded for regular graphs. This observation further strengthens our argument that indeed solely the heterogeneity of players is crucial for the selection of the dominant emotional profile.

We thus may argue that in heterogeneous networks each "dictators' dream" profile can evolve via a simple evolutionary rule. The majority may not be happy about it because the combination of moderate α and high β values is not necessarily the most coveted personality profile. Yet as our study shows, it does have its social advantages. Namely, in the absence of envy or in the presence of servility the cooperation level in the whole population can be maintained relatively very high, even if the conditions for the evolution of cooperation are extremely adverse (high T, low S). In this sense, we conclude that charity and envy are easily outperformed by competitiveness and proneness to please the dominant players and that, indeed, this profile emerges completely spontaneously. Put differently, it can be argued that it is in fact chosen by the heterogeneity among players that is introduced by an appropriate interaction network.

We would also like to emphasize that the discussed "emotional profiles" do not necessarily cover the broader psychological interpretation of the term [56]. We have used this terminology to express the liberty of each individual to act differently towards different partners dependent on the differences in social rank (or success), which traditional strategies in the context of evolutionary games do not allow. As such, and in the absence of considering further details determining our personality, our very simple model naturally cannot be held accountable for describing actual human behavior. Instead, it reveals the topology of interactions as a crucial property that determines the collective behavior of a social network. According to our observations, it is indeed the heterogeneity of the interaction network that is key in determining our willingness to help others.

Finally, we emphasize that in the present model cooperation is maintained without reciprocity. The mechanism at work here is very different from those discussed thoroughly in previous studies. Unlike direct and indirect reciprocity, network reciprocity, or even reputation, punishment, and reward, which are all deeply rooted in the fact that neighboring cooperators will help each other out while, at the same time, neighboring defectors will craft their own demise, here the nature of links determines the winner. It may well happen that cooperation and defection occur along the same link, yet the status of the population as a whole is still very robust. What players really share is the way to behave towards each other in different circumstances, which is determined within the framework of an emotional profile.

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